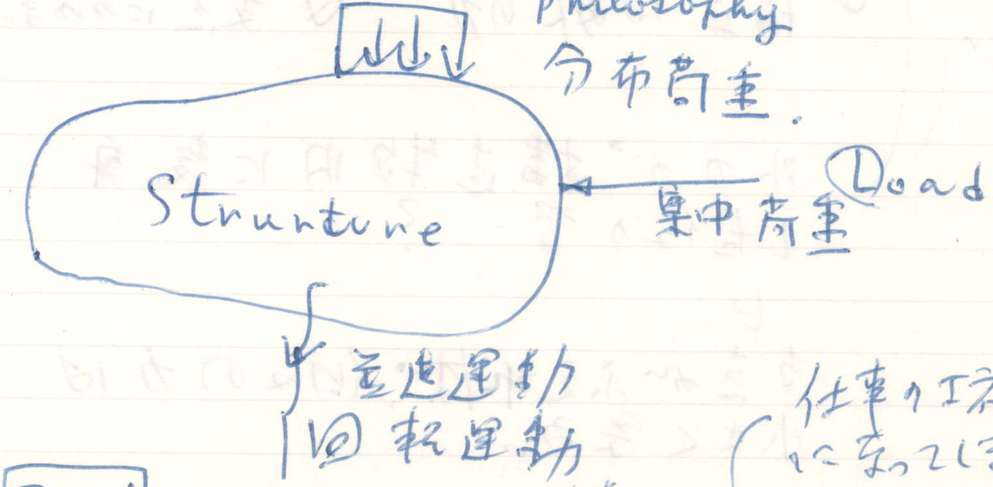
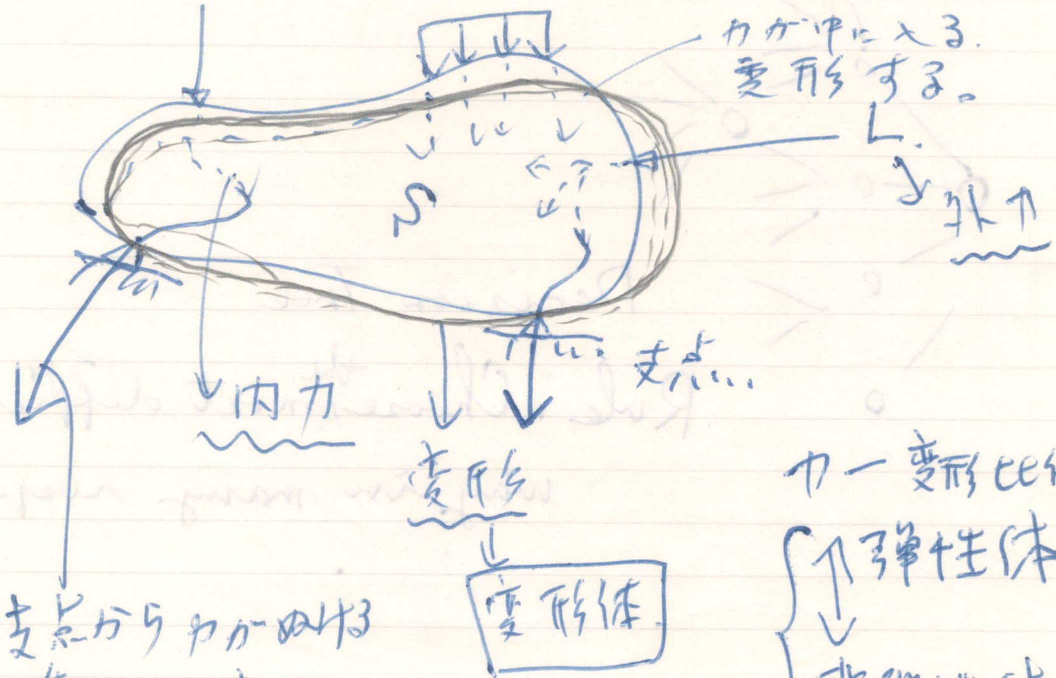
構造力学　応用力学

構造の力と変形の流れを追セキする学門



剛体：変形しない構造。

仕事のエネルギーになってしまう。



支点から力がぬける

⇒重みとして感じる。

反力

力ー変形比例

弾性体。

非弾性体。

構造物内の力

力
- 内力（曲げモーメント，せん断力，軸力）
- 外力（荷重）（反力）

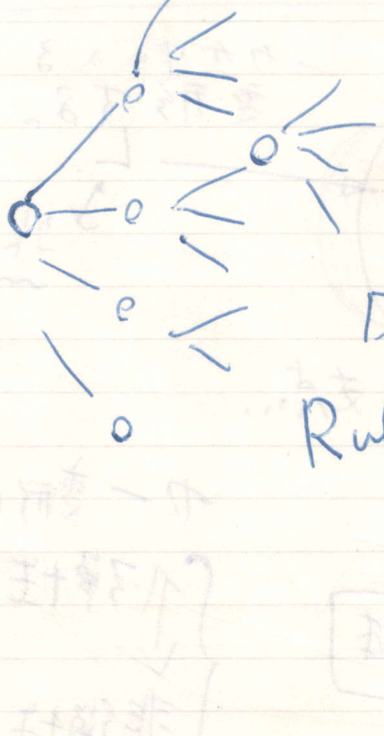
↓ 構造物外の力　　↓ 支点にかかる

外力が構造物内に変身した結果…？

↓

支点がふえれば、個々の力は小さくなる。

Turning Point



Decision Tree

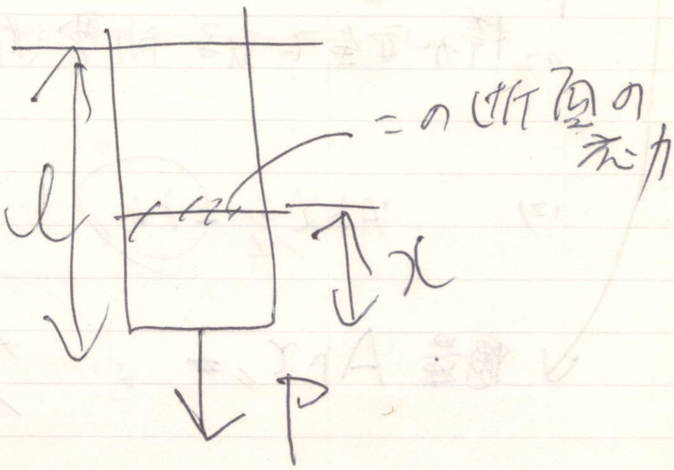
Rule: Choose the most difficult way in many respects

⑥ (A) $= \boxed{}$ $A = \dfrac{\boxed{}}{0}$

$$y = -\frac{4h}{b^2}x^2 + h \quad , \quad z = b\sqrt{\frac{h-y}{h}}$$

$$A = \int_0^h z\,dy = \frac{b}{\sqrt{h}}\int (h-y)^{\frac{1}{2}}\,dy = \frac{2}{3}bh.$$

$$I_x' = \int_0^h y^2\,dA =$$



(1) $\dfrac{P}{A} + \dfrac{A\gamma x}{A} = \dfrac{P}{A} + \gamma x$ //

(2) $A = \cancel{\dfrac{P + A\gamma l}{\sigma_a}}$ //

$$A \geqq \frac{P}{\sigma_a - \gamma l}$$

$$A(\sigma_a - \gamma l) \geqq P$$

応力が最大なのは $x = l$ のとき.

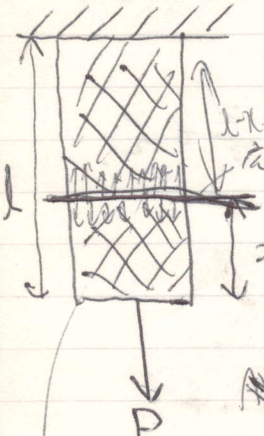
$$\sigma_{max} = \frac{P + A\gamma l}{A} \leqq \sigma_a$$

$$\cancel{\frac{P}{\gamma l - \sigma_a} \leqq A}$$

$$\frac{P}{A} + \gamma l \leqq \sigma_a$$

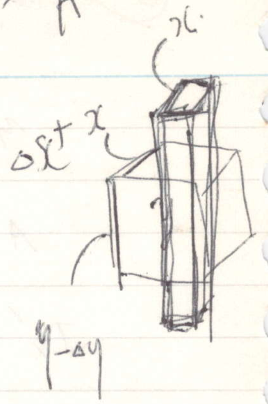
(1) γx

(2) $\frac{\sigma_a}{lA\gamma}$ $\quad \frac{P}{A}+\frac{A\gamma x}{A}=\frac{P}{A}+\gamma x$



断面積A
単位体積の重さγ
弾性係数：E
許容応力：σ_a

物体の重さ$= lA\gamma = P$



(1) ~~棒が安全である所要断面積~~ $= \frac{\sigma_a}{P}$

$= \frac{\sigma_a}{lA\gamma}$ //

$M_1 = x^2 y$

$M_2 = (x+\Delta x)^2(y-\Delta y)$

(2) $\frac{A\gamma x}{A} = \gamma x$

質量 $A\gamma x$ より $\frac{A\gamma x}{A} = \gamma x$ //

$\nu = -\frac{\beta}{\varepsilon}$

$x^2y = (x+\Delta x)^2(y-\Delta y)$

$= x^2y + 2xy\Delta x + y\Delta x^2$

$-\Delta y x^2 - 2x\Delta x\Delta y - \Delta x^2\Delta y$

$(y-\Delta y)\Delta x^2 + 2x\Delta x(y-\Delta y)$

$-\Delta y x^2$

$= -\frac{\Delta x / x}{\Delta y / y}$

$= \frac{y\Delta x}{x\Delta y}$ //

$x\Delta y$

$y\Delta x$

$2x + \Delta x - \frac{\Delta y x^2}{y\Delta x} - \frac{2x\Delta y}{y} - \frac{\Delta x\Delta y}{yB}$

$$\begin{cases} \Delta a = \dfrac{aR_1}{AE} \\ \Delta b = \dfrac{b(R_1 - P_1)}{AE} \quad \text{or} \quad \Delta b = \dfrac{b(P_2 - R_B)}{AE} \\ \Delta c = -\dfrac{cR_{\cancel{B}}}{AE} \end{cases}$$

$\Delta a + \Delta b + \Delta c = 0$ より

$$\frac{aR_1}{AE} + \frac{b(R_1 - P_1)}{AE} - \frac{cR_{\cancel{B}}}{AE} = 0$$

$\therefore$ ~~$aR_1 + a(b-c)\,T$~~

$$\therefore (a+b)R_1 - cR_2 = bP_1 \quad \text{——} \ ②$$

① より $R_2 = P_1 + P_2 - R_1$

② に代入

$$(a+b)R_1 - c(P_1 + P_2 - R_1) = bP_1$$

$$\therefore (a+b+c)R_1 = bP_1 + cP_1 + cP_2$$

$$\therefore R_1 = \frac{(b+c)P_1 + cP_2}{l}$$

$$\therefore R_2 = P_1 + P_2 - \frac{(b+c)P_1 + cP_2}{l}$$

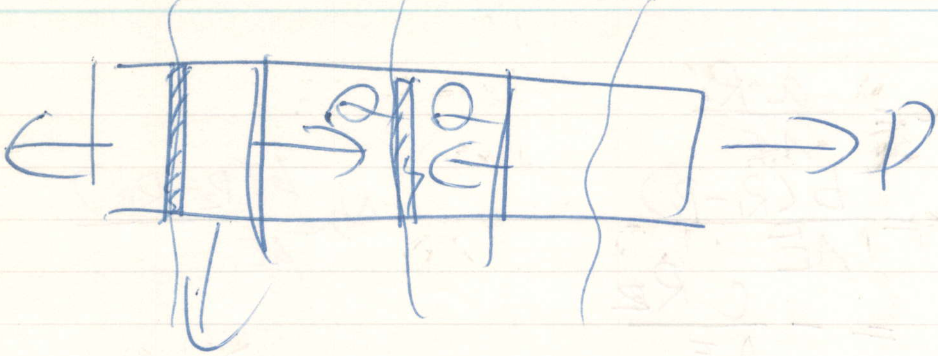
◎ $P_1 = P_2$

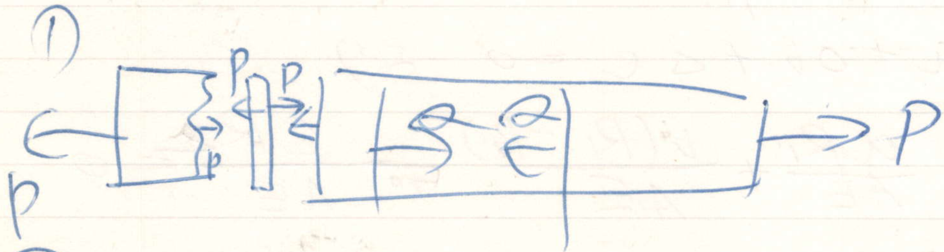
$$\begin{cases} R_1 = \dfrac{(b+2c)P}{l} \\ R_2 = \dfrac{(2a+b)P}{l} \end{cases} = \frac{aP_1 + (a+b)P_2}{l}$$

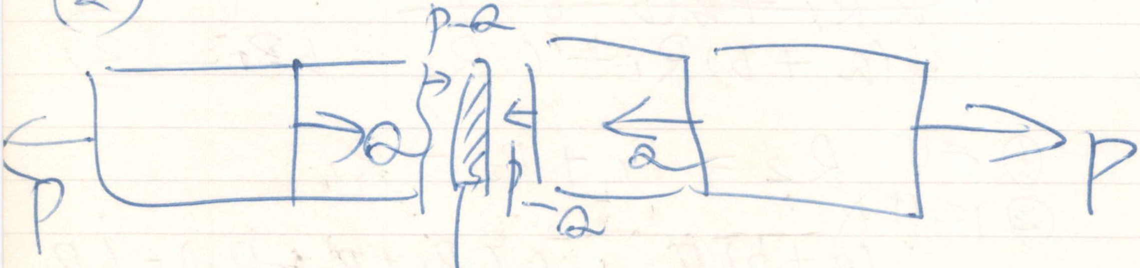
$$aR_1 + bP_2 - bR_2 - cR_2 = 0$$

~~$aR_1 = (c+b)($~~

$$a(P_1 + P_2) - aR_2 - bR_2 - cR_2 = 0$$

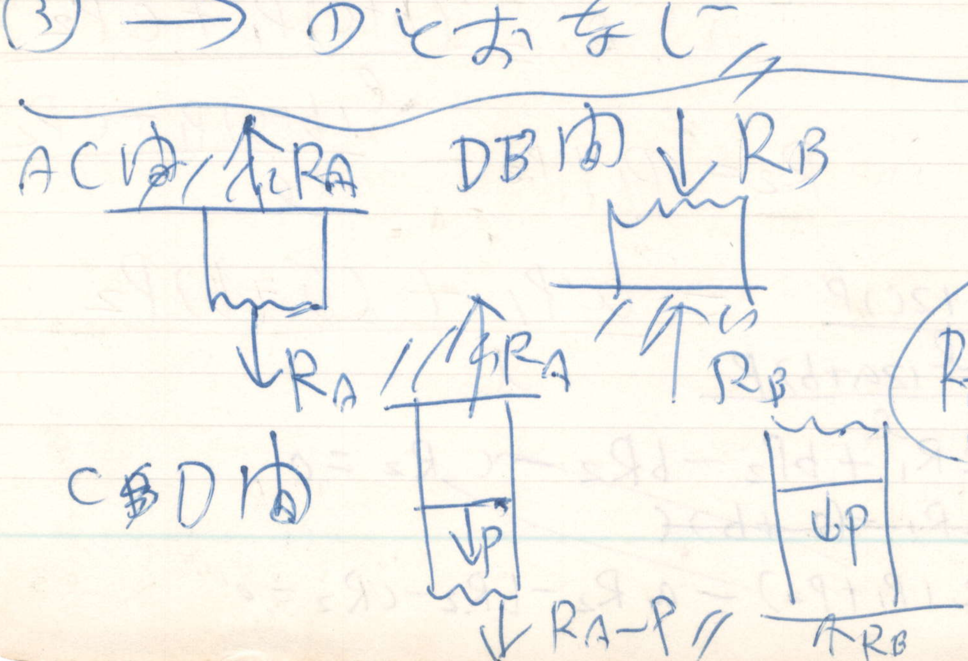




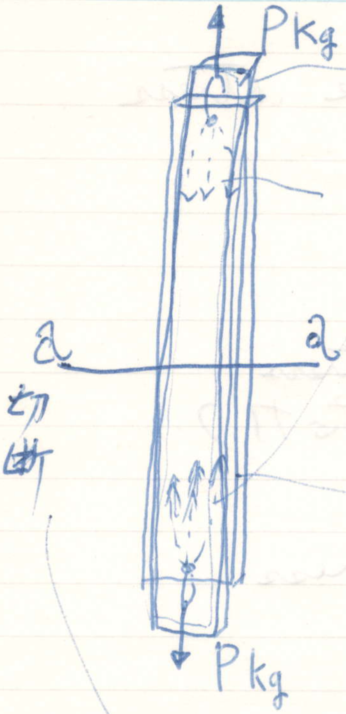


P−Q でーいっならされる

③ → ①とおなじ



$R_A - P = -(R_B - P)$



のびて、巾がせまくなる。

内力：合計零。
外力は存在しない

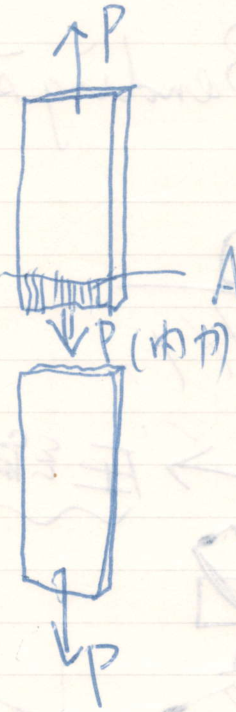
変形ノ応力。

切断する→運動とする

~~切断~~

⇓

運動しない→切断面に何かの力がはたらいている
（内力）



$$P = \sigma_t \cdot A$$

(kg/cm^2)

Tensile stress

（引張応力）

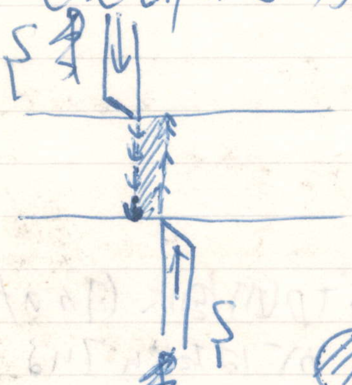
実際の力の分布は

圧縮応力 (σ_c) Compressive stress

$$P = \sigma_c \cdot A$$

Tensile stress
Compressive stress } Normal stress (垂直応力)

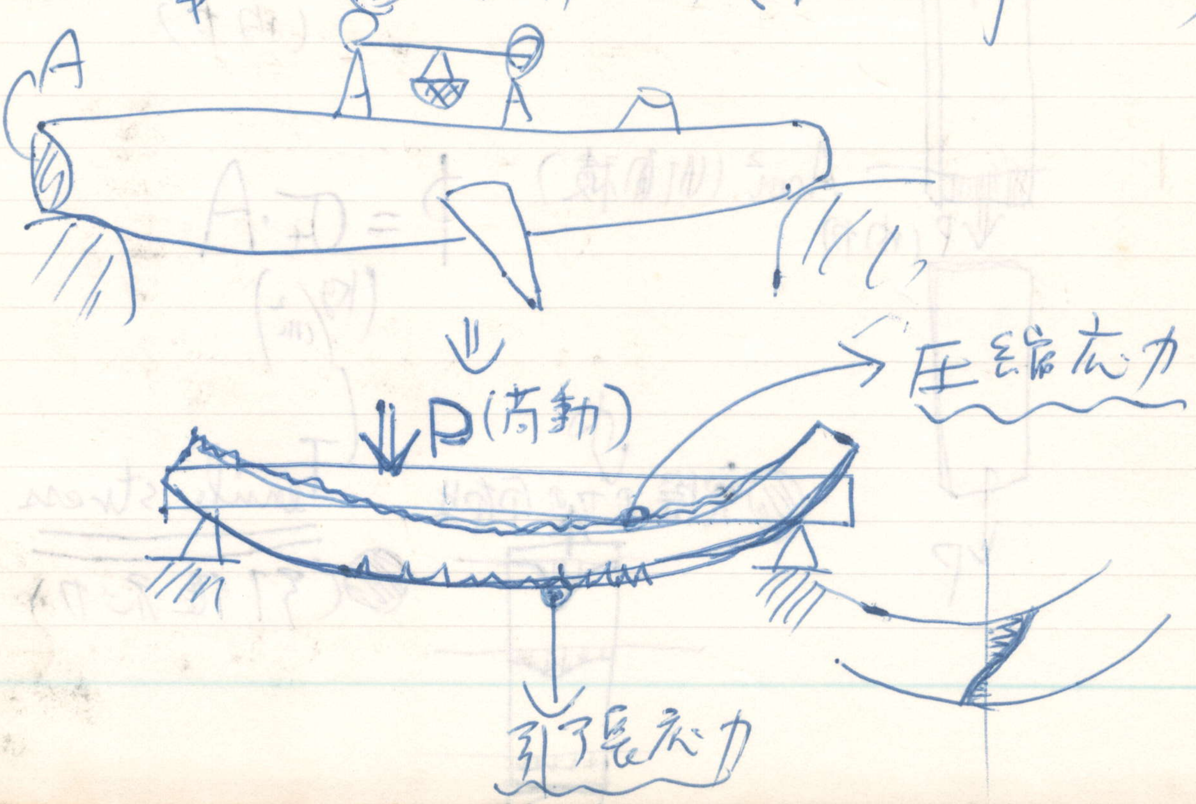
せん断応力 Shearing stress

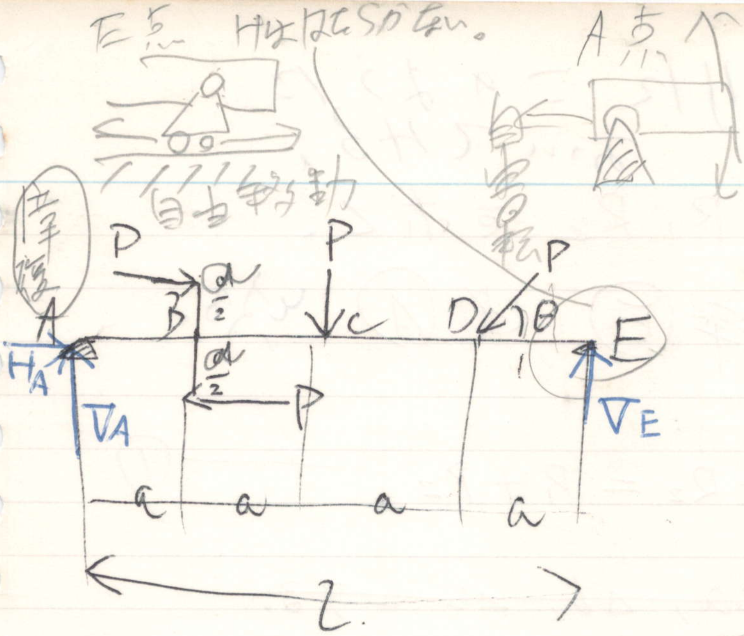


$$S = \tau \cdot A$$

(接線応力 Tangential stress)

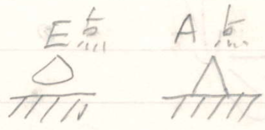
曲げ応力 (Bending stress)





A, Eにモーメントはたらかない

反力
H_A V_A V_E をもとむ。



$$\Sigma V_A = P + P\sin\theta - V_A - V_E = 0 \quad —①$$

$$\Sigma H = H_A - P\cos\theta + P - P = 0 \quad ②$$

$$\Sigma M_A = Pa + 2aP - 4aV_E + 3aP\sin\theta \quad —③$$

②より $H_A = P\cos\theta$

③より $V_E = \dfrac{\overset{3Pa}{\cancel{Pa+2aP}} + 3aP\sin\theta}{4a}$

①に代入 $V_A = P + P\sin\theta - V_E$

$= \dfrac{\overset{Pa}{\cancel{-Pa+2aP}} + aP\sin\theta}{4a}$

ΣM_E をとった方がよい

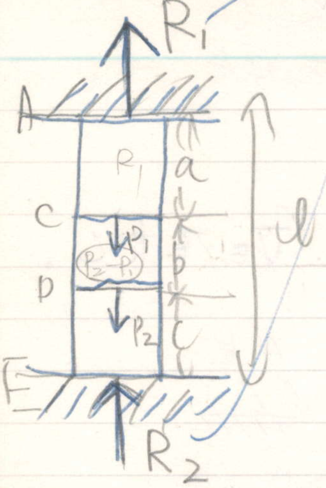
∴ $H_A = P\cos\theta$

$V_A = \dfrac{P}{2} + \dfrac{P}{4}\sin\theta - \dfrac{Pa}{4a}$ $\quad \dfrac{P}{4}(1+\sin\theta)$ (答)

$V_E = \dfrac{P}{2} + \dfrac{3P}{4}\sin\theta + \dfrac{Pa}{4a}$

$= \dfrac{3P}{4}\left(\dfrac{3}{3} + \sin\theta\right)$

力値りにこのように
おいてみる。



反力 R_1 R_2 をおく。

ヤング率 E 断面 A とする。

つりあい

$$R_1 + R_2 = P_1 + P_2 \quad \text{——①}$$

~~のびを Δa, Δb, Δc とする。~~

伸 $= \dfrac{lP}{EA}$

$= R_A - P$

$= R_B - P$

AC間ののび

$$\Delta a = \frac{aR_1}{EA}$$

CD間ののび

$$\Delta b = \frac{b(P_2 - P_1)}{EA}$$

DE間のちぢみ

$$\Delta c = \frac{cR_2}{EA}$$

より

$$\Delta a + \Delta b = \Delta c$$

$$\therefore \frac{aR_1}{EA} + \frac{b(P_2 - P_1)}{EA} = \frac{cR_2}{EA}$$

よって

$$aR_1 + b(P_2 - P_1) = cR_2 \quad \text{——②}$$

①,②をとく

①より $R_2 = P_1 + P_2 - R_1$

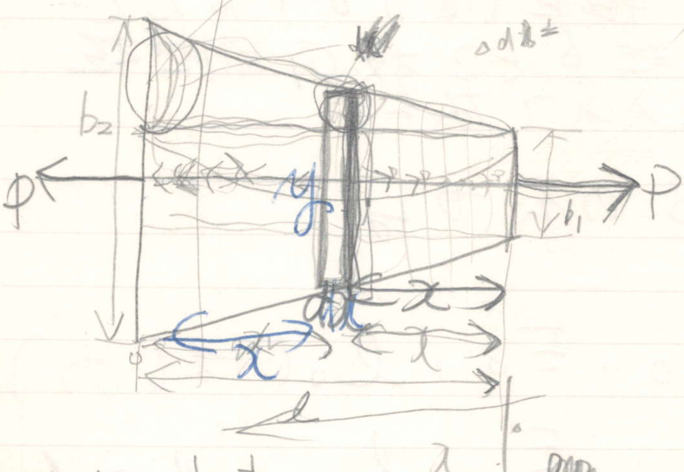
②に代入

$$aR_1 - c(P_1 + P_2 - R_1) + b(P_2 - P_1) = 0$$

$$(a + c)R_1 = b(P_1 - P_2) + c(P_1 + P_2)$$

$\frac{b_2-b_1}{2} : \bigcirc = l : x$ $\quad E = \frac{lP}{\Delta l A}$ $\quad \Delta l = \frac{lP}{AE}$

$\bigcirc = x \times \frac{b_2-b_1}{2l}$



厚さ t
ヤング率 E

伸び？

$A = t \times \{b_1 + \frac{b_2-b_1}{l}x\}$

よって

$A = t(b_1 + \frac{b_2-b_1}{l}x)$

$y = b_1 + \frac{b_2-b_1}{l}x$ $\qquad \Delta l =$

$\therefore dA = (b_1 + \frac{b_2-b_1}{l}x)t \qquad \frac{P\,dx}{t(b_1+\frac{b_2-b_1}{l})x}\,dx$

$\frac{P}{A}$

$d\lambda = \frac{P\,dx}{dA\,E}$

$\sigma = \frac{P}{A} = \int_0^l \frac{P}{t(b_1+\frac{b_2-b_1}{l}x)}\,dx$

よって
$$\boxed{\Delta l = \frac{P}{Et}\int_0^l \frac{dx}{b_1+\frac{b_2-b_1}{l}x}}$$

$$= \frac{P}{Et}\cdot\frac{l}{b_2-b_1}\left\{\log\left(b_1+\frac{b_2-b_1}{l}l\right)-\log b_1\right\}$$

$$= \frac{P}{Et}\cdot\frac{l}{b_2-b_1}\log\frac{b_2}{b_1} \quad //$$

$$\frac{Pl}{(b_2-b_1)Et}\log\frac{b_2}{b_1} \quad // \quad ?$$

$y = \frac{b_1-b_2}{l}x + b_2$

$d\lambda = \frac{P\cdot dx}{dA\cdot E}$

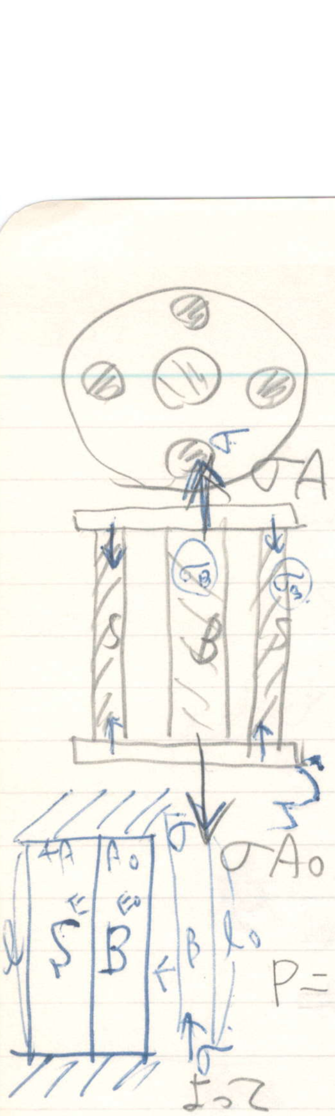
$\lambda = \frac{P}{Et}\int_0^l \frac{1}{\frac{b_1-b_2}{l}x+b_2}\,dx$

$\ln \quad b_1 \quad b_2$

$$\boxed{\lambda = \frac{Pl}{(b_1-b_2)tE}\ln\frac{b_1}{b_2}}$$

$$\frac{lP}{(b_1-b_2)Et}\log\left[\frac{b_1-b_2}{l}x+b_2\right]_0^l$$

$\cancel{\sigma = \frac{P}{A} = \frac{A_0\sigma}{A}}$



棒Bに軸圧縮力σを加えて4本の支柱Sと2枚の剛性板の間に入れてあります。

BとSの応力は？

$$\begin{cases} B : A_0,\ E_0,\ l_0 \text{（はじめ）} \\ S \text{(1本)} : A,\ E,\ l \\ \text{(4本)}\ 4A \end{cases}$$

$l_0 > l$

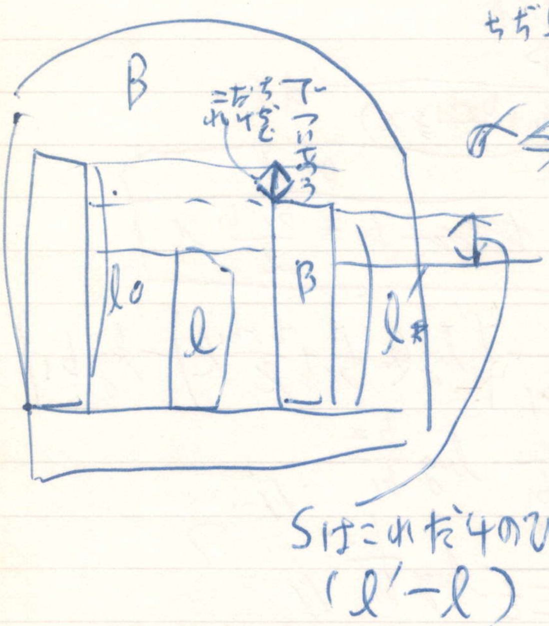
$P = \sigma A_0$ であります。

$E = \frac{l_0 P}{\Delta l\, A}$

よって Sを考えると、

ちぢむ $l_0 - l = l_0 - l$. E_0, A_0, σ.

$\cancel{\sigma = \frac{E_0 A_0 (l_0 - l)}{l_0}}$



Bについて

$$E_0 = \frac{l_0 \cdot \sigma_B}{(l_0 - l')A_0} \quad \text{— ①}$$

Sについて

$$E = \frac{l \cdot \sigma_S}{(l' - l)4A} \quad \text{— ②}$$

Sはこれだけのびる。$(l' - l)$

Bは$(l_0 - l')$ちぢむ

①〜③より l', σ_B, σ_S を求む。

①より $E_0 A_0 l_0 - E_0 A_0 l' = l_0 \sigma_B$ —①

$4AEl' - 4AEl = l\sigma_S$ —②

また

$A_0 \sigma_B = 4A\sigma_S$ —③

①× $4AEl$ $4AA_0EE_0 l l_0 - 4AA_0EE_0 l l' = 4AEl l_0 \sigma_B$

②× $A_0E_0l_0$

$\cancel{4AA_0EE_0 l_0 l'} - 4AA_0EE_0 l l_0$

$= A_0E_0 l_0 l \sigma_S$

①× $4AE$ $4AA_0EE_0 l_0 - 4AA_0EE_0 l' = 4AEl_0\sigma_B$

②× A_0E_0 $+) \; 4AEA_0E_0 l' - 4AA_0EE_0 l = A_0E_0 l \sigma_S$

$4AA_0EE_0(l_0 - l) = 4AEl_0\sigma_B + A_0E_0 l \sigma_S$

$\sigma_B = \frac{4A}{A_0}\sigma_S$

③×

$\left(\frac{16A^2El_0}{A_0} + A_0E_0 l\right)\sigma_S$

つりあっているときの応力

軸圧縮力σによるBの縮みλ_0は

$$\lambda_0 = \sigma \cdot \frac{l_0}{E_0} \quad \text{——— ①}$$

応力 σ_B, σ_S ~~とする~~、力を P_B, P_S とする。

Sの伸びλは.

$$\lambda = \sigma_S \frac{l}{E} \quad \text{——— ②}$$

Bの伸び$\lambda_0 - \lambda$は

$$\lambda_0 - \lambda = \sigma_B \cdot \frac{l_0}{E_0} \quad \text{——— ③}$$

又、$\sigma_S = \frac{P_S}{4A}$ ——— ④　$\sigma_B = \frac{P_B}{A}$ ——— ⑤

$P_B = P_S$ だから(つりあう)

④⑤より

$$\boxed{4\sigma_S A = \sigma_B A_0} \quad \text{——— ⑥}$$

①②を③に代入

$$\boxed{\sigma \frac{l_0}{E_0} - \sigma_S \frac{l}{E} = \sigma_B \cdot \frac{l_0}{E_0}} \quad \text{——— ⑦}$$

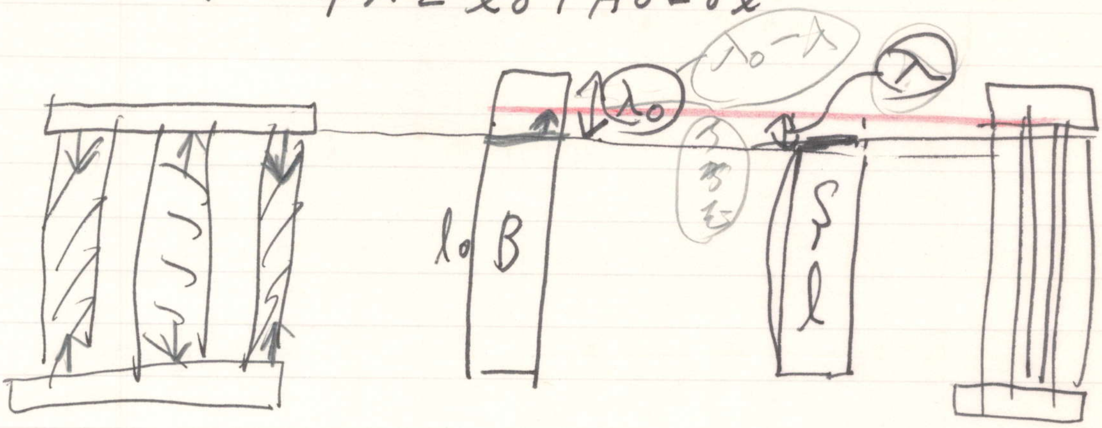
⑥, ⑦より

$$\sigma \frac{l_0}{E_0} - \frac{l}{E} \cdot \frac{\sigma_B A_0}{4A} = \sigma_B \frac{l_0}{E_0}$$

$$\therefore \sigma_B = \frac{4AEl_0}{4AEl_0 + A_0E_0l} \sigma \quad \text{——— ⑧}$$

⑧ 式 ⑥に代入し

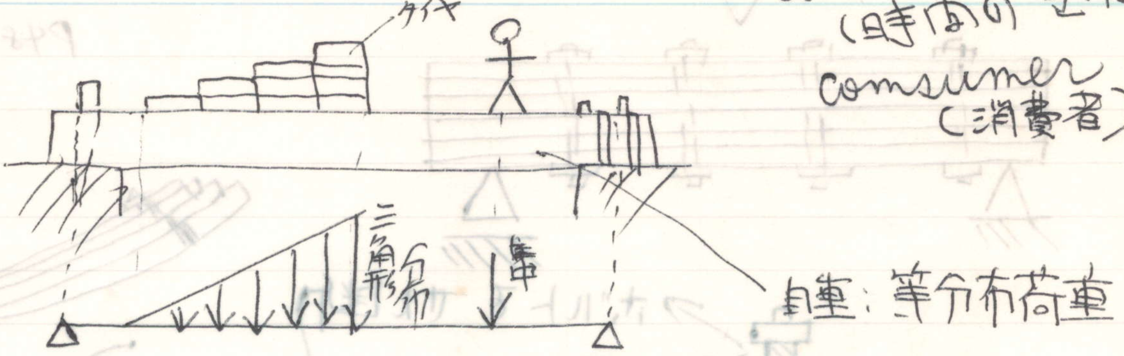
$$\sigma_S = \frac{A_0 E l_0}{4AE l_0 + A_0 E_0 l}\,\sigma \quad \text{——} ⑨$$

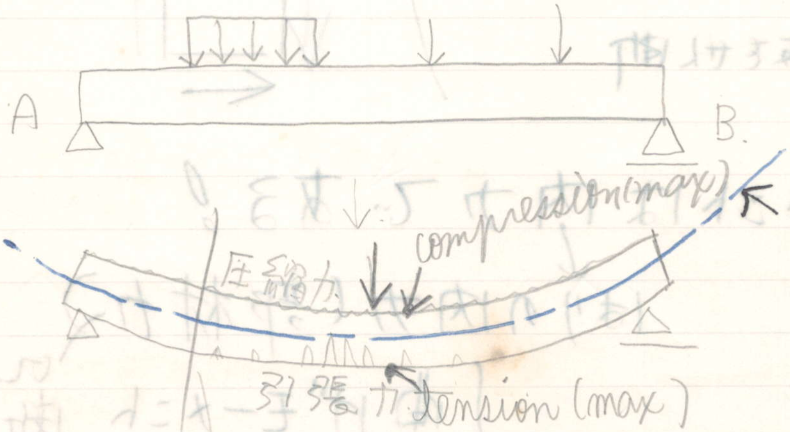


結局 長さ

$$\boxed{l' = l + \lambda = l + \frac{A A_0 l_0 \sigma}{(4AEA_0 + A_0 E_0 l)}}$$

Time wasting
consuming (時間のむだ)
consumer (消費者)

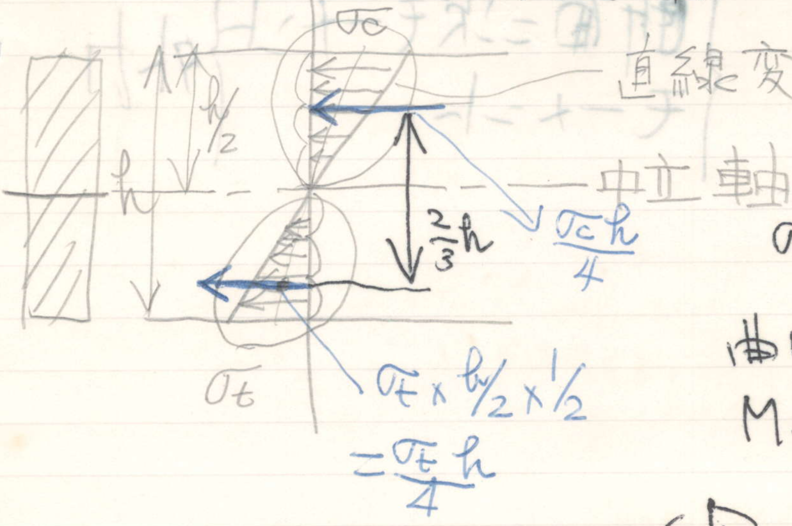




ax (おの 斧)

中立軸
~~neutral~~
neutral axis
複 axes

I do not know how to kill the time.
(どうして時間をつぶそうか。)



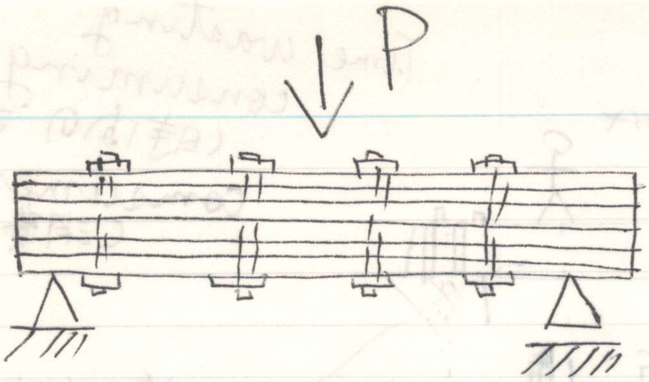
$\sigma_c = \sigma_t = \sigma$ とすると

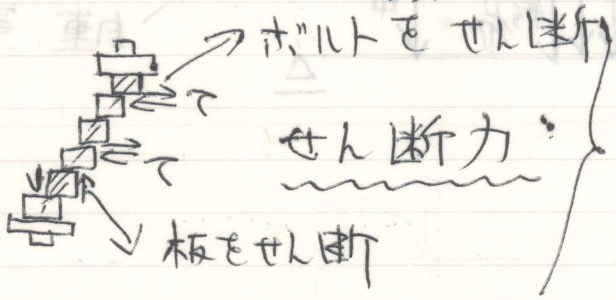
曲ゲモーメント

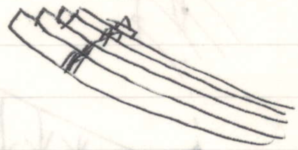
$$M = \frac{\sigma h}{4} \times \frac{2h}{3} = \frac{\sigma h^2}{6}$$

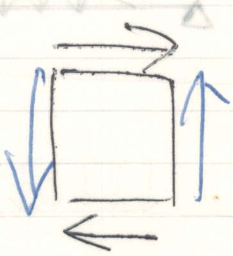
(Bending moment)

P48~P49.









曲げモーメントは内力である!

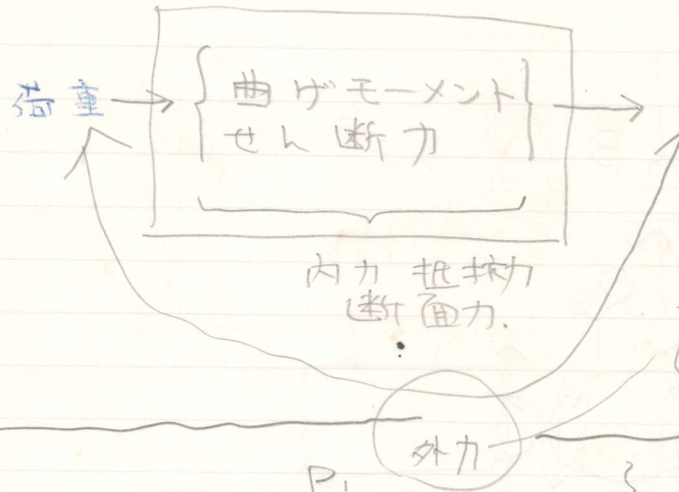
はりの内力(部材力) or 断面力

- 曲げモーメント
- せん断力

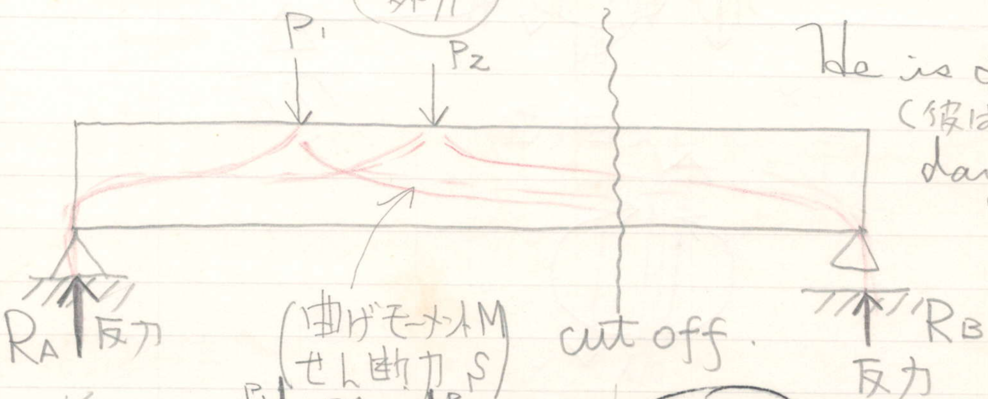
- 断面二次モーメント
- モーメント

外力

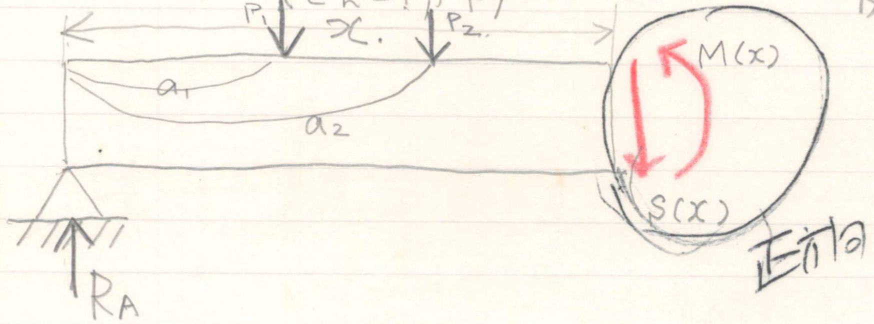
はり.



$$\begin{cases} \Sigma H = 0 \\ \Sigma V = 0 \\ \Sigma M = 0 \end{cases}$$



He is off today.
(彼は今日休みです)
day off (休日)



(注) R_A, R_B は
つりあいの式
から求まるので
既値知

$$\Sigma V = R_A - P_1 - P_2 - S(x) = 0$$

$$\Sigma M_A = P_1 a_1 + P_2 a_2 + S(x) \cdot x - M(x) = 0$$

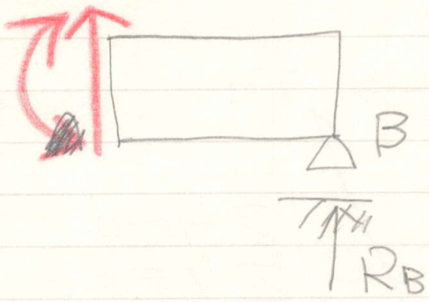
~~$(\Sigma M_x =)$~~

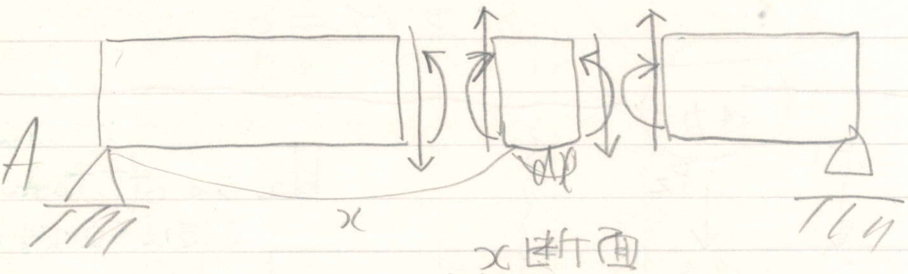
$$\boxed{\frac{dM(x)}{dx} = S(x) \quad (P56)}$$

$$S(x) = R_A - P_1 - P_2$$

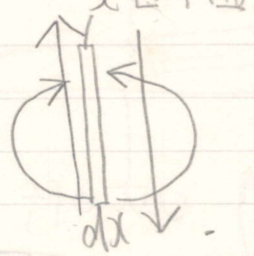
$$M(x) = R_A x - (x - a_1) P_1 - (x - a_2) P_2$$

$$= S(x) \cdot x + P_1 a_1 + P_2 a_2$$





x断面



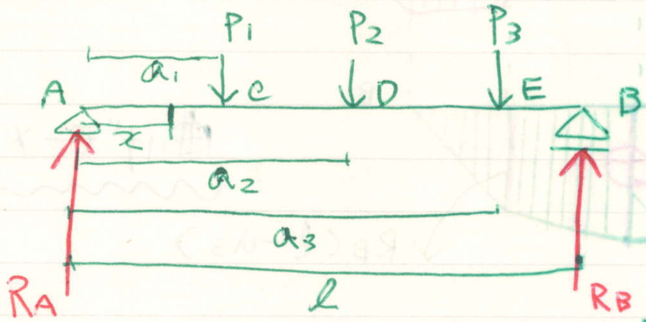
② 左を考えて 時計まわりを正とした モーメントのまわりに 正の曲げモーメント。

① 切断した点から左を考えて 上向きの力を正としたときの 集計が正のせん断力。

単純ばり。

梁の断面力；曲げモーメント，せん断力
M 　 S

(例1)

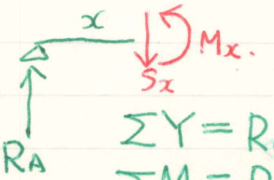


(1) 支点反力を求める。

$$\begin{cases} R_A + R_B = P_1 + P_2 + P_3 \\ M_A = a_1P_1 + a_2P_2 + a_3P_3 - lR_B = 0 \end{cases}$$

$$\begin{cases} R_B = \dfrac{P_1a_1 + P_2a_2 + P_3a_3}{l} \\ \therefore R_A = P_1 + P_2 + P_3 - \dfrac{P_1a_1 + P_2a_2 + P_3a_3}{l} \end{cases}$$

AC間 $(0 \leqq x < a_1)$

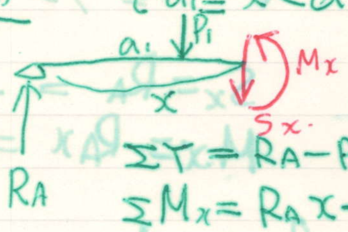


$\Sigma Y = R_A - S_x = 0 \quad \therefore S_x = R_A$

$\Sigma M_x = R_A x - M_x = 0$

$\therefore M_x = R_A \cdot x$

CD間 $(a_1 \leqq x < a_2)$



$\Sigma Y = R_A - P_1 - S_x = 0 \quad S_x = R_A - P_1$

$\Sigma M_x = R_A x - P_1(x - a_1) - M_x = 0$

$\therefore M_x = R_A x - P_1(x - a_1)$

DE間 $(a_2 \leqq x < a_3)$

$S_x = R_A - P_1 - P_2$.

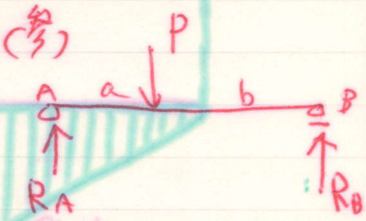
$M_x = R_A x - P_1(x - a_1) - P_2(x - a_2)$

EB間 $(a_3 \leqq x \leqq l)$

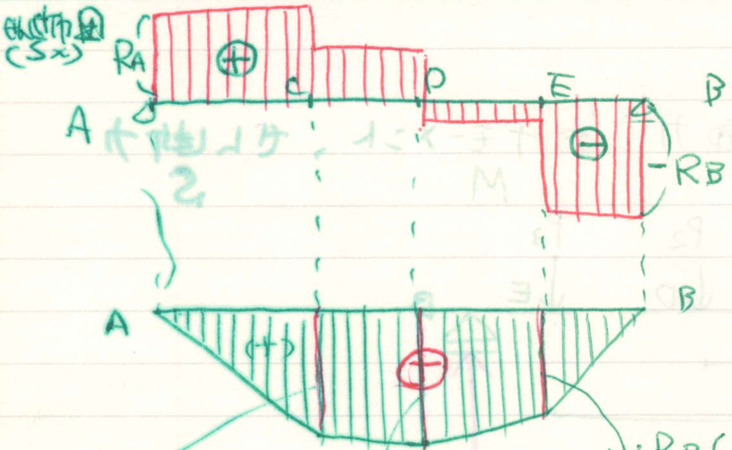
$S_x = R_A - P_1 - P_2 - P_3 \; (= -R_B)$

$M_x = R_A x - P_1(x - a_1) - P_2(x - a_2) - P_3(x - a_3)$

(参)



$M_A = Pa - R_B(a + b) = 0$ ならば，任意点のモーメントは釣合っていることを証明せよ…？



せん断力図

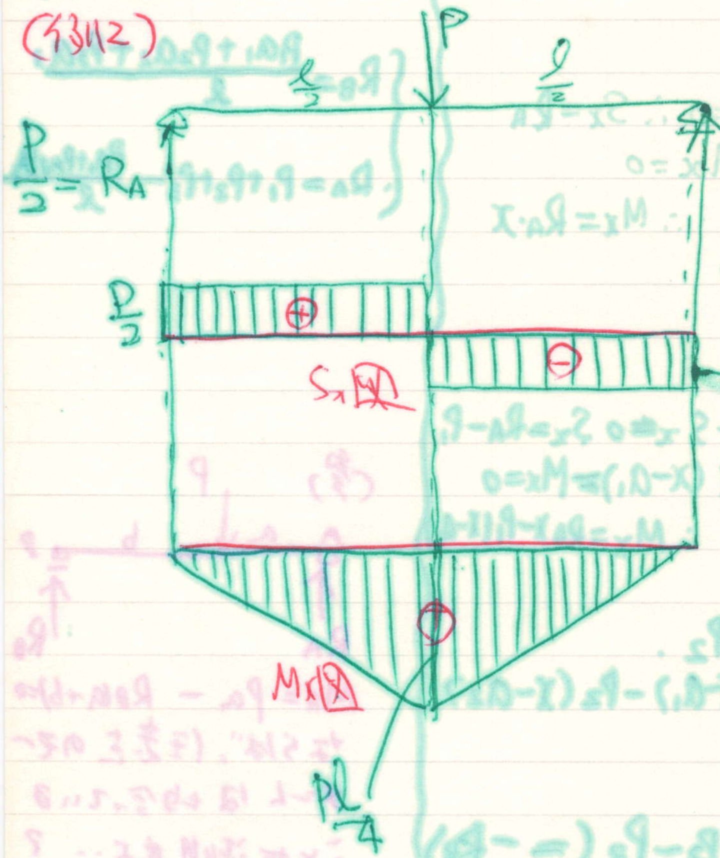
曲げモーメント図

$R_B(l-a_3)$

$R_A a_1$

$R_A a_2 - P_1(a_2-a_1)$

(例12)



$0 \leqq x \leqq l/2$

$S_x = R_A = \frac{P}{2}$

$M_x = R_A x = \frac{Px}{2}$

$\frac{l}{2} \leqq x \leqq l$

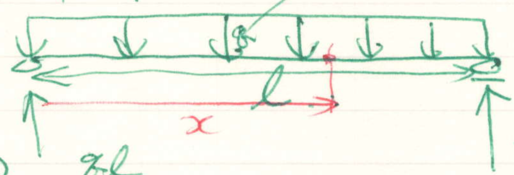
$S_x = R_A - P = -\frac{P}{2}$

$M_x = \frac{P}{2}x - P(x - \frac{l}{2})$

$= -\frac{Px}{2} + \frac{Pl}{2}$

単位長さあたり q

等分布荷重

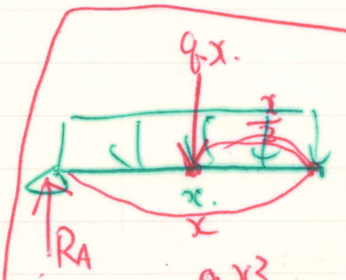


$R_A = \dfrac{ql}{2}$ $\qquad R_B = \dfrac{ql}{2}$

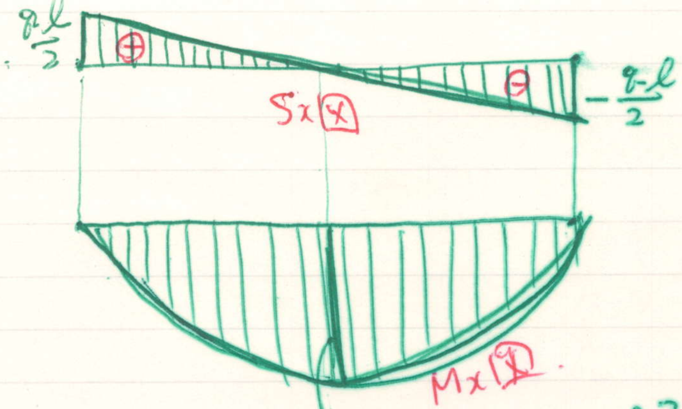
$$S_x = \frac{ql}{2} - qx = q\left(\frac{l}{2} - x\right)$$

$$M_x = \frac{ql}{2}x - \int_0^x \cancel{q}\, q x\,dx$$

$$= \frac{qlx}{2} - \frac{q}{2}x^2 = \frac{qx}{2}(l - x)$$

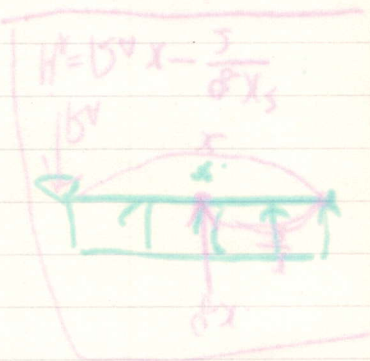


$M_x = R_A x - \dfrac{qx^2}{2}$



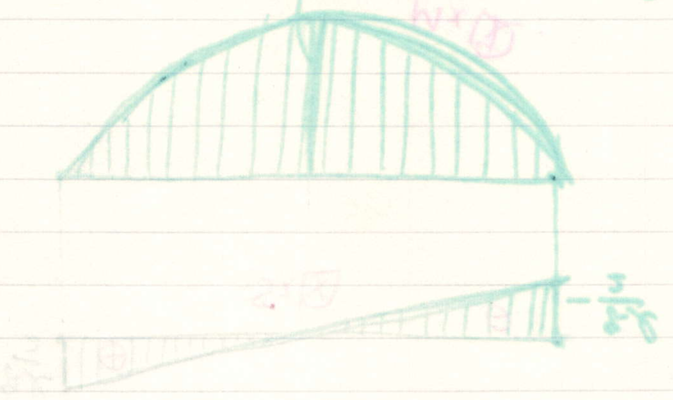
$$M_{x=\frac{l}{2}} = \frac{ql^2}{8} \quad //$$

$R_A = \frac{ql}{2}$



$M_x = R_A x - \frac{qx^2}{2}$

$M_{x=\frac{l}{2}} = \frac{ql^2}{8}$



1章の問題.

(1) 合力Rが18tの力となす角をαとすると,

$$\tan\alpha = \frac{P_1 \sin\beta}{P_2 + P_1\cos\beta} = \frac{10\sin 75^\circ}{18 + 10\cos 75^\circ}$$

$$= \frac{10 \times 0.966}{18 + 10 \times 0.259} = 0.469$$

$$\therefore \alpha = 25^\circ 08'$$

また $R = \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\beta}$

$$= \sqrt{10^2 + 18^2 + 2\cdot 10\cdot 18\cdot 0.259}$$

$$= \sqrt{517.24} = 22.7\ (t)$$

(2) $P_A = 10\sin\alpha = 10\cdot\frac{4}{5} = 8\ (kg)$

$P_B = 10\cos\alpha = 10\cdot\frac{3}{5} = 6\ (kg)$

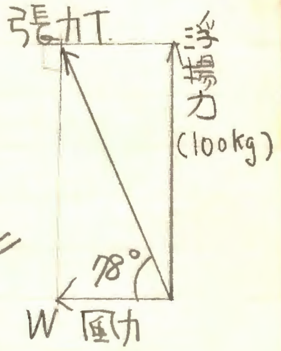
(3) 右図より.

風力をW, 張力をTとすると.

$$W = 100 \times \tan 12^\circ$$

$$= 100 \times 0.213 = 21.3\ (kg)$$

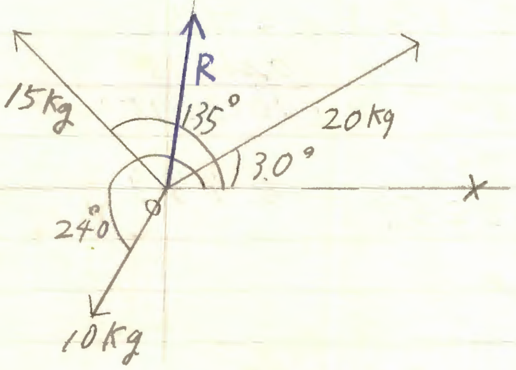
又 $T = \frac{100}{\cos 12^\circ} = \frac{100}{0.978} = 102\ (kg)$

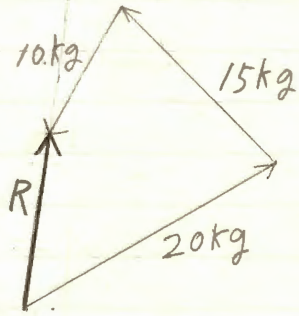


(2) $\begin{cases} \Sigma V = F_1\cos\alpha + F_2\cos\beta - 10 = 0 \\ \Sigma H = F_2\sin\beta - F_1\sin\alpha = 0 \end{cases}$

α, β の cos sin は きち, $F_1 = 8$, $F_2 = 6$

(4) 図





上図より　$R = 12.0\,\mathrm{kg}$,　$\alpha = 81^\circ \cancel{40}\,54'$

解

$$\Sigma H = 20\cos 30^\circ + 15\cos 135^\circ + 10\cos 240^\circ$$
$$= 20 \times 0.87 + 15 \times (-0.71) + 10 \times (-0.5)$$
$$= 1.75$$

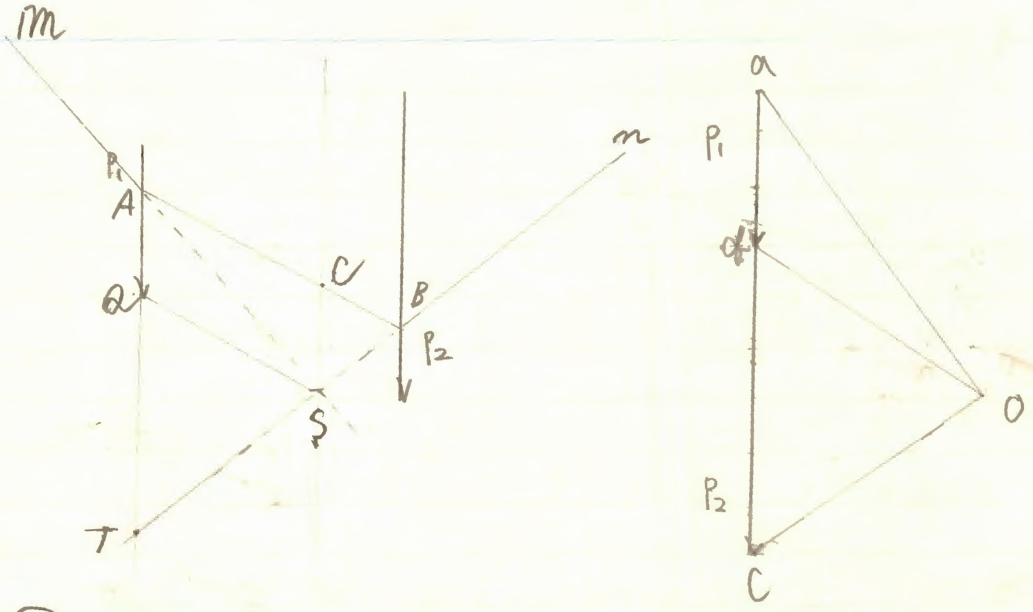
$$\Sigma V = 20\sin 30^\circ + 15\sin 135^\circ + 10\sin 240^\circ$$
$$= 20 \times 0.5 + 15 \times 0.71 + 10 \times (-0.87)$$
$$= 12.0$$

故に　$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$
$$= \sqrt{147.06} = 12.1\ (\mathrm{kg})$$

又　$\tan\alpha = \dfrac{\Sigma V}{\Sigma H} = \dfrac{12.0}{1.75} = 6.857$

$$\therefore \alpha = 81^\circ 43'$$

(5)



(証) P_1, P_2 の連力図をかき nB の延長と P_1 の作用線の交点を T とし, S から AB に平行にひいた線分と P_1 の作用線の交点を Q とする

上図の $\triangle OaC \backsim \triangle SAT$ より

$AQ : QT = P_1 : P_2$

ここで $AQ = CS$ より

$CS : QT = P_1 : P_2$

ここで $\triangle CSB \backsim \triangle QTS$ より

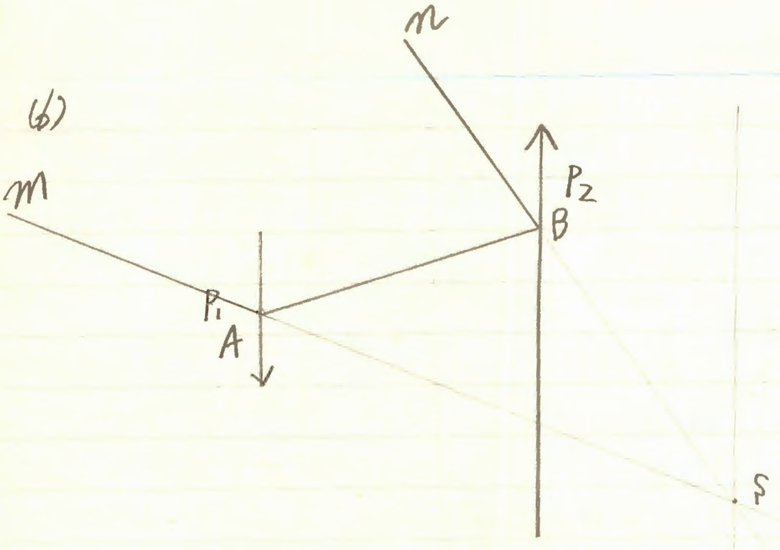
$CS : QT = CB : QS = CB : AC$

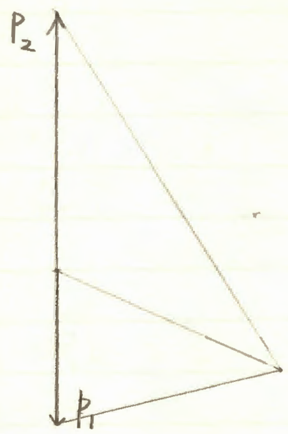
$(\because QS = AC)$

$\therefore AC : CB = P_2 : P_1$

つまり P_1 P_2 の合力は 2 力間の距離を力の大きさの逆比に内分する点を通る.

(b)





(17) $\Sigma H = \Sigma P \cos\alpha$

$$= 4\cdot\cos 60° + 6\cdot\cos 210° + 3\cdot\cos 180° + 5\cdot\cos(-45°)$$
$$= 4\times 0.5 + 6\times(-0.87) + 3\times(-1) + 5\times 0.71$$
$$= 2 - 5.22 - 3 + 3.55$$
$$= -2.67$$

$\Sigma V = \Sigma P \sin\alpha$

$$= 4\cdot\sin 60° + 6\cdot\sin 210° + 3\cdot\sin 180° + 5\sin(-45°)$$
$$= 4\times 0.87 + 6\times(-0.5) + 3\times 0 + 5\times(-0.71)$$
$$= 3.48 - 3 - 3.55$$
$$= -3.07$$

故に $R = \sqrt{(-2.67)^2 + (-3.07)^2}$

$$= \sqrt{16.5538} = 4.07\ (t)$$

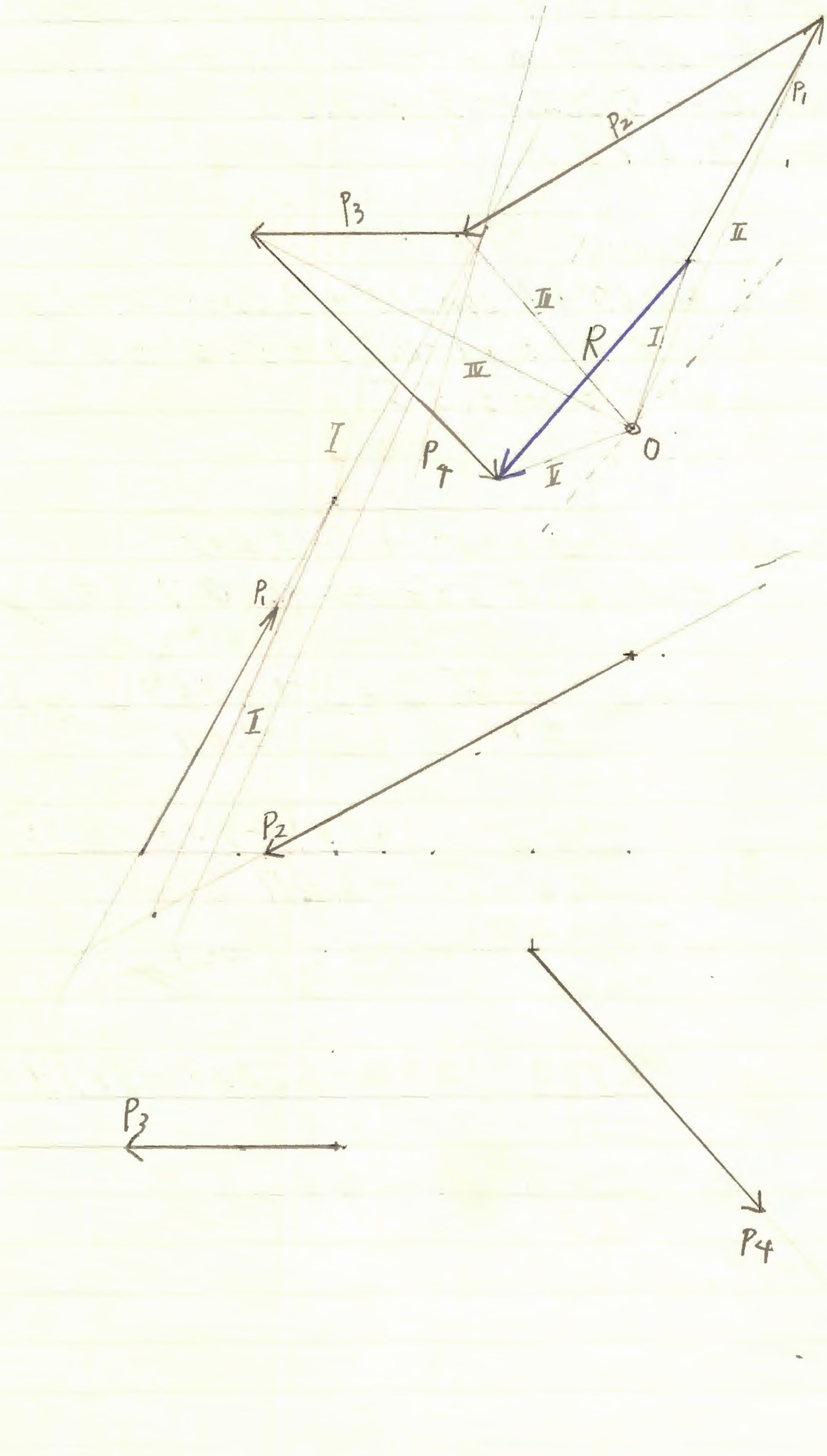
$$\alpha = \tan^{-1}\frac{\Sigma V}{\Sigma H} = \tan^{-1}\frac{-3.07}{-2.67} = 228°59'$$

又.
$$x_0 = \frac{\Sigma V\cdot x}{\Sigma V} = \frac{3.48\times 1 - 3\times 6 - 3.55\times 5}{-3.07}$$
$$= \frac{-32.27}{-3.09} = 10.5\ (m)$$

$$y_0 = \frac{\Sigma H y}{\Sigma H} = \frac{2\times 4 - 5.22\times 6 - 3\times 1 + 3.55\times 3}{-2.67}$$
$$= \frac{-15.67}{-2.67} = 5.9\ (m)$$

$M_0 =$

(8)



(9) $\Sigma V = P_1 - 10 - 4 - 5 + P_5 = 0$

$\therefore P_1 + P_5 - 19 = 0 \quad \cdots (1)$

左端の点に関するモーメント.

$\Sigma M = 3\times 10 + 8\times 4 + 10\times 5 - 14\times P_5 = 0$

$\therefore P_5 = \frac{112}{14} = 8\ (t)$ //

(1)に代入して $P_1 = 11\ (t)$ //

(10) 求めるモーメントの大きさは；P5のCに与える
モーメントの大きさと等しい

$\therefore M = 2\times 8 = 16\ (t\cdot m)$

(11) Cのまわりのモーメントを考えると.

$\Sigma M_C = 4P_1 - 6P_2 + 10 = 0 \quad \cdots (1)$

また $\Sigma V = P_1 + P_2 = 0 \quad \cdots (2)$

(1)より $2P_1 - 3P_2 = -5$

(2)より $2P_1 + 2P_2 = 0$

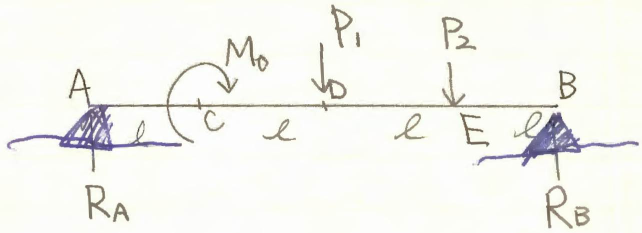
$-5P_2 = -5 \quad \therefore P_2 = 1\ (t)$ //

(2)に入れて $P_1 = -P_2 = -1\ (t)$ //

(11) $\begin{cases} \Sigma M_A = 10 - 10P_2 = 0 & \therefore P_2 = 1 \\ \Sigma M_B = 10 + 10P_1 = 0 & \therefore P_1 = -1 \end{cases}$

(未知数のある点で ΣM をとる)

(演)



ABがつりあっているから.

$$\begin{cases} \Sigma V = 0 \\ \Sigma H = 0 \\ \Sigma M = 0 \end{cases}$$

垂直方向の力の合力が零となるから.

$$\Sigma V = P_1 + P_2 - R_A - R_B = 0$$

$$\therefore \ R_A + R_B = P_1 + P_2 \quad \cdots (1)$$

またCのまわりのモーメントの和が零となるから.

$$\Sigma M = \overline{CA}\cdot R_A + \overline{CD}\cdot P_1 + \overline{CE}\cdot P_2 - \overline{CB}\cdot R_B + M_0 = 0$$

$$\therefore \ \overline{CA}\, R_A - \overline{CB}\, R_B = -\overline{CD}\cdot P_1 - \overline{CE}\cdot P_2 - M_0$$

$$l R_A - 3l R_B = -(l P_1 + 2l P_2 + M_0) \quad \cdots (2)$$

(1)より $l R_A + l R_B = l P_1 + l P_2 \quad \cdots (1)'$

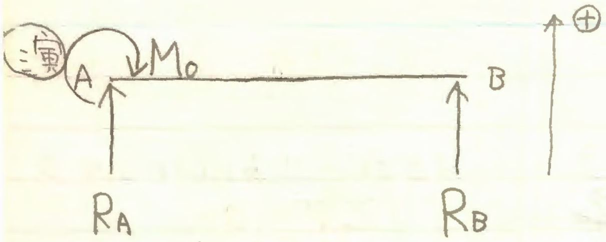
(1)′−(2)より $4 l R_B = 2l P_1 + 3l P_2 + M_0$

$$\therefore \ R_B = \frac{2l P_1 + 3l P_2 + M_0}{4l} \quad //$$

(1)に代入して

$$R_A = P_1 + P_2 - \frac{2l P_1 + 3l P_2 + M_0}{4l}$$

$$\therefore R_A = \frac{2l P_1 + l P_2 - M_0}{4l} \quad //$$



垂直方向の合力は零となるから

$\Sigma V = R_A + R_B = 0$ ……(1)

Aのまわりのモーメントの和が零であるから

$\Sigma M_A = M_0 - \overline{AB} \cdot R_B = 0$ ……(2)

(2)より $R_B = \dfrac{M_0}{\overline{AB}}$

(1)に代入して $R_A = -R_B = -\dfrac{M_0}{\overline{AB}}$ ……(答)

$\overline{AB} = l$ とすれば

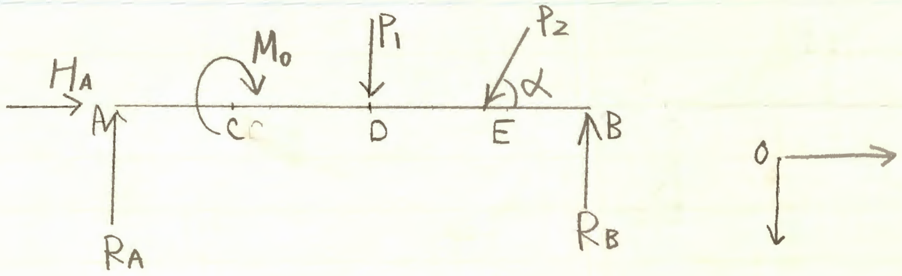
$$\begin{cases} R_A = -\dfrac{M_0}{l} \\ R_B = \dfrac{M_0}{l} \end{cases}$$ (上向きを正とする) ……(答)

[釣合] $\begin{cases} \Sigma H = 0 \\ \text{and} \\ \Sigma V = 0 \\ \text{and} \\ \Sigma M = 0 \end{cases}$ これですべて解ける。

$\Sigma M_B = M_0 + l R_A = 0$

$\therefore R_A = -\dfrac{M_0}{l}$ でも可 //

演



$$\Sigma H = H_A - P_2 \cos\alpha = 0 \quad (1)$$

$$\Sigma V = P_1 + P_2 \sin\alpha - R_A - R_B = 0 \quad (2)$$

$$\Sigma M_A = M_0 + 2l P_1 + 3l P_2 \sin\alpha - 4l R_B = 0 \quad (3)$$

(3) より $R_B = \dfrac{M_0 + 2lP_1 + 3lP_2\sin\alpha}{4l}$

(2) より $R_A = P_1 + P_2\sin\alpha - R_B$

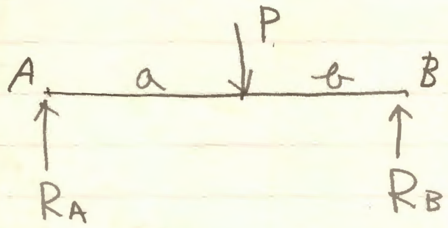
$= \dfrac{-M_0 + 2lP_1 + lP_2\sin\alpha}{4l}$ ･･･(答)

(1) より $H_A = P_2\cos\alpha$

(参 $\Sigma M_B = 4lR_A + M_0 - 2lP_1 - lP_2\sin\alpha$

より $R_A = \dfrac{-M_0 + 2lP_1 + lP_2\sin\alpha}{4l}$

でも可)



(イ) $\Sigma M_A = ap - (a+b)R_B = 0$

$$\therefore\ R_B = \frac{aP}{a+b} \quad \text{---(1)}$$

(ロ) $\Sigma M_B = (a+b)R_A - bP = 0$

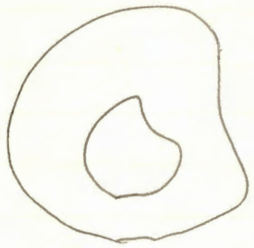
$$\therefore\ R_A = \frac{bP}{a+b} \quad \text{---(2)}$$

(ハ) $\Sigma V = P - R_A - R_B = 0.$

(1), (2) はこれを満たしている。

〈(イ), (ロ), (ハ) のうちの2つの式より求める。〉

(3)



$\left\{\begin{array}{l}\text{中空を含む } A \\ \text{中空だけ } B\end{array}\right.$

$$\begin{cases} G_A = \int_A y\,dA = y_A \cdot A \\ G_B = \int_B y\,dB = y_B \cdot B \end{cases}$$

(y_A, y_B は軸から図心までのキョリ)

ここで中空をのぞいた部分の断面——を G_0、す~~るを~~、図心までの位置を y_0 とすると

$$y_0 = \frac{y_A A - y_B B}{A - B} \quad \cdots\cdots (1)$$

ここで

$$G_A - G_B = y_A A - y_B B \quad \cdots\cdots (2)$$

(1) (2) をくらべると

$$G_0 = y_0 (A - B) = y_A A - y_B B = G_A - G_B$$

$= z$

(4) $I_x = \int_{-\frac{1}{3}h}^{\frac{2}{3}h} y^2 \left(\frac{b}{h} \frac{y_2 - y}{h} \right) dy$

$= \frac{b}{h} \left[\frac{2}{9} h y^3 - \frac{y^4}{4} \right]_{-\frac{h}{3}}^{\frac{2}{3}h}$

$= \frac{bh^3}{36}$ $\cancel{I_{x'} = I_x + \left(\frac{h}{3}\right)^2 \frac{bh}{2}}$

上縁 $W_1 = \frac{I}{y_2} = \frac{bh^2}{24}$ // 下縁 $W_2 = \frac{I_x}{y_1} = \frac{bh^2}{12}$ //

$r_x = \sqrt{\frac{I_x}{\frac{bh}{2}}} = \sqrt{\frac{bh^3}{36} \cdot \frac{2}{bh}} = \sqrt{\frac{h^2}{18}} = \frac{h}{\sqrt{18}} = 0.236h$

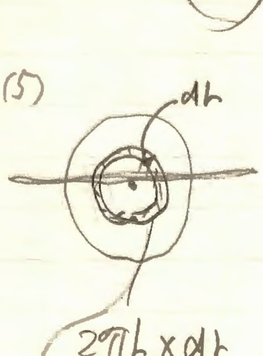
または

$I_{x'} = \frac{bh^3}{36} + \left(\frac{h}{3}\right)^2 \cdot \frac{bh}{2} = \frac{bh^3}{36} + \frac{bh^3}{18}$

$= \frac{3bh^3}{36} = \frac{bh^3}{12}$

よって $r_{x'} = \sqrt{\frac{\frac{bh^3}{12}}{\frac{bh}{2}}} = \sqrt{\frac{bh^3}{12} \cdot \frac{2}{bh}} = \sqrt{\frac{h^2}{6}} = \frac{h}{\sqrt{6}} = 0.408h$

(5)



断面二次極モーメント

$I_p = \int_A r^2 dA = \int_0^{\frac{d}{2}} r^2 (2\pi r\, dr)$

$= 2\pi \left[\frac{r^4}{4} \right]_0^{\frac{d}{2}}$

$= \frac{\pi}{2} \times \frac{d^4}{16} = \frac{\pi d^4}{32}$

$\left(\because \int_A r^2 dA = \int_A (x^2 + y^2) dA = \int_A x^2 dA + \int_A y^2 dA \right)$

ここで $I_p = I_x + I_y$

又 $I_x = I_y$ より

$I_x = \frac{\pi d^4}{64}$

$$\begin{cases} G_A = y_A \cdot A \\ G_B = y_B \cdot B \end{cases}$$

y_A, y_B は軸から図心までの距離
A, B は面積.

ここで、中空部をのぞく図形の断面一次モーメントを G_0、図心までの距離を y_0 とすると.

[定理2]より $y_0 = \dfrac{y_A A - y_B B}{A - B}$ ・・・(1)

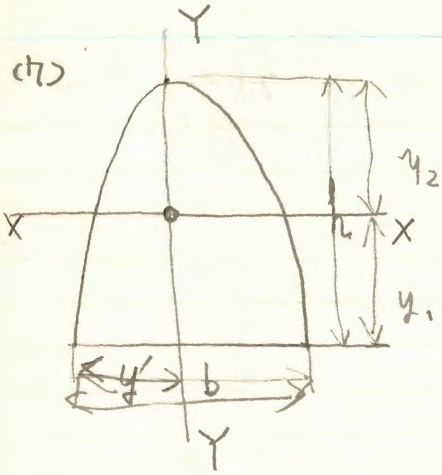
又、$G_A - G_B = y_A A - y_B B$ ・・・(2)

(1), (2) をくらべると.

$$\underbrace{G_0 = y_0(A-B)}_{\text{定義}} = \underbrace{y_A A - y_B B}_{\text{(1)より}} = \underbrace{G_A - G_B}_{\text{(2)より}}$$

$\therefore G_0 = G_A - G_B$ //

(7)



$Y = -aX^2 + y_2$

ここで

$Y = y_1$ のとき $X = \pm y'$

より

$y_1 = -a y'^2 + y_2$

$\therefore a = \frac{y_2 - y_1}{y'^2} = \frac{4h}{5b^2}$

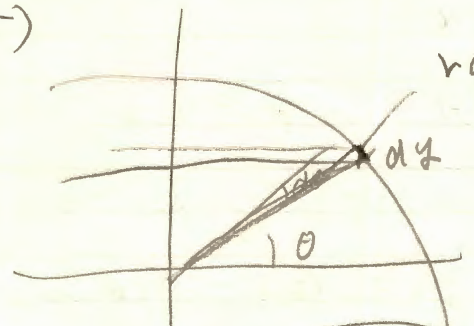
$= \sqrt{\frac{h}{5} \cdot \frac{4}{b^2}} = \sqrt{\cancel{\frac{4h}{5b^2}}} \cancel{\frac{2}{b}\sqrt{\frac{h}{5}}}$

$\therefore X = \cancel{Y - y_2}$

$\pm \sqrt{\frac{(y_2 - Y)}{\cancel{a}} \cancel{\frac{b}{2}\sqrt{\frac{5}{h}}} \frac{5b^2}{4h}}$

$\therefore I_x = \int_{y_1}^{y_2} 2y^2 \sqrt{(y_2 - Y)\frac{5b^2}{4h}}$

(5-)



$dy = \left(\frac{d}{2} d\theta\right) \cos\theta$

$dA = 2 \cdot \frac{d}{2} \cos\theta \cdot dy$

$= \frac{d^2}{2} \cos^2\theta \, d\theta$

$= \int \frac{1 - \cos 4\theta}{2} d\theta$

$\left(\frac{\sin 2\theta}{2}\right)^2$

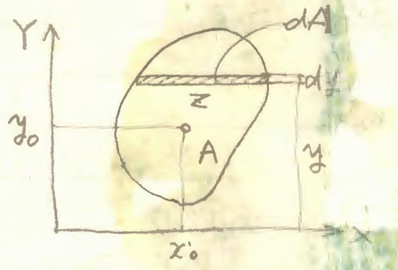
$I_x = 2\int_0^{\frac{\pi}{2}} \frac{d^2}{4} \sin^2\theta \cdot \frac{d^2}{2} \cos^2\theta \, d\theta$

$\cancel{= \frac{1}{4} \cdot \frac{1}{3} \sin^3 2\theta} \cdot \cos 2\theta \cdot 2$

$= \frac{d^4}{4} \left[\cancel{\frac{1}{4} \cdot \frac{1}{3} \sin^3 2\theta} \cos 2\theta \right]$

2章

図心，断面1次モーメント．



$$G_X = \int_A y\,dA = \int_{y_1}^{y_2} y z\,dy$$

$$G_Y = \int_A x\,dA = \int_{x_1}^{x_2} x z\,dx$$

$$x_0 = \frac{G_Y}{A} = \frac{\int_A x\,dA}{A}, \quad y_0 = \frac{G_x}{A} = \frac{\int_A y\,dA}{A}$$

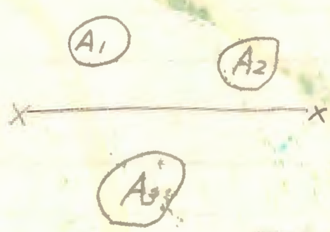
〈定1〉図心を通る軸に対する断面1次モーメントは零

〈定2〉 $x_0 = \dfrac{\Sigma A x}{\Sigma A}$, $y_0 = \dfrac{\Sigma A y}{\Sigma A}$

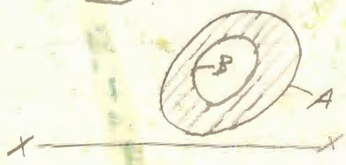
断面2次モーメント

$$I_X = \int_A y^2\,dA = \int_{y_1}^{y_2} y^2 z\,dy$$

$$I_Y = \int_A x^2\,dA = \int_{x_1}^{x_2} x^2 z\,dy$$

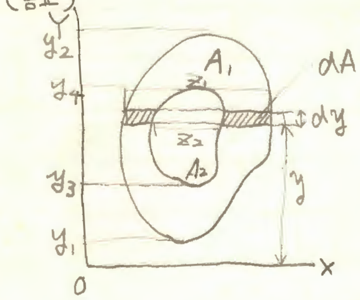


〈定1〉 $I_X = I_{X1} + I_{X2} + \cdots$



〈定2〉 $I_X = I_{XA} - I_{XB}$

(証)

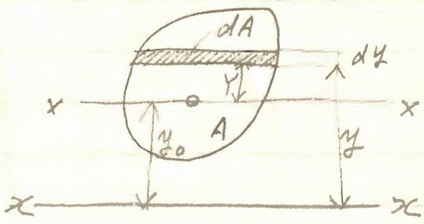


$$I_X = \int_A y^2\,dA = \int_{y_1}^{y_2} y^2 (z_1 - z_2)\,dy$$

$$= \int_{y_1}^{y_2} y^2 z_1\,dy - \int_{y_1}^{y_2} y^2 z_2\,dy$$

$$= \int_{y_1}^{y_2} y^2 z_1\,dy - \int_{y_3}^{y_4} y^2 z_2\,dy = I_{XA_1} - I_{XA_2}$$

〈定3〉 $I_x = I_X + y_0^2 A$



$$I_X = \int Y^2 dA$$
$$= \int (y - y_0)^2 dA$$
$$= \int y^2 dA - 2y_0 \int y dA + y_0^2 \int dA$$
$$= I_x - 2y_0 \times y_0 A + y_0^2 A$$
$$= I_x - y_0^2 A$$

$\left(y_0 = \frac{\int y dA}{A} \text{ " } \right)$

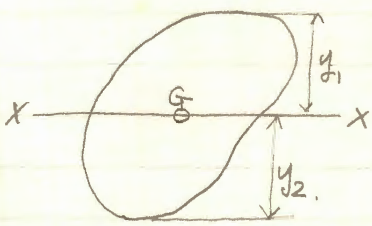
or

$$I_x = \int y^2 dA = \int (Y + y_0)^2 dA$$
$$= \int Y^2 dA + 2y_0 \int Y dA + y_0^2 \int dA$$
$$= I_X + y_0^2 A$$

◎ 断面係数.

上縁に対する $W_1 = I_X / y_1$

下縁に対する $W_2 = I_X / y_2$



◎ 断面2次半径.

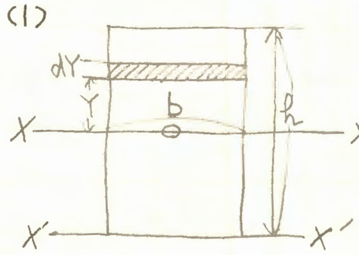
$$r_X = \sqrt{\frac{I_X}{A}}, \quad r_Y = \sqrt{\frac{I_Y}{A}}$$

— X軸に対する図形の断面2次半径.

▨ 種々の図形の各常数を求める。

（断面2次モーメント，断面係数，断面2次半径）

(1)



$$I_x = \int_{-h/2}^{h/2} Y^2 dA = \int_{-h/2}^{h/2} Y^2 \cdot b\,dY$$

$$= b\left[Y^3/3\right]_{-h/2}^{h/2} = bh^3/12.$$

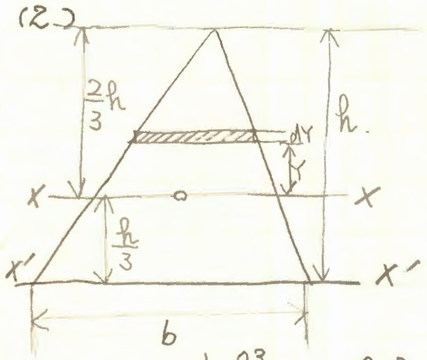
$$I_{x'} = I_x + (h/2)^2 \cdot bh$$

$$= \frac{bh^3}{12} + \frac{bh^3}{4} = \frac{bh^3}{3}$$

$$W_x = \frac{I_x}{h/2} = \frac{bh^2}{6}$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{bh^3/12}{bh}} = \frac{h}{\sqrt{12}}, \quad r_{x'} = \sqrt{\frac{bh^3/3}{bh}} = \frac{h}{\sqrt{3}}$$

(2)



$$I_x = \int_{-h/3}^{\frac{2}{3}h} Y^2 dA = \int_{-\frac{h}{3}}^{\frac{2}{3}h} Y^2 \cdot b \cdot \frac{\frac{2}{3}h - Y}{h}\, dY$$

$$= \frac{b}{h}\int_{-\frac{h}{3}}^{\frac{2}{3}h} \left(\frac{2}{3}hY^2 - Y^3\right) dY$$

$$= \frac{b}{h}\left[\frac{2}{9}hY^3 - \frac{1}{4}Y^4\right]_{-\frac{h}{3}}^{\frac{2}{3}h} = \frac{bh^3}{36}$$

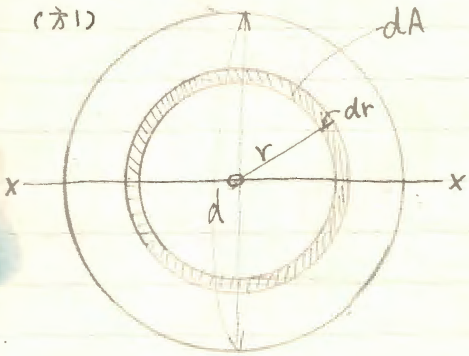
$$I_{x'} = \frac{bh^3}{36} + \left(\frac{h}{3}\right)^2 \cdot \frac{bh}{2} = \frac{bh^3}{12}$$

上縁 $W_{x1} = bh^3/36 \Big/ \frac{2}{3}h = \frac{bh^2}{24}$ 下縁 $\frac{bh^3}{36} \Big/ \frac{h}{3} = \frac{bh^2}{12}$

$$r_x = \sqrt{\frac{\frac{bh^3}{36}}{\frac{bh}{2}}} = \frac{h}{\sqrt{18}}, \quad r_{x'} = \sqrt{\frac{\frac{bh^3}{12}}{\frac{bh}{2}}} = \frac{h}{\sqrt{6}}$$

(3)

(オ1)



断面2次極モーメント

$$I_p = \int_A r^2 dA$$

$$= \int_0^{d/2} r^2 \cdot 2\pi r dr = \int_0^{d/2} 2\pi r^3 dr$$

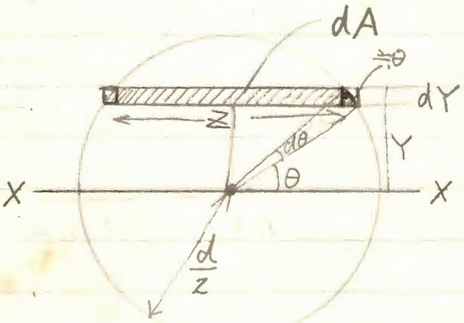
$$= \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

ここで $\begin{cases} I_p = I_x + I_Y \\ I_x = I_Y \end{cases}$

$$\therefore I_x = \frac{\pi d^4}{64}$$

$$W_x = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$$

$$r_x = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}}} = \frac{d}{\sqrt{16}} = \frac{d}{4}$$



又は.

$$I_x = \int_A Y^2 dA = \int_A y^2 Z \cdot dy$$

ここで $dy = \left(\frac{d}{2} d\theta\right)\cos\theta$

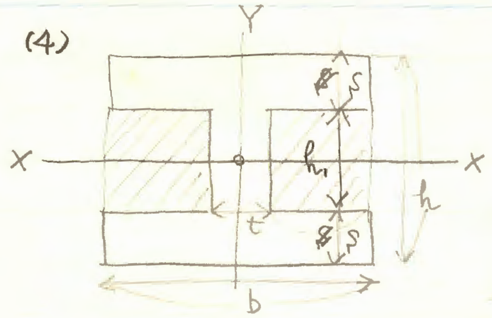
又 $Z = \cancel{2} \cdot \frac{d}{2}\cos\theta$, $Y = \frac{d}{2}\sin\theta$

よって $I_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d^2}{4}\sin^2\theta \cdot d\cos\theta \cdot \frac{d}{2}\cos\theta d\theta$

$$= \frac{2 \times d^4}{8}\int_0^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2} \cdot \frac{1+\cos 2\theta}{2} d\theta = \frac{d^4}{4}\int_0^{\frac{\pi}{2}} \frac{\sin^2 2\theta}{4} d\theta$$

$$= \frac{d^4}{16} \times \frac{1}{2}\int_0^{\frac{\pi}{2}} (1-\cos 4\theta) d\theta = \frac{\pi d^4}{64} \;/\!/$$

(4)



$$I_x = \frac{bh^3}{12} - \frac{1}{12}\cdot(b-t)\cdot(h-2S)^3$$

$$= \frac{bh^3 - h_1^3(b-t)}{12}$$

$$I_Y = \frac{2Sb^3 + h_1 t^3}{12}$$

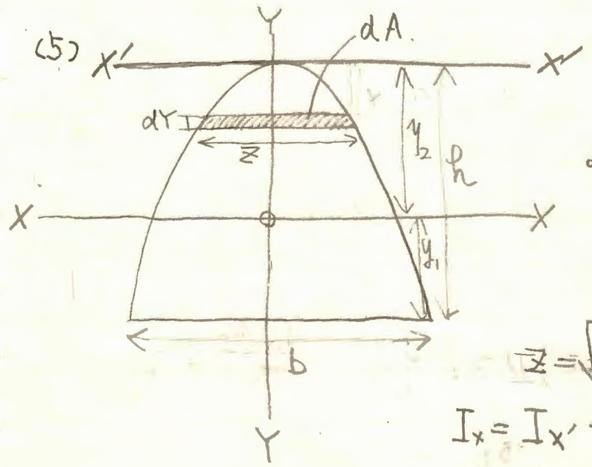
$$\left(\begin{array}{l} A = bh - h_1(b-t) \\ \text{or } A = t h_1 + 2Sb \end{array}\right)$$

$$W_X = \frac{I_X}{h/2} = \frac{bh^3 - h_1^3(b-t)}{6h}$$

$$W_Y = \frac{I_Y}{b/2} = \frac{2Sb^3 + h_1 t^3}{6b}$$

$$r_X = \sqrt{\frac{bh^3 - h_1^3(b-t)}{12\{bh - h_1(b-t)\}}} \qquad r_Y = \sqrt{\frac{2Sb^3 + h_1 t^3}{12\{bh - h_1(b-t)\}}}$$

(5)



$$y = ax^2$$

$$x = \pm b/2 \longrightarrow y = -h \quad \therefore a = -\frac{4h}{b^2}$$

$$\text{よって } A = bh - 2\int_0^{\frac{b}{2}}\left(+\frac{4h}{b^2}x^2\,dx\right)$$

$$= bh - \frac{8h}{b^2}\cdot\frac{1}{3}\cdot\frac{b^3}{8} = \frac{2bh}{3}$$

$$z = \sqrt{\frac{b^2 y}{4h}} = \frac{\cancel{2}b}{\cancel{2}\sqrt{h}}\,y^{\frac{1}{2}}$$

$$I_x = I_{x'} - \frac{2}{3}bh\cdot\left(\frac{3}{5}h\right)^2 = \int_0^h \frac{2b}{2\sqrt{h}}\,y^{\frac{1}{2}}\,y^2\,dy - \frac{6}{25}bh^3$$

$$= \frac{2b}{2\sqrt{h}}\cdot\frac{2}{7}\cdot h^{\frac{7}{2}} - \frac{6}{25}bh^3 = \left(\frac{2}{7} - \frac{6}{25}\right)bh^3 = \frac{8bh^3}{175}$$

$$y = \frac{4h}{b^2}x^2$$

$$I_Y = 2\int_0^{\frac{b}{2}} x^2 \cdot (h - y)\,dx$$

$$= 2\int_0^{\frac{b}{2}} x^2\left(h - \frac{4h}{b^2}x^2\right)dx$$

$$= 2h\left[\frac{x^3}{3} - \frac{4}{5b^2}x^5\right]_0^{b/2}$$

$$= 2h\left(\frac{b^3}{24} - \frac{b^3}{40}\right) = \frac{5-3}{60}hb^3 = \frac{hb^3}{30} \quad //$$

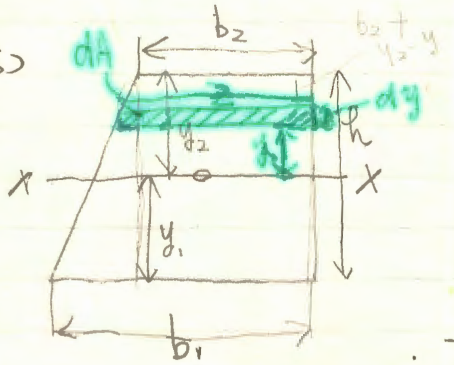
又, $$W_{X2} = \frac{I_X}{y_2} = \frac{8bh^{\cancel{3}2}}{\cancel{175}_{35}}\cdot\frac{\cancel{5}}{3\cancel{h}} = \frac{8bh^2}{105}$$

$$W_Y = \frac{I_Y}{\frac{b}{2}} = \frac{hb^3}{30}\cdot\frac{2}{b} = \frac{hb^2}{15}$$

$$r_X = \sqrt{\frac{I_X}{A}} = \sqrt{\frac{\cancel{8}^4 \cancel{b} h^{\cancel{3}2}}{175}\cdot\frac{3}{\cancel{2}\cancel{b}\cancel{h}}} = \sqrt{\frac{12}{175}}\,h$$

$$r_Y = \sqrt{\frac{I_Y}{A}} = \sqrt{\frac{hb^3}{30}\cdot\frac{3}{2bh}} = \frac{b}{\sqrt{20}} = \frac{b}{2\sqrt{5}}$$

(6)



$$\begin{cases} A = \dfrac{h}{2}(b_1 + b_2) \\ y_1 = \dfrac{h}{3}\cdot\dfrac{b_1 + 2b_2}{b_1 + b_2}, \quad y_2 = \dfrac{h}{3}\cdot\dfrac{2b_1 + b_2}{b_1 + b_2} \end{cases}$$

$$z = b_2 + (b_1 - b_2)\cdot\frac{y_2 - y}{h}$$

$$\therefore I_X = \int_A y^2 dA = \int y^2 z\,dy = \int_{-y_1}^{y_2} y^2\left\{b_2 + (b_1 - b_2)\frac{y_2 - y}{h}\right\}dy$$

$$= \left[\left\{b_2 + (b_1 - b_2)\cdot\frac{y_2}{h}\right\}\frac{y^3}{3} - \frac{b_1 - b_2}{4h}y^4\right]_{-y_1}^{y_2}$$

$$= \left\{b_2 + (b_1 - b_2)\frac{\frac{h}{3}\frac{2b_1 + b_2}{b_1 + b_2}}{h}\right\}\frac{y_2^3 + y_1^3}{3} - \frac{b_1 - b_2}{4h}\cdot(y_2^4 - y_1^4)$$

2章の問題.

(1) 図心と重心のちがいは.

(2)

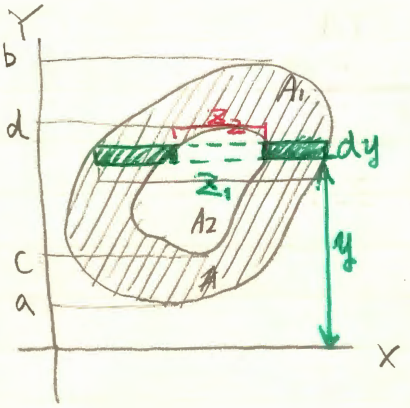
$$x_0 = \frac{\Sigma Ax}{\Sigma A} = \frac{3\cdot 12 + 1\cdot 18}{30} = \frac{54}{30} = 1.8\,\mathrm{cm}$$

$$y_0 = \frac{\Sigma Ay}{\Sigma A} = \frac{1\cdot 12 + 6.5\cdot 18}{30} = 4.3\,\mathrm{cm}$$

$$I_x = \frac{6\cdot 2^3}{12} + 1^2\cdot 12 + \frac{2\cdot 9^3}{12} + 6.5^2\cdot 18 = 898\,\mathrm{cm}^4$$

$$I_Y = \frac{2\cdot 6^3}{12} + 3^2\cdot 12 + \frac{9\cdot 2^3}{12} + 1^2\cdot 18 = 168\,\mathrm{cm}^4$$

(3)



$$G_X = \int_A y\,dA$$

$$= \int_a^b y\cdot(z_1 - z_2)\,dy$$

$$= \int_a^b y z_1\,dy - \int_a^b y z_2\,dy$$

$$= \int_a^b y z_1\,dy - \int_c^d y z_2\,dy$$

$$= G_{X_1} - G_{X_2}$$

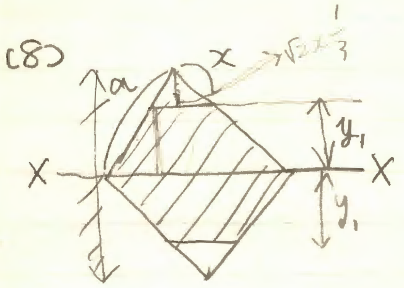
(6)

$$I_x = \frac{\pi(2r)^4}{64} - \frac{\pi r^4}{64} = \frac{15\pi r^4}{64}$$

$$I_y = \frac{16\pi r^4}{64} - \left\{\frac{\pi r^4}{64} + \left(\frac{r}{2}\right)^2\cdot \pi\left(\frac{r}{2}\right)^2\right\}$$

$$= \frac{15\pi r^4}{64} - \frac{\pi r^4}{16} = \frac{11\pi r^4}{64}$$

(8)



$$W_x = \frac{I_x}{y_1}$$

より.
図形を切りとれば面積が小さくなるから当然 I_x は小さくなるが y_1 を小さくすればそうともかぎらない。
そこで、図のように切りとらねばならない。

一辺をa 図の部分を x とすると、

二つの三角形を切りとって減じる断面二次モーメント I は.

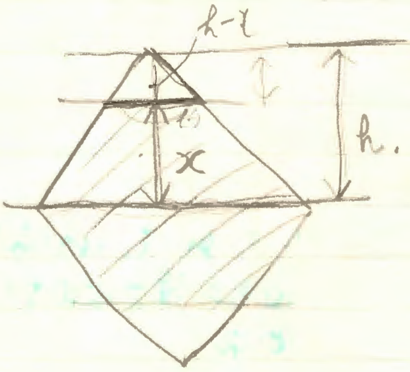
$$I = 2\left[\frac{\sqrt{2}x\cdot(\sqrt{2}x)^3}{12\,6} + \left(\frac{\sqrt{2}}{3}x + \frac{a-x}{\sqrt{2}}\right)^2 \frac{x^2}{2}\right]$$

$$= \frac{2}{3}x^4 + \frac{x^2}{18}(2x+3a-3x)^2$$

$$= \frac{2}{3}x^4 + \frac{x^2}{18}(3a-x)^2$$

また $y = \dfrac{a-x}{\sqrt{2}}$

よって $W = \dfrac{\sqrt{2}}{a-x}\left\{\dfrac{2}{3}x^4 + \dfrac{x^2}{18}(3a-x)^2\right\}$



左のようにおく

減じる I.

$$I = 2\left[\frac{2(h-x)(h-x)^3}{12} + \left(\frac{h}{3}+\frac{2}{3}x\right)^2 (h-x)^2\right]$$

$x+(h-x)\frac{1}{3}$

$$= \frac{(h-x)^4}{3} + \frac{2}{9}(h+2x)^2(h-x)^2$$

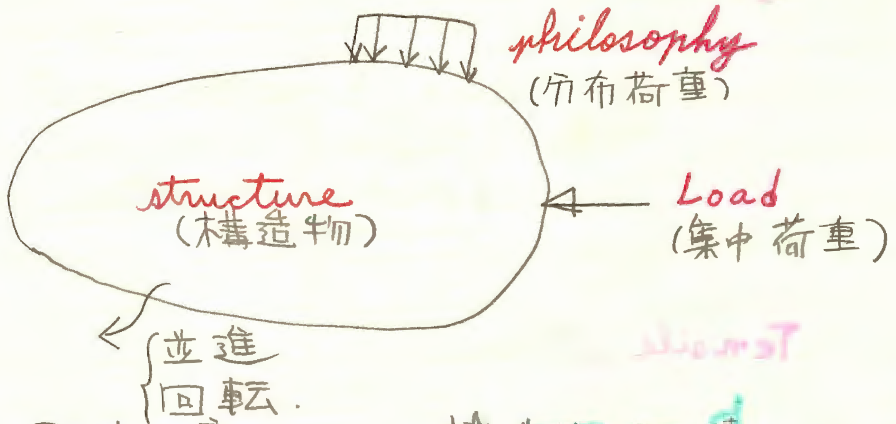
$$= \frac{(h-x)^2(11h^2+2hx+11x^2)}{9}$$

$$W = \frac{I_x - I}{x}$$

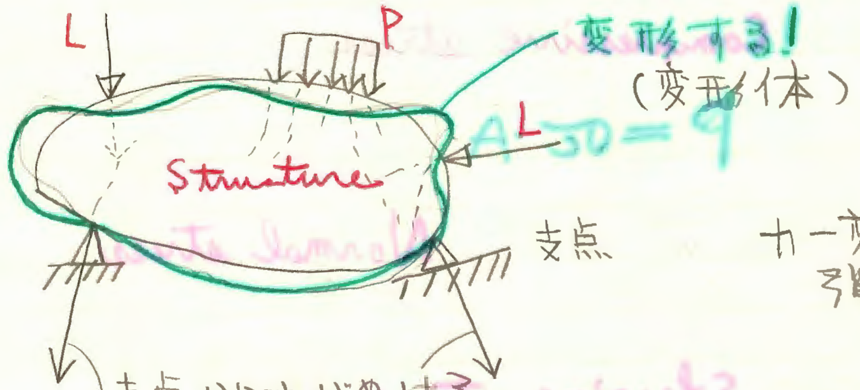
3章 材料の性質と強さ.

● 力と応力

〈構造の力と変形の流れ〉



剛体：変形しない構造物 → 仕事のエネルギーになる。



支点から力がぬける

⇓

重みとして感じる(反力)

力ー変形比例.
弾性体

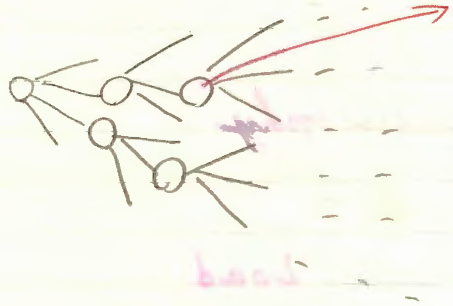
⇕

非弾性体

力 { 内力（曲げモーメント，せん断力，軸力）
外力（荷重，反力）→ 支点がふえれば個々の反力は小さくなる!

→ 構造物内に仮想的に働く力

Decision Tree



Turning Point

$\boxed{\text{Rule: Choose the most difficult way in many respects.}}$

◎引張応力 Tensile stress

$$P = \sigma_t \cdot A$$

◎圧縮応力 Compressive stress

$$P = \sigma_c \cdot A$$

σ_t, σ_c は共に Normal stress
(垂直応力)

◎せん断応力 Shearing stress

$$S = \tau \cdot A$$

τ : 接線応力 Tangential stress

◎曲げ応力 Bending stress

私にはよくわからない It is Greek to me.
= I cannot make head ~~of~~ or tail out of it.

ひずみ

〈[illegible]直〉

縦ひずみ Longitudinal strain

$$\varepsilon = \frac{\Delta l}{l} \quad \left(\begin{array}{l}\text{引張} > 0 \\ \text{圧縮} < 0\end{array}\right)$$

横ひずみ Lateral strain

$$\beta = \frac{\Delta d}{d} \quad \left(\begin{array}{l}\text{引張} < 0 \\ \text{圧縮} > 0\end{array}\right)$$

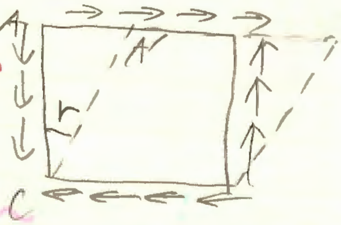
ポアソン比 Poisson's ratio

$$\beta = -\nu\varepsilon = -(1/m)\varepsilon$$

m : Poisson's number.

せん断ひずみ Shearing strain

$$\gamma = \frac{AA'}{AC} = \angle ACA' \ (\text{ラジアン})$$



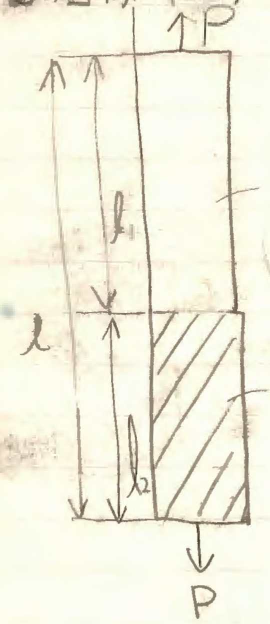
Hooke's Law

$E = \sigma/\varepsilon$, $G = \tau/\gamma$

- E : Modulus of elasticity or Young's modulus
- G : Shear modulus

$$\boxed{E = \frac{\sigma}{\varepsilon} = \frac{l P}{\Delta l A}}$$ 重要

◎合成柱 ＜直列＞



E_1

E_2

断面積一定

この場合 応力は一定（$\sigma = \frac{P}{A}$）

（Δl と ε を求めてみる。）

物体1において伸び Δl_1 は

$$\Delta l_1 = \frac{l_1 P}{E_1 A}$$

物体2において伸び Δl_2 は

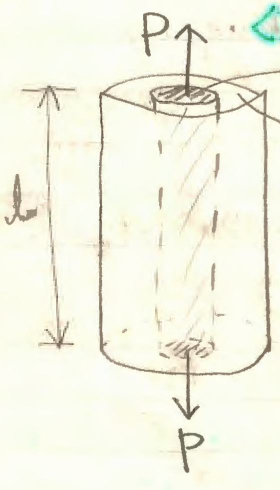
$$\Delta l_2 = \frac{l_2 P}{E_2 A}$$

よって全体の伸び

$$\Delta l = \Delta l_1 + \Delta l_2 = \frac{P}{A}\left(\frac{l_1}{E_1} + \frac{l_2}{E_2}\right)$$

ここで $\varepsilon = \frac{\Delta l}{l} = \frac{P}{lA}\left(\frac{l_1}{E_1} + \frac{l_2}{E_2}\right)$

＜並列＞



E_2, A_2

E_1, A_1

この場合

伸び Δl は一定.

応力は異なる.

応力を σ_1, σ_2 とすると、ひずみ ε は.

$$\varepsilon = \frac{E_1}{\sigma_1} = \frac{E_2}{\sigma_2} \quad \text{——①}$$

また $P = \sigma_1 A_1 + \sigma_2 A_2$ ——②

①より $\sigma_2 = \frac{E_2}{E_1}\sigma_1$

②に代入. $\sigma_1\left(A_1 + A_2\frac{E_2}{E_1}\right) = P$

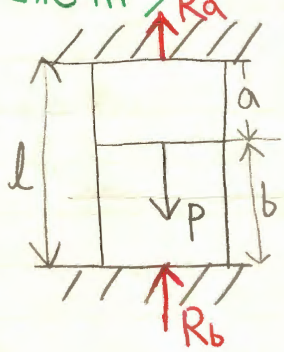
$$\therefore \sigma_1 = \frac{P}{A_1 + A_2\frac{E_2}{E_1}}, \quad \sigma_2 = \frac{P\frac{E_2}{E_1}}{A_1 + A_2\frac{E_2}{E_1}}$$

or $\sigma_1 = \frac{P}{A_1 + A_2\frac{E_2}{E_1}}$ $\quad \sigma_2 = \frac{P}{\frac{E_1}{E_2}A_1 + A_2}$

又 $\varepsilon = E_1 \cdot \dfrac{A_1 + A_2\frac{E_2}{E_1}}{P} = \dfrac{E_1A_1 + E_2A_2}{P}$

$\Delta l = \dfrac{l(E_1A_1 + E_2A_2)}{P}$

<応用>



Ra, Rb なる反力がかかるとする。
(ヤング率 E, 断面積 A とおく)
つりあいより $R_a + R_b = P$ ---- ①

また、P と反力による下半分の伸び Δb は、

$\Delta b = \dfrac{b(R_b - P)}{EA}$

P と反力 RA による上半分の伸び Δa は

$\Delta a = \dfrac{a(P - R_a)}{EA}$

全体の伸びは零だから

$\Delta a + \Delta b = \dfrac{P(a-b) - aR_a + bR_b}{EA} = 0$

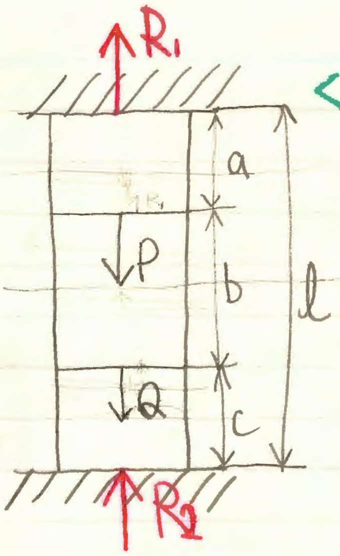
$\therefore aR_a - bR_b = P(a-b)$ ---- ②

①を代入して

$R_a(a+b) = b\{P + P(a-b)\} = Pa$

$\therefore R_a = \dfrac{Pa}{l}$

①に代入して $R_b = \dfrac{Pb}{l}$



〈応用〉 E・A

つりあいから

$$R_1 + R_2 = P + Q \quad \cdots ①$$

$$\begin{cases} \Delta a = \dfrac{a \cdot R_1}{AE} \\ \Delta b = \dfrac{b(R_1 - P)}{AE} \quad \text{or} \quad \Delta b = \dfrac{b(Q - R_2)}{AE} \\ \Delta c = \dfrac{-c\,R_2}{AE} \end{cases}$$

全体の伸びは零であるから、

$$\Delta a + \Delta b + \Delta c = \frac{1}{AE}\left\{(a+b)R_1 - cR_2 - bP\right\} = 0$$

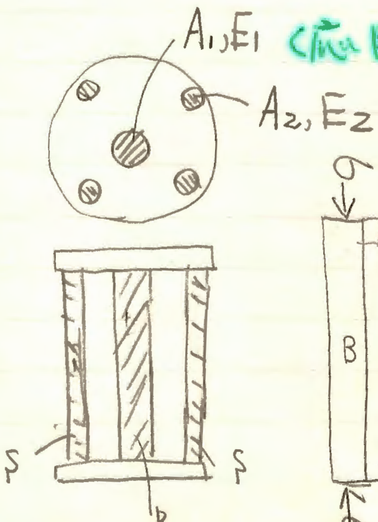
$$\therefore (a+b)R_1 - cR_2 = bP \quad \cdots ②$$

①×c+② より

$$(a+b+c)R_1 = Pc + Qc + Pb$$

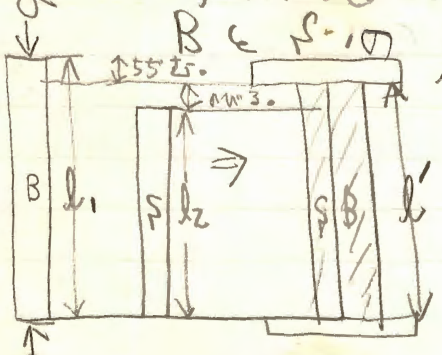
$$\therefore R_1 = \frac{(b+c)P + cQ}{l}$$

①に代入 $R_2 = P + Q - R_1 = \dfrac{aP + (a+b)Q}{l}$



〈応用〉

棒Bに軸圧縮応力σを加えて4本の支柱Sと2枚の剛性板の間に入れてはなす。
BとSの応力は？，結局の長さ？



軸圧縮応力によるBの縮みλ_0は．〈y書〉

$$\lambda_1 = \sigma \cdot \frac{l_1}{E_1} \quad \text{------} \quad ①$$

応力σ_1 σ_2，力P_1 P_2とする（つりあったときの）

Sの伸びλは

$$\lambda = \sigma_2 \cdot \frac{l_2}{E_2} \quad \text{---} \quad ②$$

又，Bの元の長さよりちぢんだ分$\lambda_1 - \lambda$は

$$\lambda_1 - \lambda = \sigma_{\cancel{B}} \cdot \frac{l_1}{E_1} \quad \text{---} \quad ③$$

?

①，②，③より

$$\sigma \frac{l_1}{E_1} - \sigma_2 \frac{l_2}{E_2} = \sigma_1 \frac{l_1}{E_1} \quad \text{------} \quad ④$$

ここで $\sigma_2 = \dfrac{P_2}{4A_2}$，$\sigma_1 = \dfrac{P_1}{A_1}$

より $P_2 = P_1$ から

$$4A_2\sigma_2 = A_1\sigma_1 \quad \text{---} \quad ⑤$$

④，⑤より

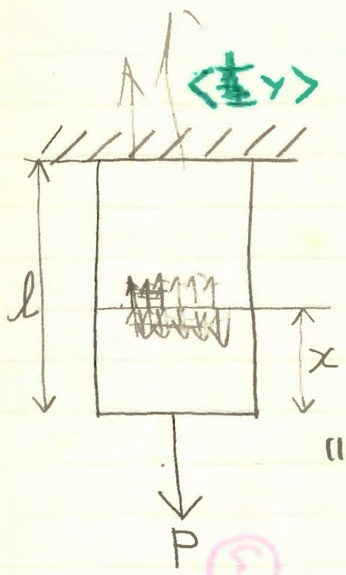
$$\sigma_1 = \frac{4A_2E_2l_1}{4A_2E_2l_1 + A_1E_1l_2}\sigma$$

$$\sigma_2 = \frac{A_1E_2l_1}{4A_2E_2l_1 + A_1E_1l_2}\sigma$$

結局

長さ $l' = l + \lambda = l + \dfrac{\cancel{EA}\,A_1l_2l_1}{4A_2E_2l_1 + A_1E_1l_2}\sigma$

<重γ>



断 A，弾 E，許応 σ_a
単位体積の重さ．

(2) 棒が安全である所要断面積．
(1) 位置 x の応力．
(3) ？

(1)
$$\sigma = \frac{P + A\gamma x}{A} = \frac{P}{A} + \gamma x$$

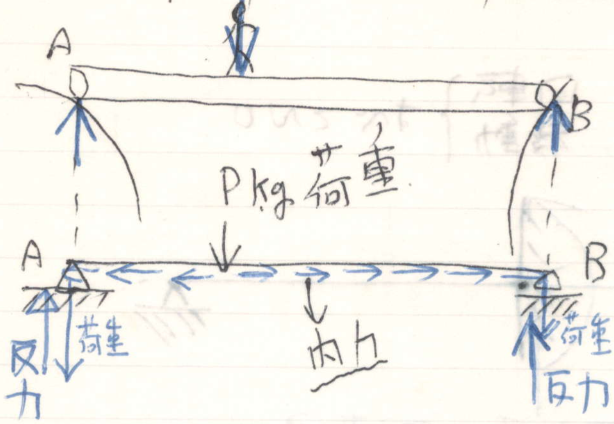
(2) $\sigma \leqq \sigma_a$ より

$$\sigma_a \geqq \frac{P}{A} + \gamma l$$

(∵ 応力最大は $x = l$)

$$A \geqq \frac{P}{\sigma_a - \gamma l}$$

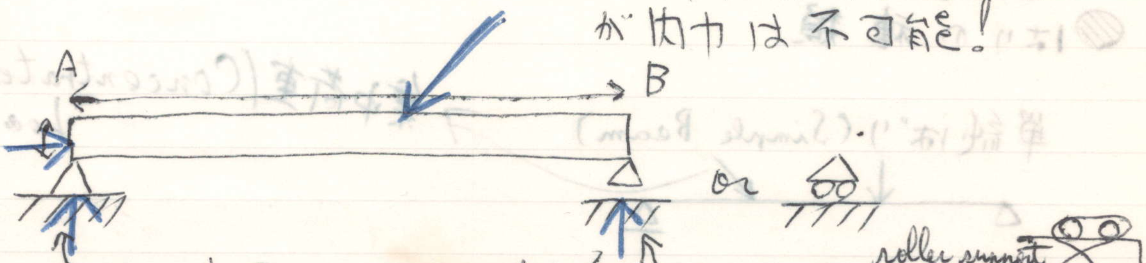
4章　はり (Beam)　12/8



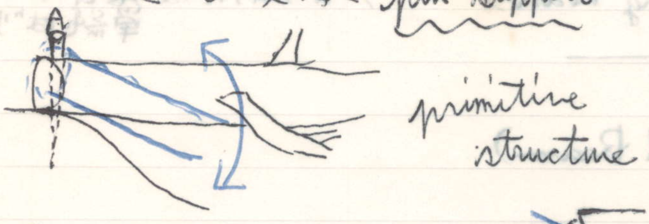
定義
垂直方向の荷重(外力)を
水平方向に伝達する
構造・・・(梁)

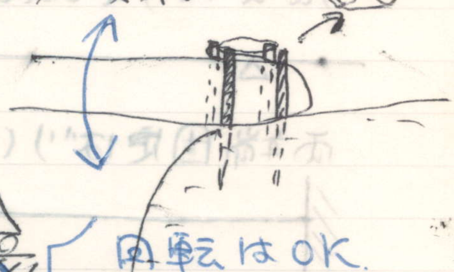
力 { 外力(荷重, 反力)
　　 内力(　?　)

外力は実測可能であるが内力は不可能!



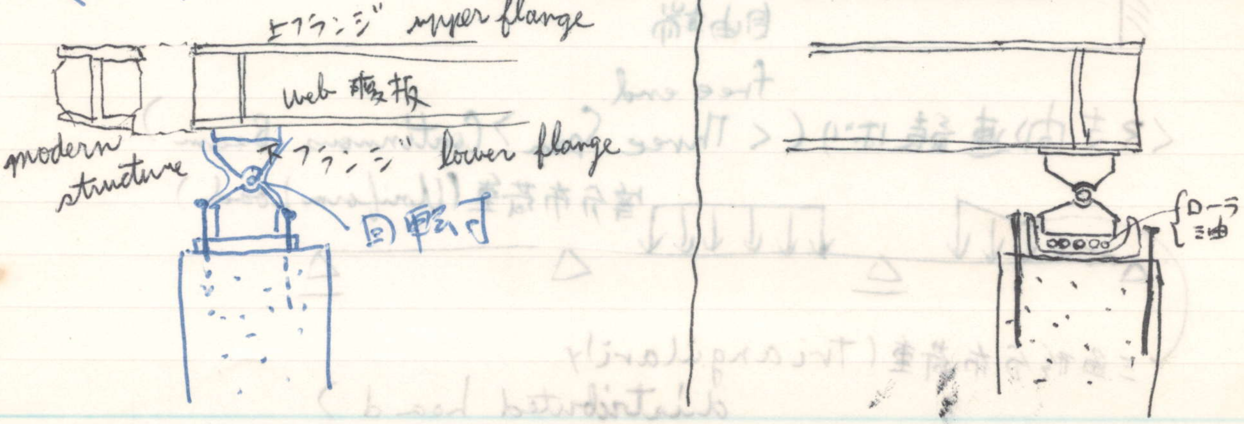
ピン支承 pin support　　ローラー支承

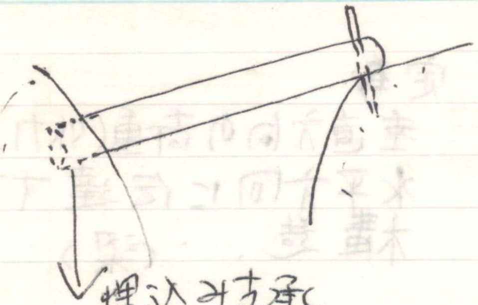




回転はOK
水平・垂直 方向の移動はNO

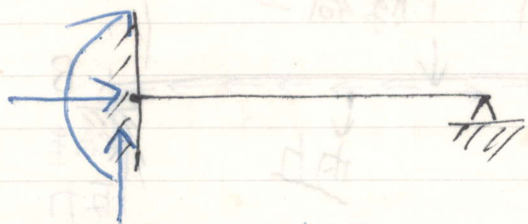
回転はOK
水平方向移動OK
垂直方向移動NO





埋込み支承
固定支承
(fixed end)

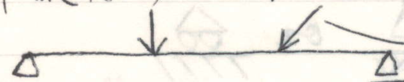
回転
移動 } すべてNO.



反力すべてある。

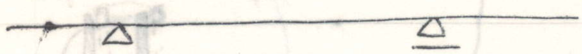
◎はりの種類

単純ばり(Simple Beam)

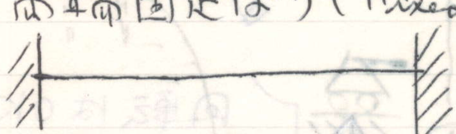


集中荷重(Concentrated Load)

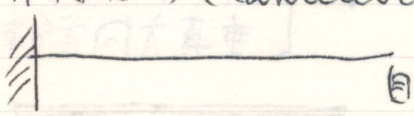
張出ばり(Overhanging Beam) → 力学的には ~~張出ばり~~ 単純ばり。



両端固定ばり(Fixed Beam)

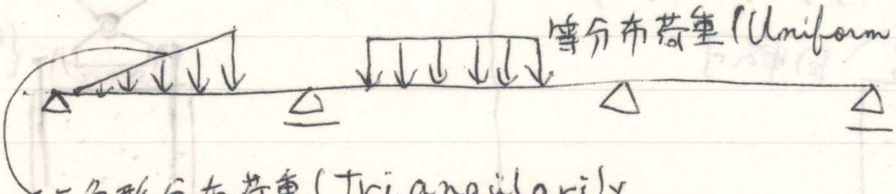


片持ばり(Cantilever Beam)



自由端
free end

<3支間>連続ばり(<Three Span>Continuous Beam)

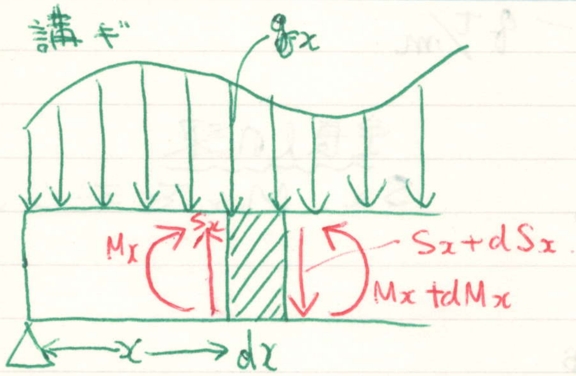


等分布荷重(Uniform Load)

三角形分布荷重(Triangularily distributed Load)

1/19 講ギ

P55



$$\Sigma V = S_x - q_x dx - (S_x + dS_x) = 0.$$

Vertical direction equilibrium

$$\therefore \quad dS_x = -q_x dx$$

$$\Sigma M = M_x + \frac{q_x}{2}dx^2 + (S_x + dS_x)dx - (M_x + dM_x) = 0$$

$$\therefore \quad dM_x = \frac{q_x}{2}dx^2 + S_x dx + dS_x dx$$

2次の微少量を無視すると,

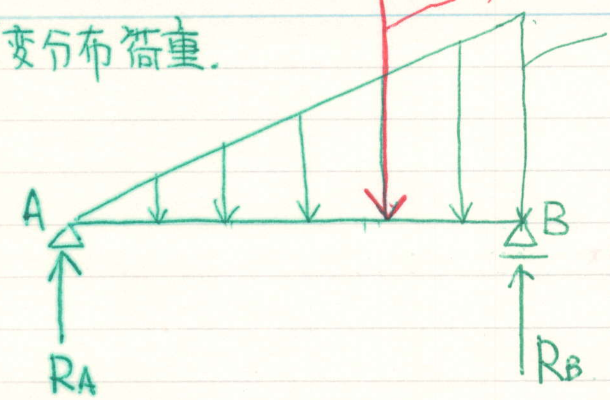
$$\boxed{\begin{gathered} \frac{dM_x}{dx} = S_x \\ \frac{dS_x}{dx} = -q_x \\ \left(\therefore \frac{d^2M_x}{dx^2} = \frac{dS_x}{dx} = -q_x\right) \end{gathered}}$$

計算のcheckに用いる.

教P56 定理をおぼえる.

集中荷重 $\frac{ql}{2}$ t（反力を求める場合だけに分ける）

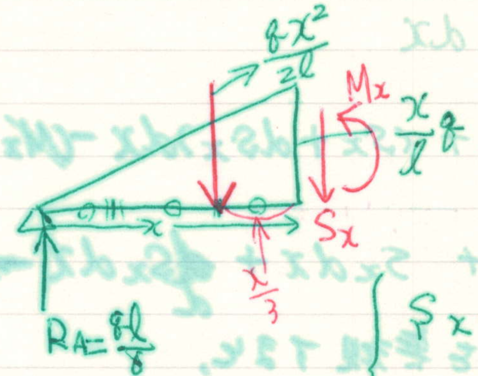
等変分布荷重.



全長 l の梁
S_x, M_x を求む。

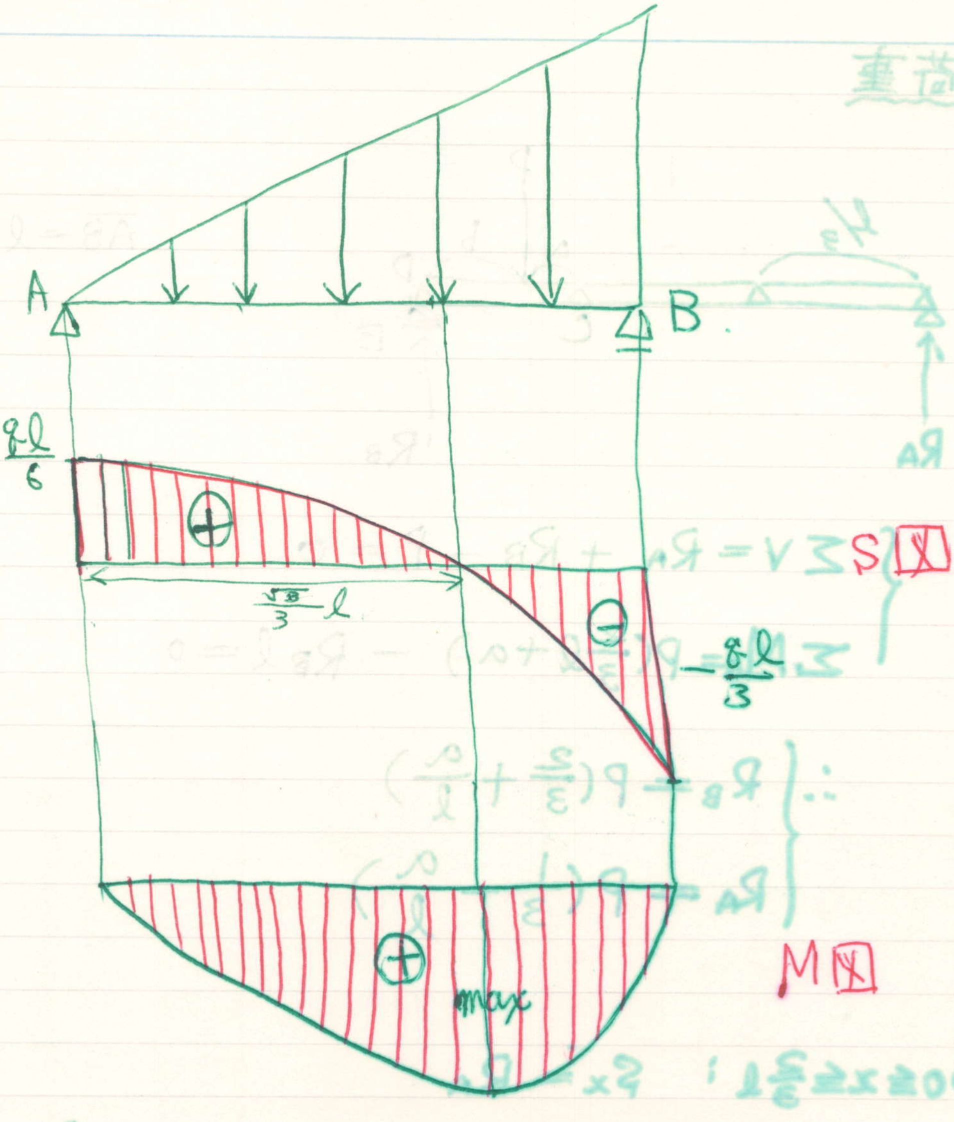
$$\Sigma M_B = R_A l - \frac{ql}{2}\cdot\frac{l}{3} = 0 \qquad \therefore R_A = \frac{ql}{6}$$

$$\therefore R_B = \frac{ql}{2} - \frac{ql}{6} = \frac{ql}{3}$$



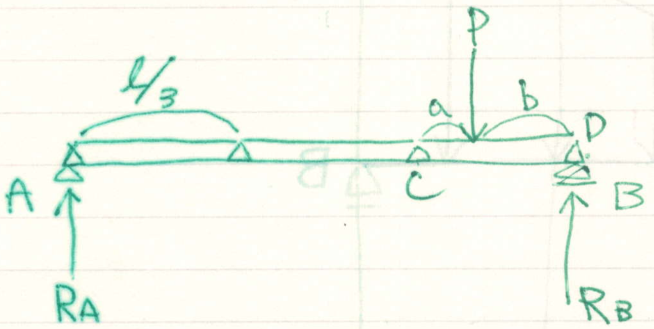
$$\begin{cases} S_x = R_A - \dfrac{qx^2}{2l} \\ M_x = R_A x - \dfrac{qx^3}{6l} \end{cases}$$

$$\therefore \begin{cases} S_x = \dfrac{ql}{6} - \dfrac{qx^2}{2l} \\ M_x = \dfrac{ql}{6}x - \dfrac{q}{6l}x^3 \end{cases}$$



A
B
ql/6
√3/3 l
−ql/3
S図
M図
max

間接荷重



$\overline{AB} = l.$

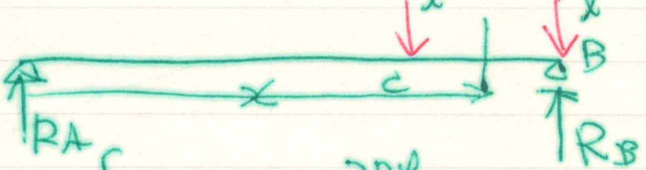
$$\begin{cases} \Sigma V = R_A + R_B - P = 0. \\ \Sigma M_A = P\left(\frac{2}{3}l + a\right) - R_B l = 0 \end{cases}$$

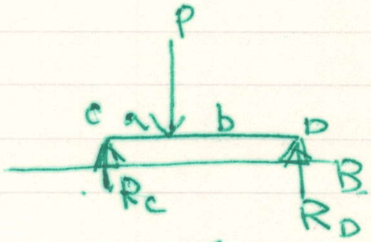
$$\therefore \begin{cases} R_B = P\left(\frac{2}{3} + \frac{a}{l}\right) \\ R_A = P\left(\frac{1}{3} - \frac{a}{l}\right) \end{cases}$$

(i) $0 \leqq x \leqq \frac{2}{3}l$; $S_x = R_A$

$$M_x = R_A x$$

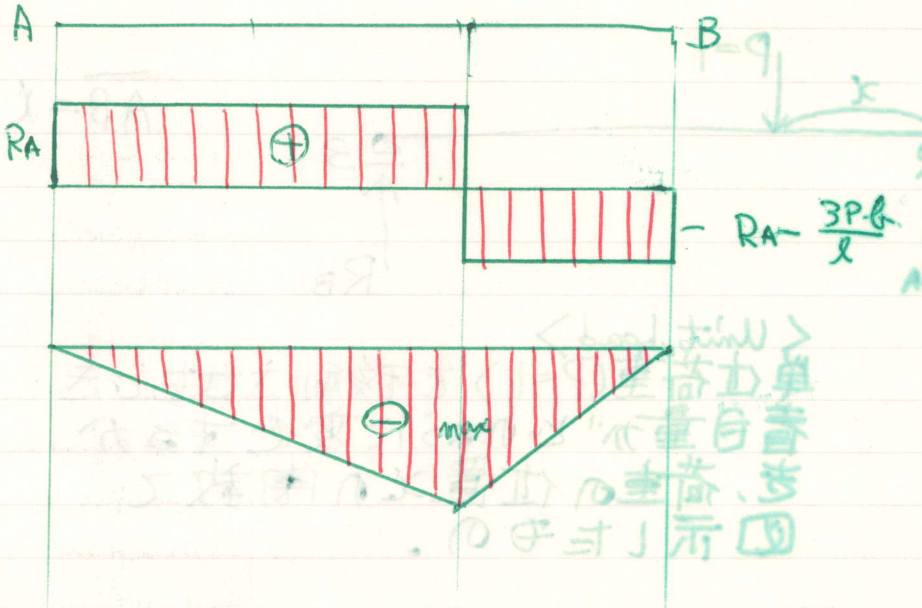
(ii) $\frac{2}{3}l \leqq x \leqq l$;



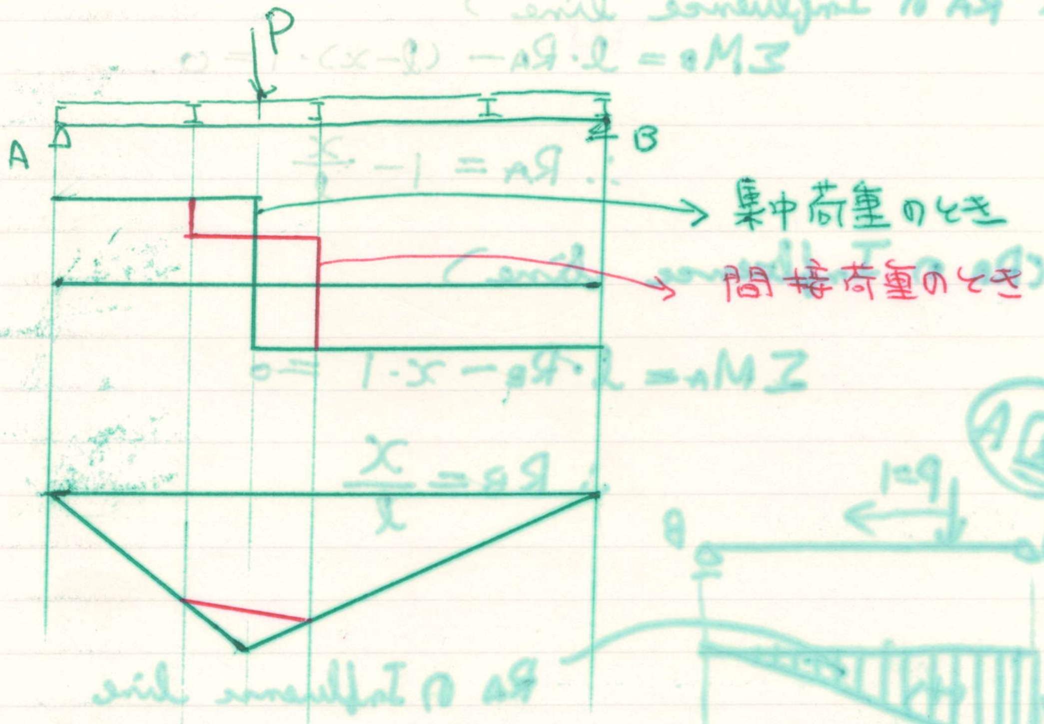


$$\begin{cases} R_C = \frac{Pb}{l/3} = \frac{3bP}{l} \\ R_D = \frac{3aP}{l} \end{cases}$$

$$\begin{cases} S_x = R_A - \frac{3Pb}{l} \\ M_x = R_A x - \frac{3Pb}{l}x \end{cases}$$

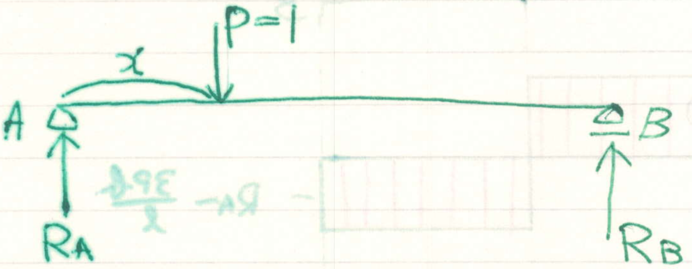


A
B
RA
⊕
RA− 3P·b / l
⊖
max



P
A
B
集中荷重のとき
間接荷重のとき

影響線（Influence line）



$\overline{AB} = l.$

<Unit load>
単位荷重($P=1$)を移動させたとき着目量がどのように変化するかを，荷重の位置xの関数で図示したもの。

（R_A の Influence line）

$$\Sigma M_B = l\cdot R_A - (l-x)\cdot 1 = 0$$

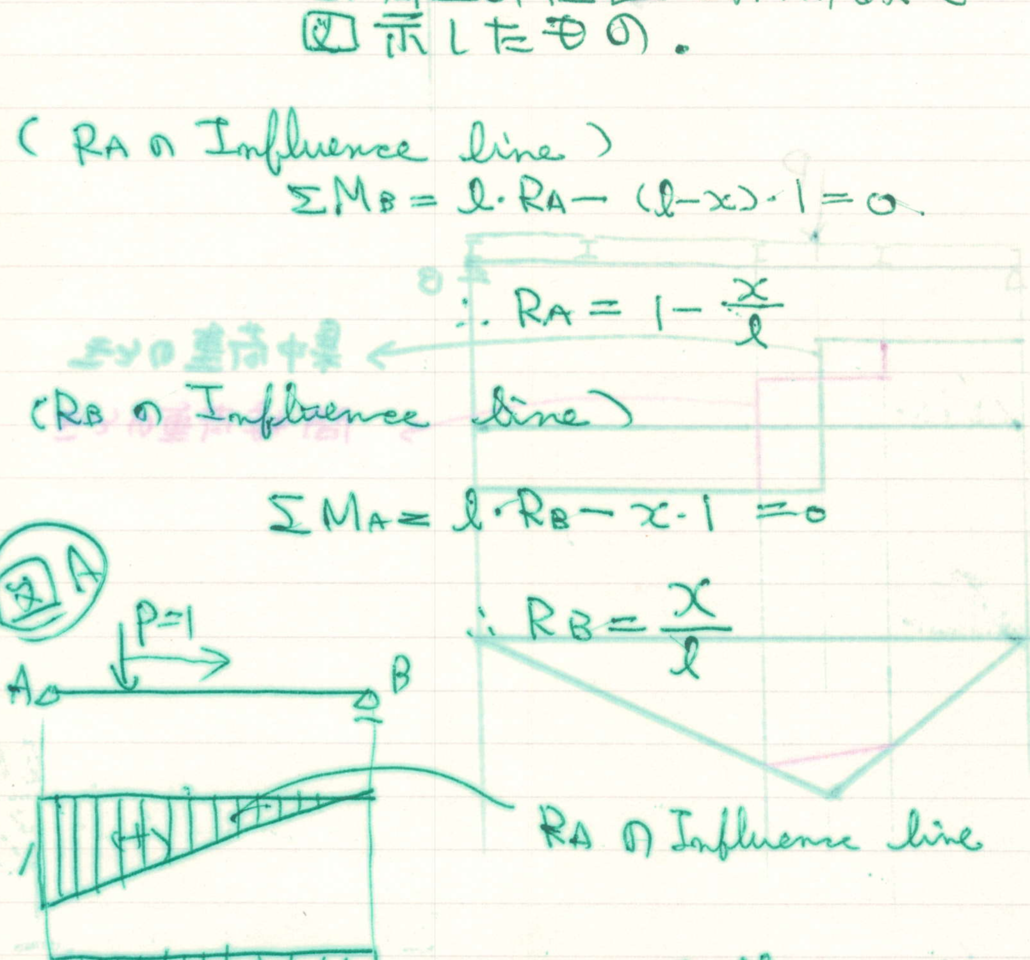
$$\therefore R_A = 1 - \frac{x}{l}$$

（R_B の Influence line）

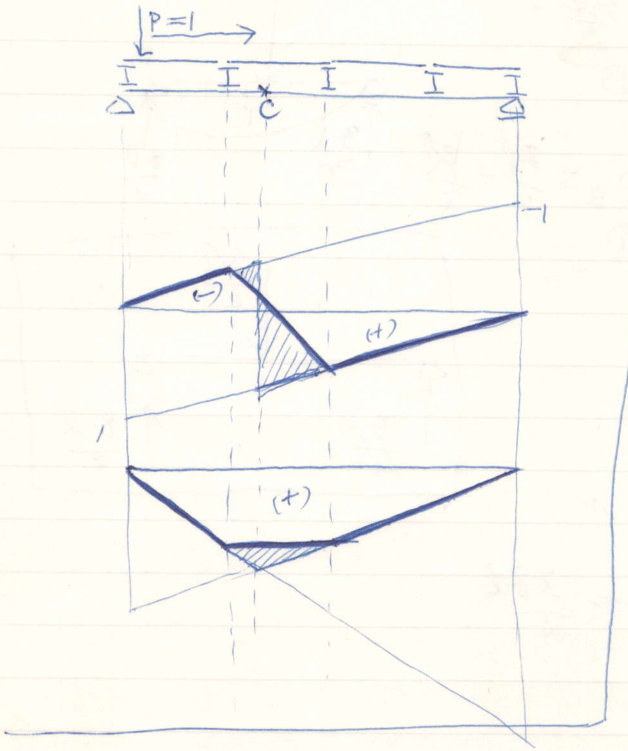
$$\Sigma M_A = l\cdot R_B - x\cdot 1 = 0$$

$$\therefore R_B = \frac{x}{l}$$

図A



2/2 講義



$0 \leqq x \leqq 2$

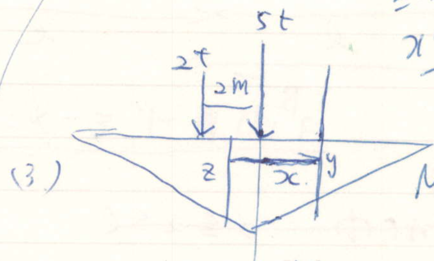
$$M_C = 2 \times \frac{3+x}{2} + 5 \times \frac{5-x}{2}$$

$$= \frac{31-3x}{2}$$

減少

$x=0$ で M_C^{max}

(3)



$M_{C\,max} = 15.5$ t·m

$2.5 : 5 = y : 5-x$

$y = 2.5(5-x) \div 5$

$= 2.5 - \frac{x}{2} = \frac{5-x}{2}$

~~よって x=0 のとき y max~~

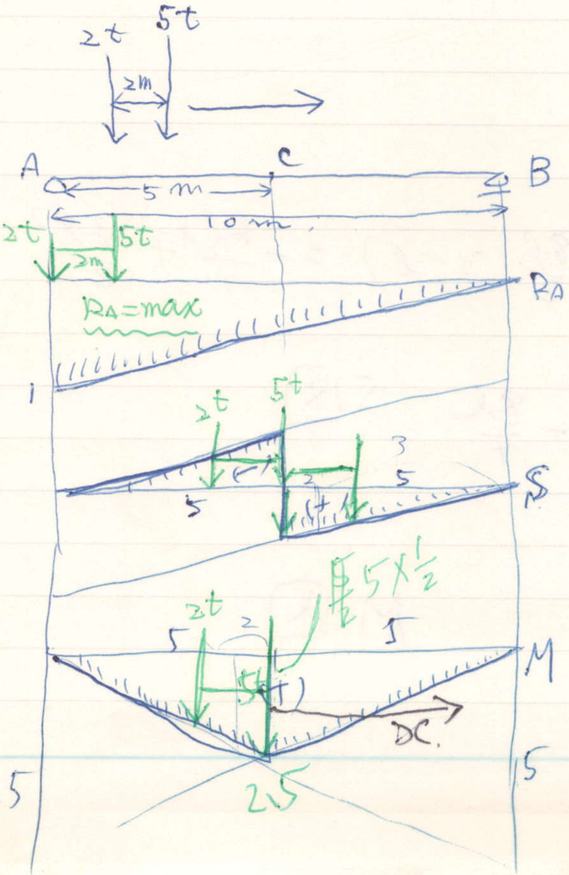
$5 : 2.5 = 5-2+x : z$

$z = 2.5(3+x) \div 5$

$= \frac{3+x}{2}$

$y + \cancel{z = \frac{8}{2} = 4}$ //

EX)



(1) A点反力の最大値(max)

(2) 中央点Cのせん断力のmax

(3) 〃 曲げモーメントのmax

(1) R_A の Infl line

① A点に5tのとき

$R_A = 1 \times 5 = 5^t$

② A点に2tのとき

$R_A = 1 \times 2 + \frac{8}{10} \times 5 = 6^t$ ‥ max

(2) 2^t がC点にあるとき 正max.

$S = 2 \times \frac{1}{2} + 5 \times \frac{3}{10} = 2.5^t$ ‥ max

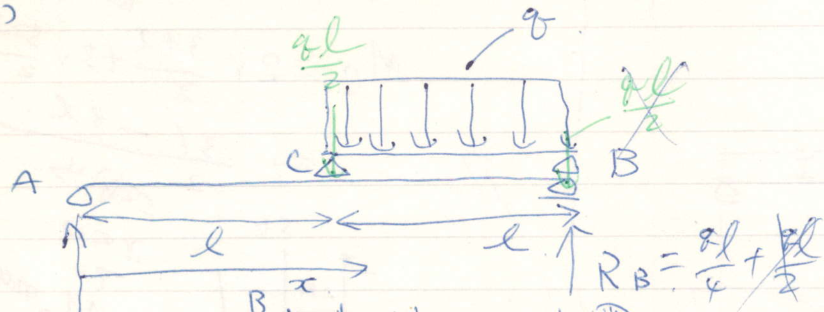
(3) 5^t がC点にあるとき max

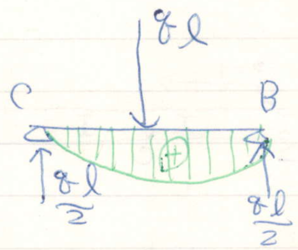
$M = 5 \times 1 + 2 \times \frac{3}{10} = 5.6^{tm}$

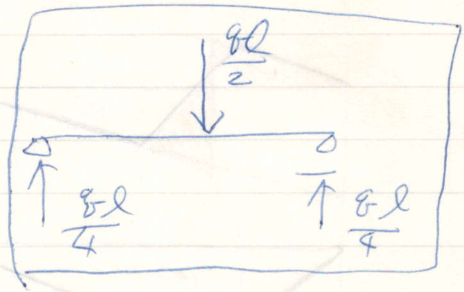
5^t がC てんのとき 負のmax

$S = 5 \times (-\frac{1}{2}) + 2 \times (-\frac{3}{10}) = -3.1^t$

(1)







$\overset{B}{\text{A}}$間曲げモーメント図

(i) ~~AC間~~ ~~$0 \leqq x \leqq l$.~~

~~$S_x =$~~

~~$M_x =$~~

$$\begin{cases} R_A = \frac{ql}{2} \times \frac{1}{2} = \frac{ql}{4} \\ R_B = \frac{ql}{2} \times 1 + \frac{ql}{2} \times \frac{1}{2} = \frac{3ql}{4} \end{cases}$$

(i) $0 \leqq x \leqq l$.

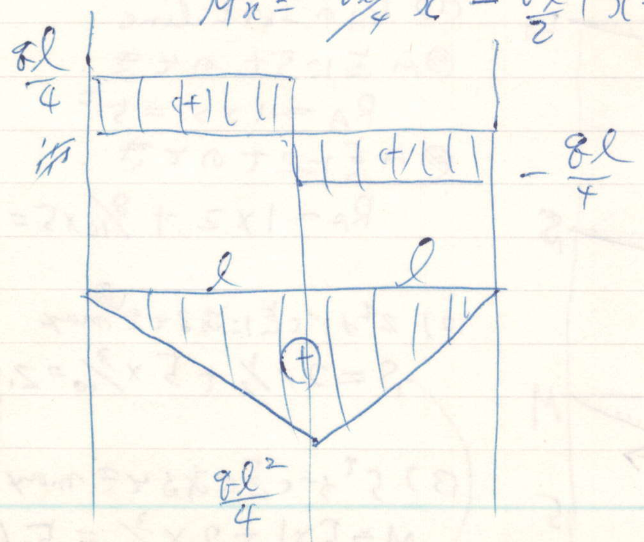
$$S_x = ql/4$$

$$M_x = \frac{ql}{4} \cdot x$$

(ii) $l \leqq x \leqq 2l$.

$$S_x = -ql/4$$

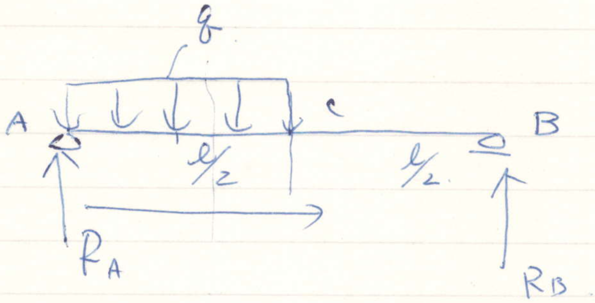
$$M_x = ql/4\, x - \frac{ql}{2}(x - l) = -\frac{qlx}{4} + \frac{ql^2}{2}$$



S図

M図.

(2)



つりあいより
影響線より

$$R_A = \frac{ql}{2} \times \frac{3}{4} = \frac{3ql}{8}$$

$$R_B = \frac{ql}{8}$$

$\frac{18ql^2}{64}$ $-\frac{9ql^2}{128}$

$0 \leqq x \leqq \frac{l}{2}$ のとき

$$M_x = \frac{3ql}{8} \times x - qx \cdot \frac{x}{2} = \frac{3ql}{8}x - \frac{q}{2}x^2$$

$$= -\frac{q}{2}\left(x^2 - \frac{3l}{4} + \frac{9l^2}{64}\right) + \frac{9ql^2}{128}$$

$$= \frac{q}{2}\left(x - \frac{3}{8}l\right)^2 + \frac{9ql^2}{128}$$

(ii) $\frac{l}{2} \leqq x \leqq l$ のとき

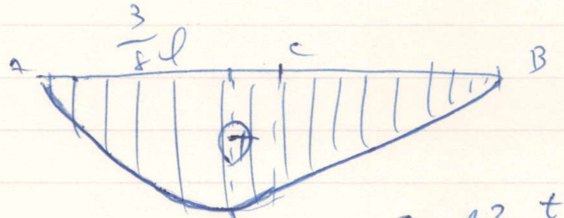
$$M_x = R_A \frac{3ql}{8} \times 8 - \frac{ql}{2} \cdot x$$

ビブンしてもよい。

$$M_x' = \frac{3ql}{8} - qx = 0.$$

$\therefore x = \frac{3}{8}l$ で M_{max}

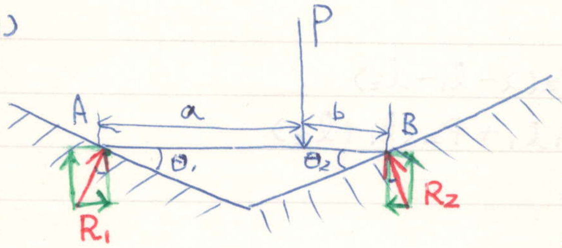
$\therefore M_{max} = \frac{9ql^2}{128}$ t·m



$$M_{max} = \frac{9ql^2}{128} \ \text{t·m} \longleftarrow \left(x = \frac{3}{8}l^m\right)$$

P177

(1)



A, B点の垂直抗力を R_1, R_2 とする。

$\Sigma H = R_1 \sin\theta_1 - R_2 \sin\theta_2 = 0$ —— ①

$\Sigma V = P - R_1 \cos\theta_1 - R_2 \cos\theta_2 = 0$ —— ②

$\Sigma M_A = aP - (a+b) R_2 \cos\theta_2 = 0$ —— ③

③より

$$R_2 = \frac{aP}{(a+b)\cos\theta_2} \quad —— ④$$

これを②に代入して

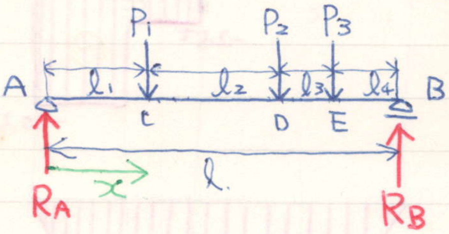
$$R_1 = \frac{P - R_2\cos\theta_2}{\cos\theta_1} = \frac{bP}{(a+b)\cos\theta_1} \quad —— ⑤$$

④ ⑤ を ①に代入して

$$\frac{bP}{(a+b)\cos\theta_1}\sin\theta_1 - \frac{aP}{(a+b)\cos\theta_2}\sin\theta_2 = 0$$

$$\therefore\ b\tan\theta_1 = a\tan\theta_2 \quad \cdots\cdots (答)$$

(2)



$$R_A = \frac{1}{l}\left\{(l_2+l_3+l_4)P_1 + (l_3+l_4)P_2 + l_4 P_3\right\}$$

$$R_B = \frac{1}{l}\left\{l_1 P_1 + (l_1+l_2)P_2 + (l_1+l_2+l_3)P_3\right\}$$

i) $0 \leqq x \leqq l_1$ $S_x = R_A$

$M_x = R_A x$

ii) $l_1 \leqq x \leqq l_1 + l_2$ $S_x = R_A - P_1$

$M_x = R_A x - P_1(x - l_1)$

$= (R_A - P_1)x + P_1 l_1$

iii) $l_1+l_2 \leqq x \leqq l_1+l_2+l_3$

$$S_x = R_A - P_1 - P_2$$

$$M_x = R_A x - P_1(x-l_1) - P_2(x-l_1-l_2)$$

$$= (R_A - P_1 - P_2)x + P_1 l_1 + P_2(l_1+l_2)$$

iv) $l_1+l_2+l_3 \leqq x \leqq l$

$$S_x = R_A - P_1 - P_2 - P_3 \quad = -R_B$$

$$M_x = \quad = \cancel{R_B(x-l)}\ R_B(l-x)$$

$$R_A x - P_1(x-l_1) - P_2(x-l_1-l_2) - P_3(x-l_1-l_2-l_3)$$

$$= (R_A - P_1 - P_2 - P_3) + P_1 l_1 + P_2(l_1+l_2) + P_3(l_1+l_2+l_3)$$

$$\begin{cases} P_1 = 4t, \quad P_2 = 6t, \quad P_3 = 8t \\ l_1 = 3m, \quad l_2 = 4m, \quad l_3 = 2m, \quad l_4 = 3m. \end{cases}$$

$R_A = 7.5t, \quad R_B = 10.5t$

i) $S_x = 7.5^t$

$M_x = 7.5x^{t\cdot m}$

ii) $S_x = 3.5^t$

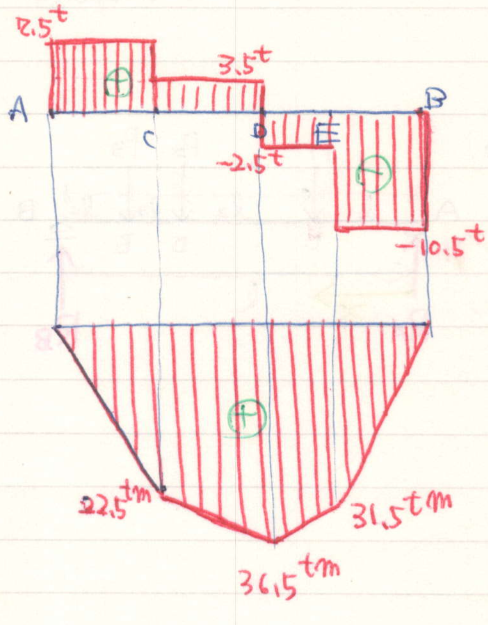
$M_x = 3.5x + 12^{t\cdot m}$

iii) $S_x = -2.5^t$

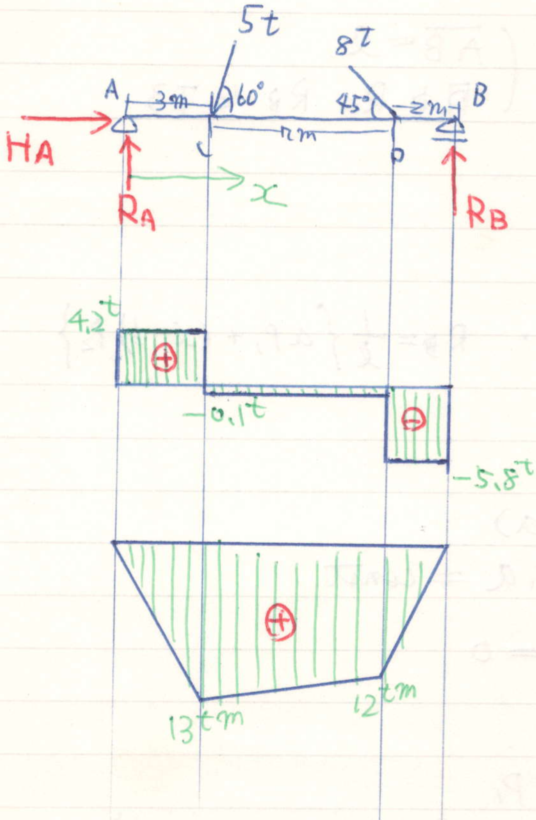
$M_x = -2.5\overset{t}{x} + 54^{t\cdot m}$

iv) $S_x = -10.5^t$

$M_x = -10.5x + 136^{t\cdot m}$



(3)



$\Sigma H = H_A + 8\cos 45° - 5\cos 60° = 0$

$\therefore H_A = 2.5 - 4\sqrt{2} = -3.2^t$

$\Sigma M_A = 3 \cdot 5\sin 60° + 10 \cdot 8\sin 45° - 12 R_B = 0$

$\therefore R_B = \frac{1}{12}(7.5\sqrt{3} + 40\sqrt{2}) = 5.8^t$

$\Sigma V = R_A + R_B - 5\sin 60° - 8\sin 45° = 0$

$\therefore R_A = 2.5\sqrt{3} + 4\sqrt{2} - 5.8 = 4.2^t$

i) $0 \leqq x \leqq 3$

$S_x = R_A = 4.2^t$

$M_x = R_A x = 4.2x^{tm}$

$N_x = -H_A = 3.2^t$

ii) $3 \leqq x \leqq 10$

$S_x = R_A - 5\sin 60° = -0.1^t$

$M_x = R_A x - 5\sin 60°(x-3)$

$= -0.1x + 13^{t \cdot m}$

$N_x = -H_A + 5\cos 60° = 5.7^t$

iii) $10 \leqq x \leqq 12$

$S_x = R_A - 5\sin 60° - 8\sin 45°$

$= -5.8^t$

$M_x = 0.1x + 13 - 8\sin 45°(x-10)$

$= -5.8x + 70^{tm}$

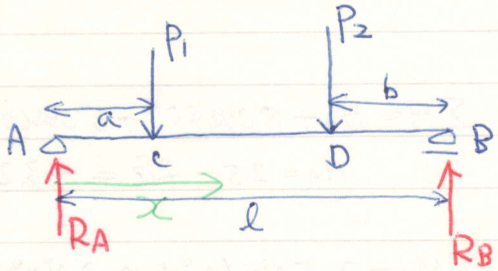
$N_x = 5.7^t - 8\cos 45°$

$= 0$

$M_C = 4.2 \times 3 = 12.6^{tm} \fallingdotseq 13^{tm}$

$M_D = -0.1 \times 10 + 13 = 12^{tm}$

(4)



$\left(\begin{array}{l}\overline{AB}=l.\\ 反力\ R_A\ R_B\ とする\end{array}\right.$

$$R_A=\frac{1}{l}\left\{(l-a)P_1+bP_2\right\}\quad,\quad R_B=\frac{1}{l}\left\{aP_1+(l-b)P_2\right\}$$

CD間において

$$M_x=R_A x-P_1(x-a)$$
$$=(R_A-P_1)x+P_1 a=\text{const}.$$

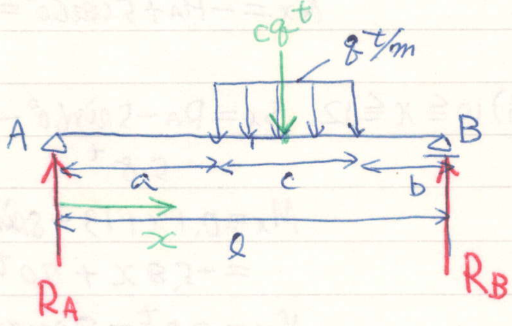
故に $\dfrac{dM_x}{dx}=R_A-P_1=0$

$\therefore\ (l-a)P_1+bP_2=lP_1$

$\therefore\ bP_2=aP_1$ ・・・ (答)

$(P_1/P_2=b/a)$

(5)



$\Sigma M_A=cq(a+c/2)-lR_B=0$

$\therefore\ R_B=\dfrac{cq}{2l}(2a+c)$

$\Sigma M_B=cq(b+c/2)-lR_A=0$

$\therefore\ R_A=\dfrac{cq}{2l}(2b+c)$

i) $0\leqq x\leqq a$ $\quad S_x=R_A$

$M_x=R_A x.$

ii) $a\leqq x\leqq a+c$

$S_x=R_A-(x-a)q$

$M_x=R_A x-(x-a)q\cdot\dfrac{x-a}{2}$

$=R_A x-\dfrac{q}{2}(x-a)^2$

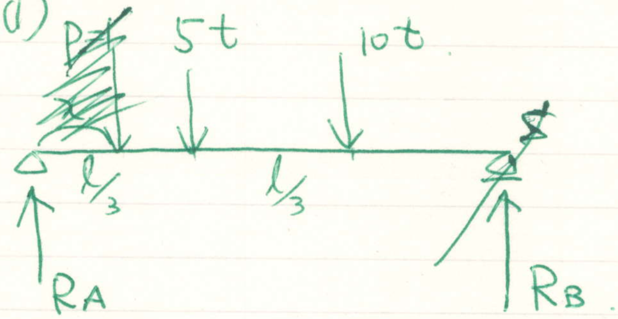
iii) $a+c\leqq x\leqq l$

$S_x=-R_B$

$M_x=-R_B(x-l)$

(図Aを使う)

例(1)



$$\cancel{\Sigma M_B = l \cdot R_A - x \cdot 1 - 5 \cdot \frac{l}{3} - 10 \cdot \frac{2l}{3} = 0}$$

$$\cancel{\therefore R_A = \frac{x}{l} + \frac{5}{3} + \frac{20}{3} = \frac{x}{l} + \frac{25}{3}}$$

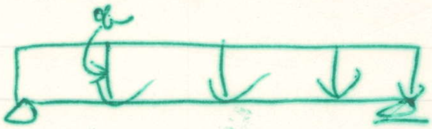
$$\cancel{\Sigma M_A = l \cdot R_B - \frac{2}{3} l \cdot 10 - \frac{l}{3} \cdot 5 - x \cdot 1}$$

$$\cancel{\therefore R_B =}$$

Influence line を使って R_A、R_B を求める.

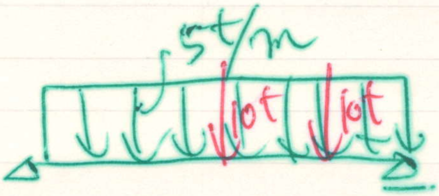
$$\begin{cases} R_A = \frac{2}{3} \times 5 + \frac{1}{3} \times 10 = 6.7t \\ R_B = \frac{1}{3} \times 5 + \frac{2}{3} \times 10 = 8{,}3t \end{cases}$$

例2.



$$R_A = \left(\frac{1}{2} \times 1 \times l\right) \times q = \frac{lq}{2}$$

例3.



$$R_A = \frac{5 \cdot l}{2} + 10 \times \frac{1}{2} + 10 \times \frac{1}{4}$$

~~$\Sigma M_B = l \cdot R_A - x \cdot 1 - 5 \cdot \frac{l}{3} - 10 \cdot \frac{2l}{3} = 0$~~

~~$R_A = \frac{x}{l} + \frac{5}{3} + \frac{20}{3} = \frac{x}{l} + \frac{25}{3}$~~

~~$\Sigma M_A = l \cdot R_B - \frac{2}{3}l \cdot 10 - \frac{l}{3} \cdot 5 - x \cdot 1$~~

~~$\therefore R_B =$~~

Influence line を使って R_A, R_B を求める.

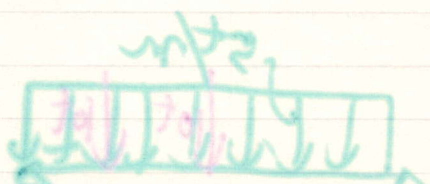
$$R_A = \frac{2}{3} \times 5 + \frac{1}{3} \times 10 = 6.7t$$

$$R_B = \frac{1}{3} \times 5 + \frac{2}{3} \times 10 = 8.3t$$

〈例2〉

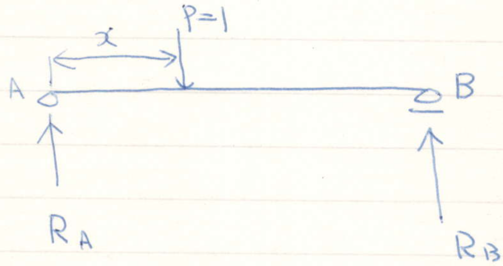
$$R_A = \left(\frac{1}{2} \times l \times 1\right) \times 8 = \frac{8l}{2}$$

〈例3〉

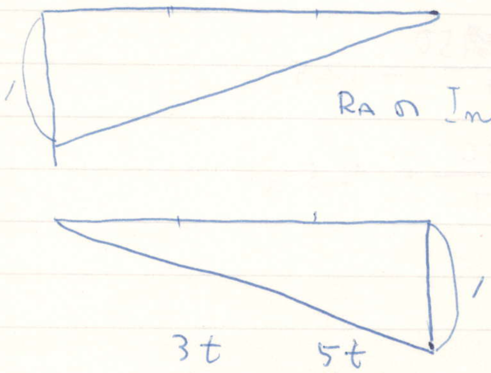


$$R_A = \frac{5l}{2} + 10 \times \frac{1}{2} + 10 \times \frac{l}{4}$$

反力の Influence line

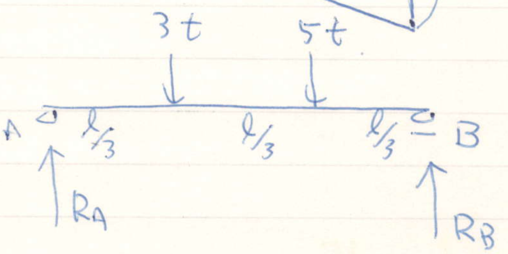


R_B Influence line.

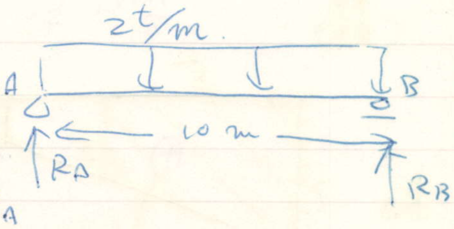


R_A の Influence line.

(例1)



(例2)



$$R_A = \frac{2}{3}\cdot 3_t + \frac{1}{3}\cdot 5_t = \frac{11}{3}t$$

$$R_B = \frac{1}{3}\cdot 3t + \frac{2}{3}\cdot 5_t = \frac{13}{3}t$$

$$R_A l - 3\cdot\frac{2}{3}l - 5\cdot\frac{l}{3} = 0$$

$$R_A = \frac{11}{3}t$$

$$R_B l - 5\cdot\frac{2}{3}t - 3\cdot\frac{1}{3}l = 0$$

$$R_B = \frac{13}{3}t.$$

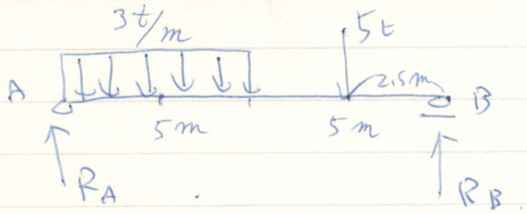
$$R_A = A\cdot 2\,t/m.$$

$$= \frac{1}{2}\cdot 1\times 10\times 2$$

$$= 10\,t.$$

$$R_B = \frac{1}{2}\times 1\times 10\times\frac{1}{2} = 10_t$$

(3113)



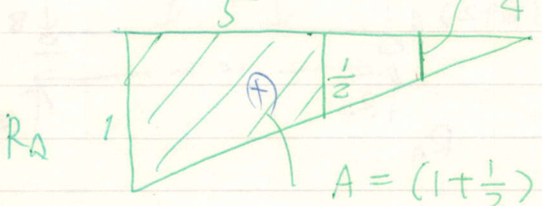
$$R_A = \cancel{\frac{3}{4} \times 15 \times 2.5 + \frac{1}{4} \times 5}$$

$$R_A = 15 \times \frac{7.5}{10} + 5 \times \frac{2.5}{10} = \frac{50}{4} = 12.5\ t$$

$$R_B = 15 \times \frac{1}{4} + 5 \times \frac{3}{4} = \frac{30}{4} = 7.5\ t$$

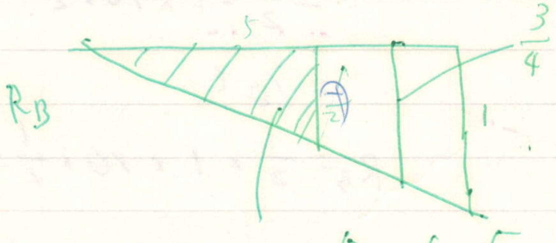
(別解)

$$\frac{3 \times 15}{4} + \frac{5}{4}$$



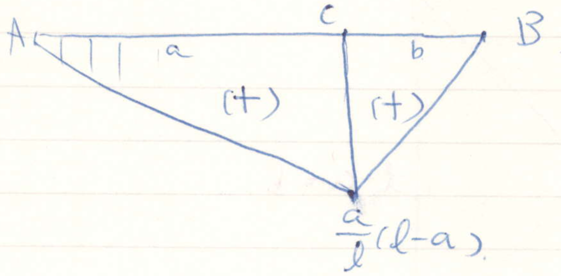
$$A = (1 + \frac{1}{2}) \times 5 \times \frac{1}{2} = \frac{15}{4}$$

$$R_A = A \cdot q + \frac{1}{4} \times 5 = \frac{15}{4} \times 3 + \frac{5}{4} = \frac{50}{4}$$



$$A = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$$

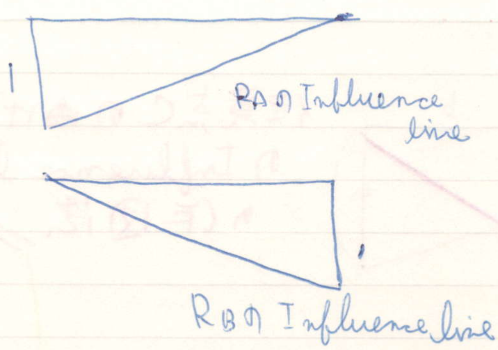
$$\therefore R_B = A \cdot q + \frac{3}{4} \times 5 = \frac{5}{4} \times 3 + \frac{15}{4} = \frac{30}{4}$$

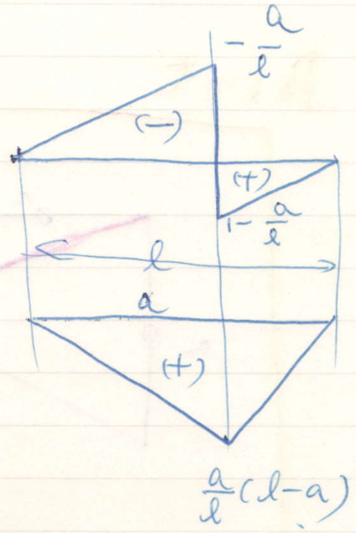


M_c の Influence line

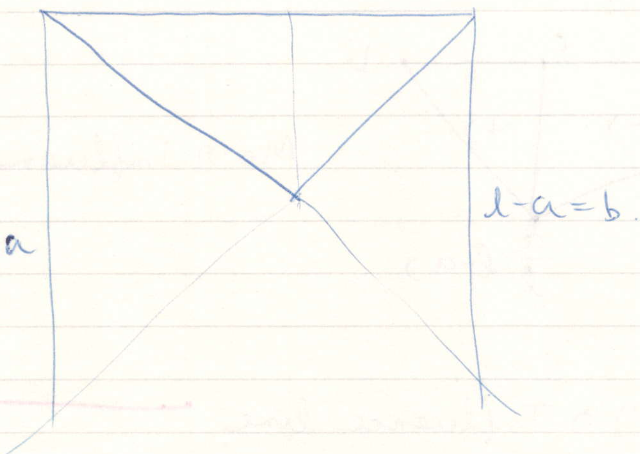
Point

単純ばりの Influence line

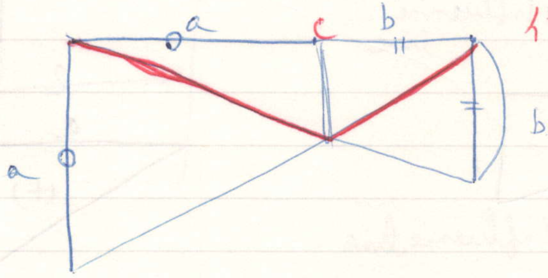




一般に.



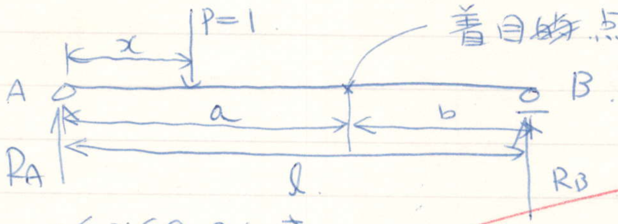
Point



任意点Cの曲げモーメントのInfluence lineの作図法

間接荷重 → P86 ヨム.

せん断力の Influence line



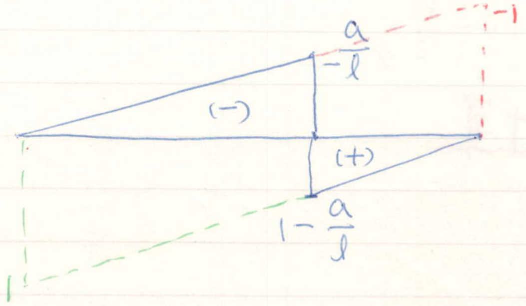
R_A の Influence line より求めても可。

$0 \leqq x \leqq a$ のとき

$$S_a = R_A - 1 = \frac{l-x}{l} - 1 = -\frac{x}{l}$$

$a \leqq x \leqq l$ のとき

$$S_a = R_A = \frac{l-x}{l} = 1 - \frac{x}{l}$$

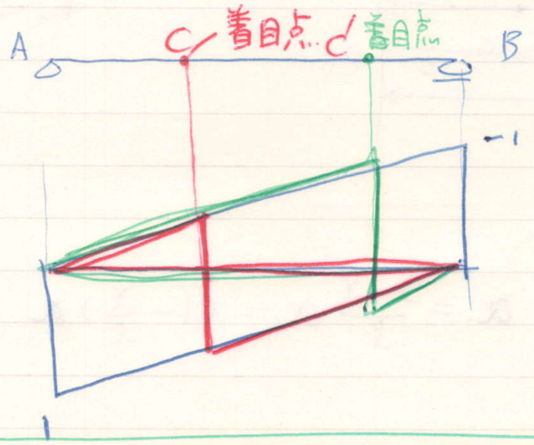


S_a の Influence line

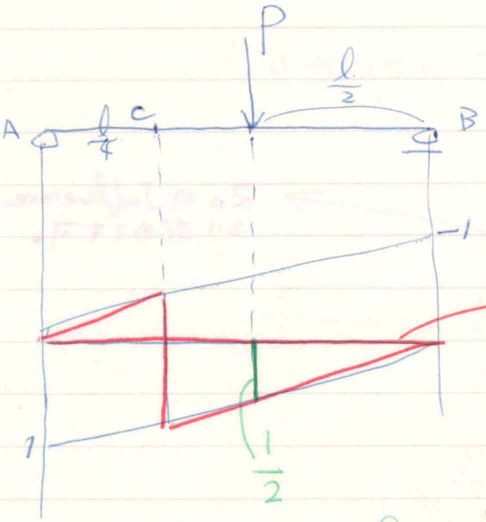
下部は R_A の影響線
上部は R_B の影響線の逆を加える

Point

一般に



任意点Cのせん断力の Influence line の作図法.

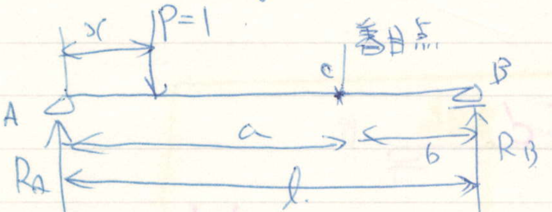


C点のせん断力(S_c)のInfluence line

$$S_c = \frac{1}{2} \times p = \frac{P}{2} \quad //$$

分布荷重 ⟶ 面積計算.

曲げモーメントのInfluence line



$$R_A = \frac{l-x}{l}, \quad R_B = \frac{x}{l}$$

(ii) $a \leqq x \leqq l$

~~$0 \leqq x \leqq a$ のとき.~~

$\cancel{M_a =}$

$$M_c = R_A a = \frac{l-x}{l} \cdot a = \left(1 - \frac{x}{l}\right) a.$$

~~$a \leqq x \leqq l$ のとき.~~

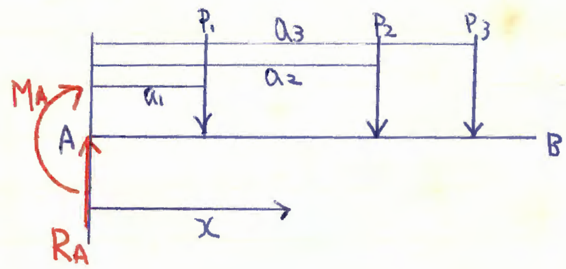
$\cancel{M_c = R_A x}$

(i) $0 \leqq x \leqq a$

$$M_c = R_A a - 1 \cdot (a - x)$$

$$= \frac{l-x}{l} a - (a - x) = \cancel{(1 - \frac{a}{l}) x}. \ \left(1 - \frac{a}{l}\right) x.$$

片持梁



$R_A = P_1 + P_2 + P_3$

$M_A = -(P_1 a_1 + P_2 a_2 + P_3 a_3)$

i) $0 \leqq x \leqq a_1$

$M_x = M_A + R_A x = P_1(x-a_1) + P_2(x-a_2) + P_3(x-a_3)$

$S_x = \cancel{M}.R_A = P_1 + P_2 + P_3$

ii) $a_1 \leqq x \leqq a_2$

$M_x = P_2(x-a_2) + P_3(x-a_3)$

$S_x = P_2 + P_3$

iii) $a_2 \leqq x \leqq a_3$

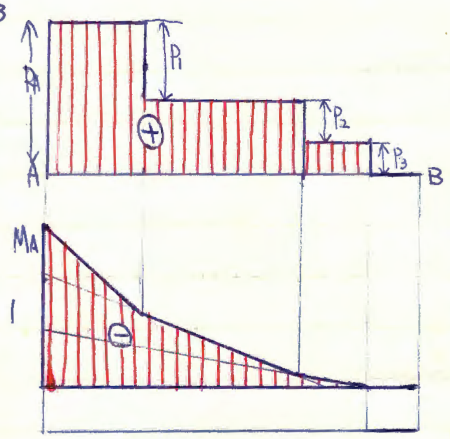
$M_x = P_3(x-a_3)$

$S_x = P_3$

iv) $a_3 \leqq x \leqq l$

$M_x = 0$

$S_x = 0$



i) $0 \leqq x \leqq a_1$ $M_x = -\sum_1^3 P(a-x)$

$S_x = \sum_1^3 P$

ii) $a_1 \leqq x \leqq a_2$ $M_x = -\sum_1^2 P(a-x)$

$S_x = \sum_1^2 P$

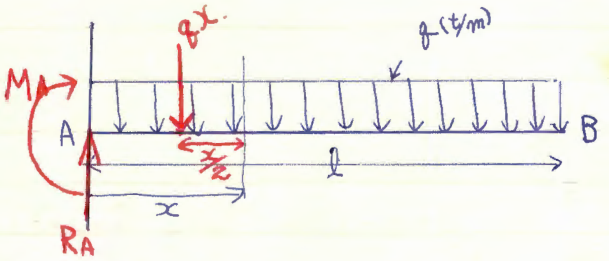
iii) $a_2 \leqq x \leqq a_3$ $M_x = -P_3(a_3 - x)$

$S_x = P_3$

iv) $a_3 \leqq x \leqq l$ $M_x = 0$

$S_x = 0$

等分布荷重〜



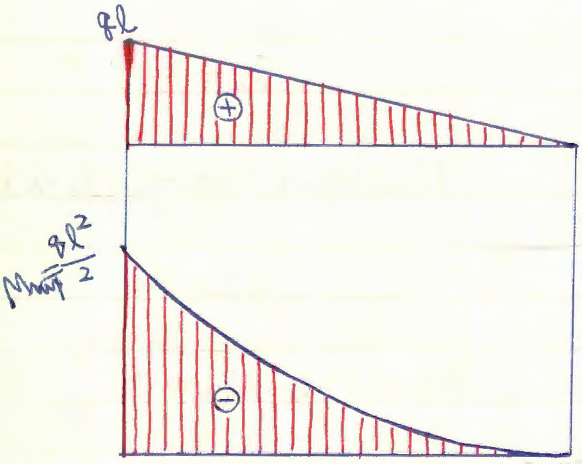
$$\begin{cases} R_A = ql \\ M_A = -ql^2/2 \end{cases}$$

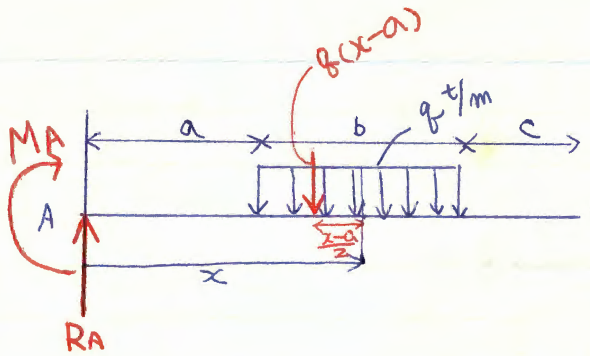
$$M_x = M_A + R_A x - qx^2/2$$

$$= -ql^2/2 + qlx - qx^2/2$$

$$= -\frac{q}{2}(x^2 - 2lx + l^2) = -\frac{q(x-l)^2}{2}$$

$$S_x = -q/2 \cdot 2(x-l) = -q(x-l) = q(l-x)$$





$$\begin{cases} R_A = qb \\ M_A = -qb(a + b/2) \end{cases}$$

i) $0 \leqq x \leqq a$ $\quad M_x = M_A + R_A x = -qb(a + \frac{b}{2} - x)$

$S_x = R_A = qb$

ii) $a \leqq x \leqq a+b$ $\quad M_x = -qb(a + \frac{b}{2} - x) - \frac{q}{2}(x-a)^2$

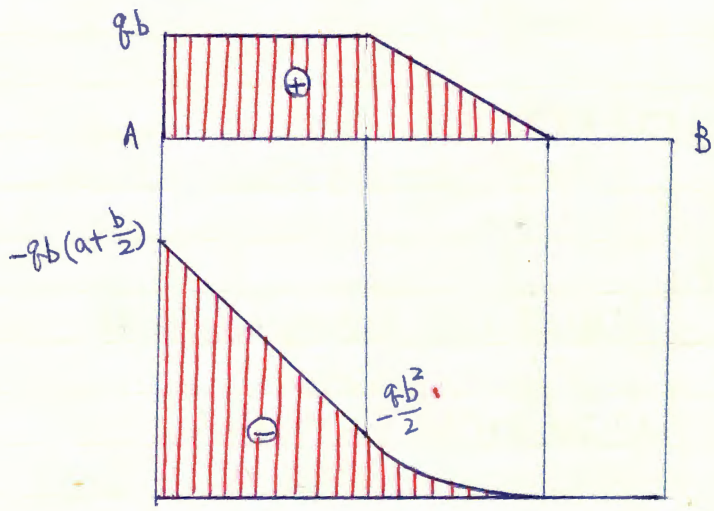
$$= -\frac{q}{2}\left(x^2 - (2a+2b)x + a^2 + 2ab + b^2\right)$$

$$= -\frac{q}{2}(a+b-x)^2$$

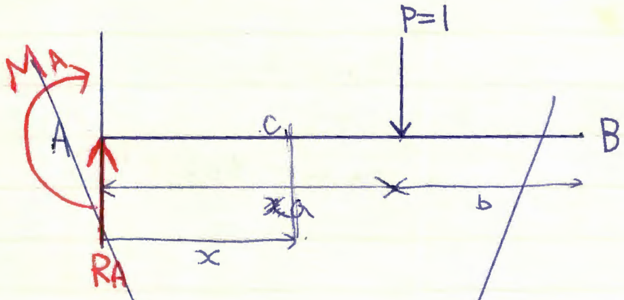
$S_x = q(a+b-x)$

iii) $a+b \leqq x \leqq l$ $\quad M_x = 0$

$S_x = 0$



片持梁の Influe line 〜



$R_A = P = 1$

$\cancel{M_A = -x}$ $M_A = -Pa = -a$

$\cancel{M_x =}$

$\cancel{\text{i)}\ 0 \leqq x \leqq a}$

$\cancel{M_x = M_A + R_A x}$

$\cancel{= x - a}$

$\cancel{S_x = 1}$

$\cancel{\text{ii)}\ a \leqq x \leqq l}$

$\cancel{M_x = x - a - 1(x-a)}$

$\cancel{= 0}$

M_x

AC間にPがある $M_x = M_A + R_A x - 1(x-a)$
$= 0$

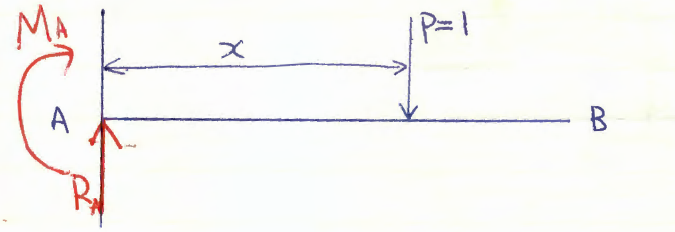
CB間にPがある $M_x = M_A + R_A x$
$= x - a$

S_x

AC間にPがある $S_x = R_A - 1 = 0$

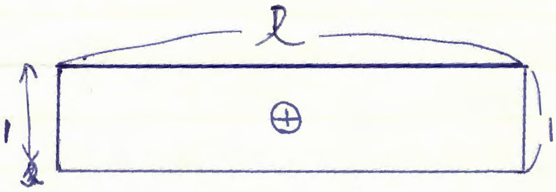
CB間 〃 $S_x = R_A = 1$

片持梁　(支点反力の Influ line)

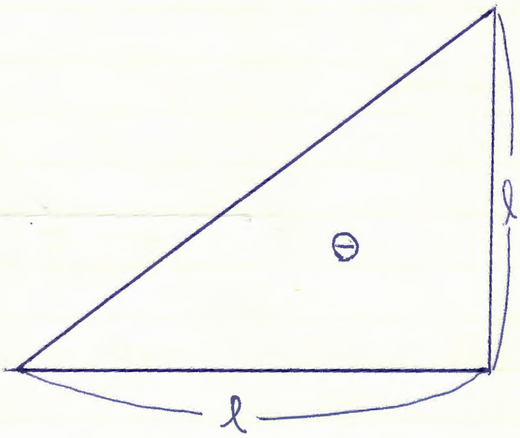


$R_A = 1$

$M_A = -Px = -x$

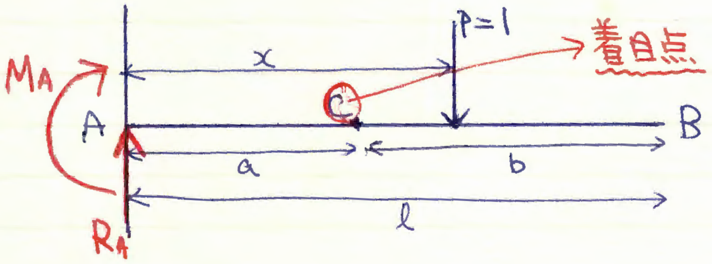


R_A の
Influ line



M_A の
Influ line

片持梁 (S_x, M_x の InfluLine)



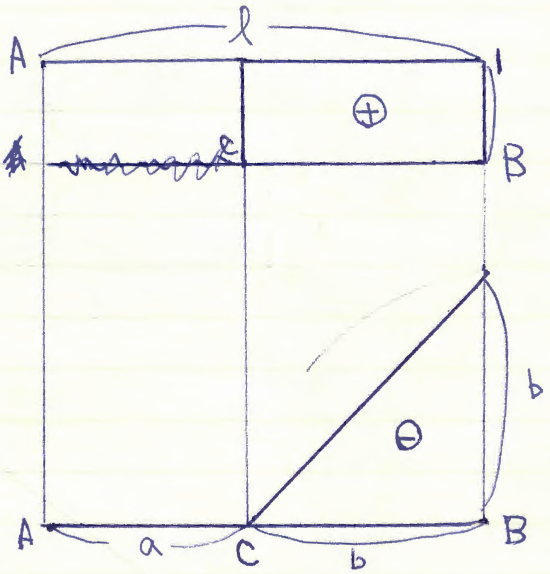
i) $0 \leqq x \leqq a$

$$M_x = M_A + R_A \cancel{x}^{\,a} - 1(\cancel{x}\,a - x)$$
$$= \cancel{-x + \overset{a}{x}} + x - a$$
$$= 0$$

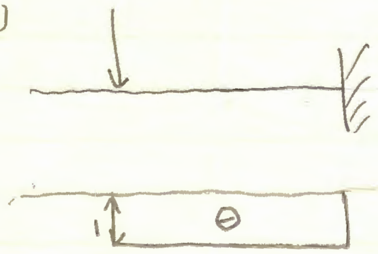
$$S_x = 0$$

ii) $a \leqq x \leqq l$

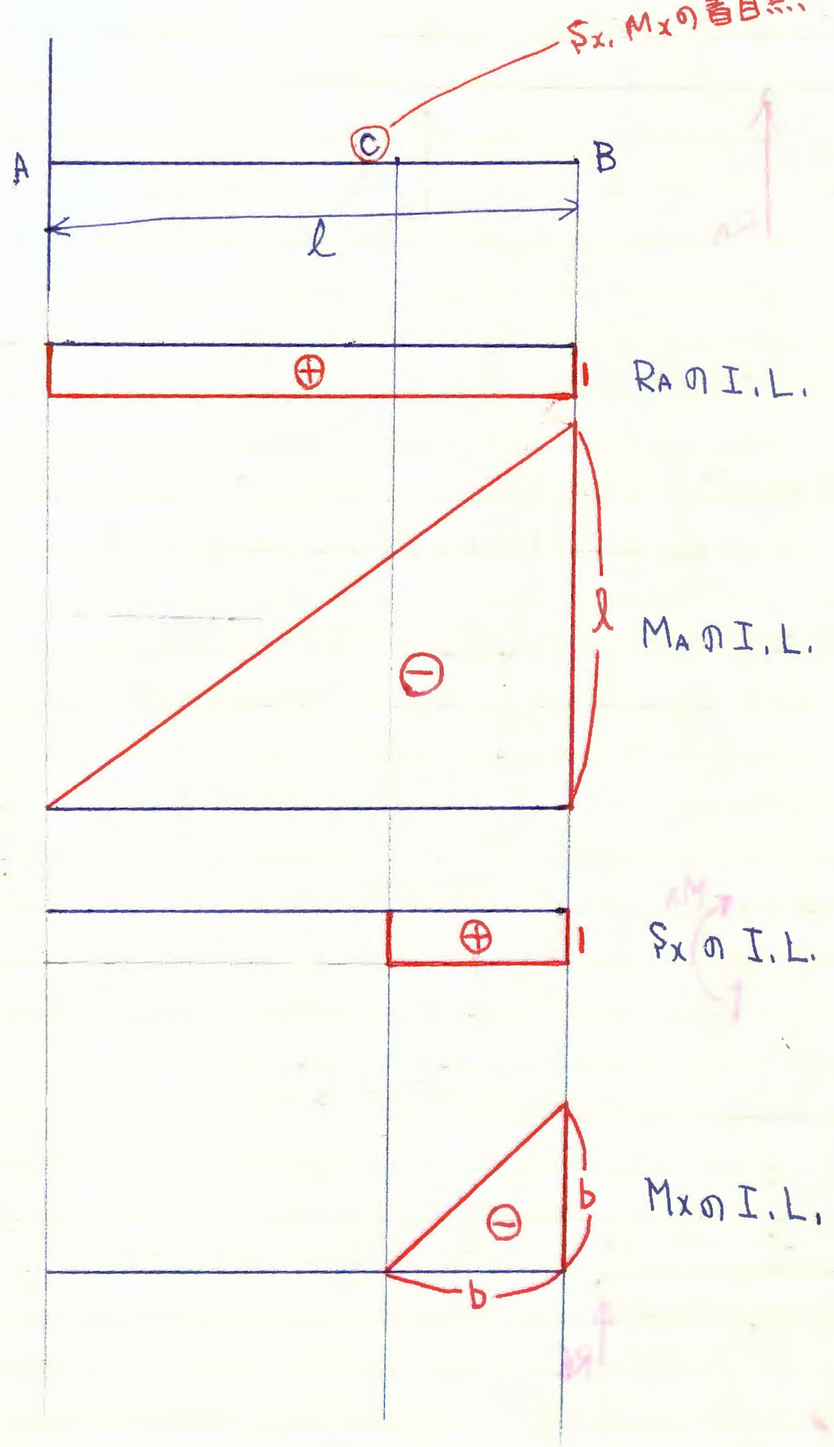
$$M_x = \cancel{M_A + R_A a - a - x}$$
$$= a - x$$
$$S_x = 1$$



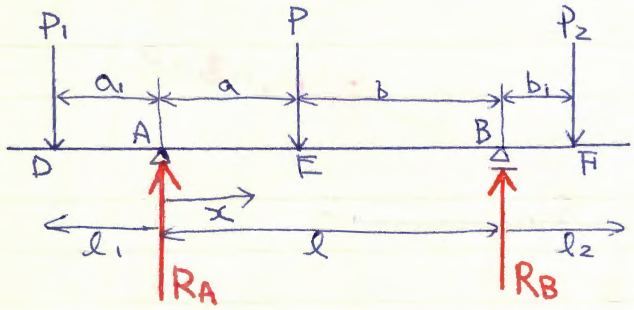
(注)



片持梁の Influence Line



張出梁



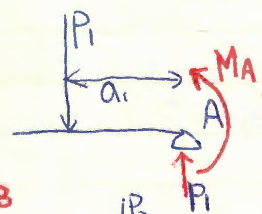
〈支点反力 R_A, R_B〉

$$\Sigma M_B = \underbrace{(a+b)}_{\ell} R_A + b_1 P_2 - bP - (a_1 + \underbrace{a+b}_{\ell}) P_1 = 0$$

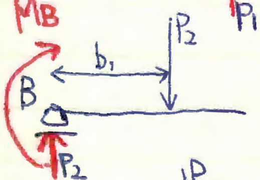
$$\therefore\ R_A = \frac{1}{\ell}\left\{P_1(a_1+\ell) + Pb - P_2 b_1\right\}$$

$$\Sigma M_A = Pa + (b_1+\ell)P_2 - P_1 a_1 - \ell R_B = 0$$

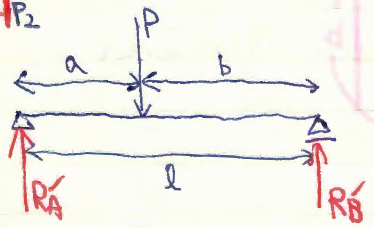
$$\therefore\ R_B = \frac{1}{\ell}\left\{P_2(b_1+\ell) + Pa - P_1 a_1\right\}$$



$$M_A = -P_1 a_1$$



$$M_B = -P_2 b_1$$



$$R_A' = \frac{bP}{\ell}, \quad R_B' = \frac{aP}{\ell}$$

$$\begin{cases} R_A = \frac{1}{l}(-M_A + M_B) + P_1 + R_A' = R_A' + P_1 + \frac{M_B - M_A}{l} \\ R_B = \frac{1}{l}(M_A - M_B) + P_2 + R_B' = R_B' + P_2 + \frac{M_A - M_B}{l} \end{cases}$$

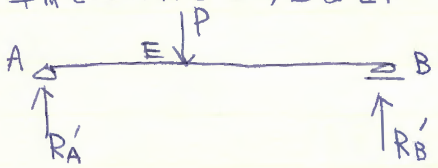
〈せん断力〉

$$\begin{cases} \text{DA間} & S_x = -P_1 \\ \text{BF間} & S_x = P_2 \end{cases} \quad \text{(片持梁)}$$

$$\begin{cases} \text{AE間} & S_x = -P_1 + R_A \left(= R_A' + \frac{M_B - M_A}{l} \right) \\ \text{EB間} & S_x = -P_1 + R_A - P \left(= R_A' + \frac{M_B - M_A}{l} - P \right) \end{cases}$$

(参)

単純バリABを考えると・・・



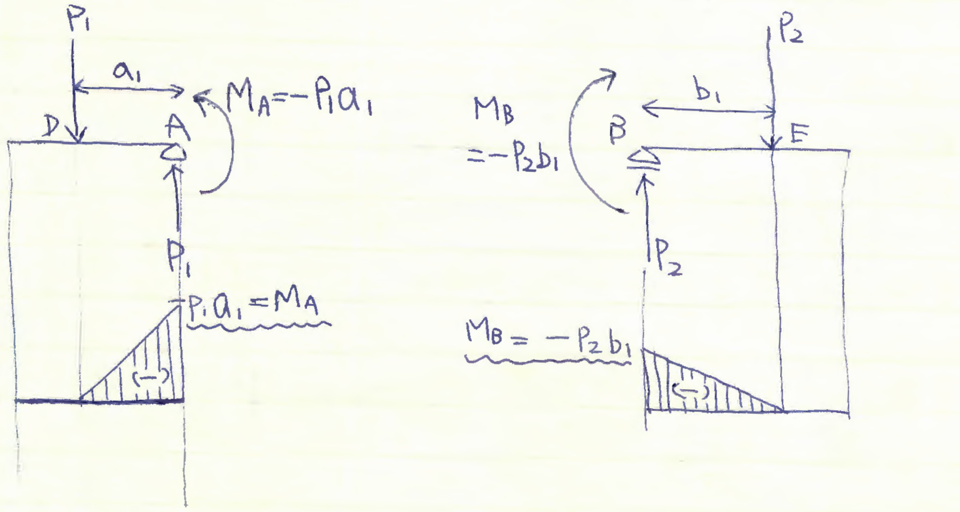
$$\begin{cases} \text{AE間} & S_x' = R_A' \\ \text{EB間} & S_x' = R_A' - P \end{cases}$$

故に

$$\boxed{S_x = S_x' + \frac{M_B - M_A}{l}}$$

(曲げモーメント)

DA間(片持)　　　　BF間(片持)



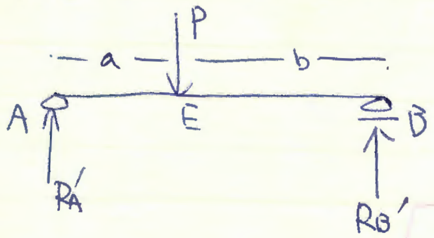
AE間　　$M_x = -P_1(a_1+x) + R_A x$

EB間　　$M_x = -P_1(a_1+x) + R_A x - P(x-a)$

or

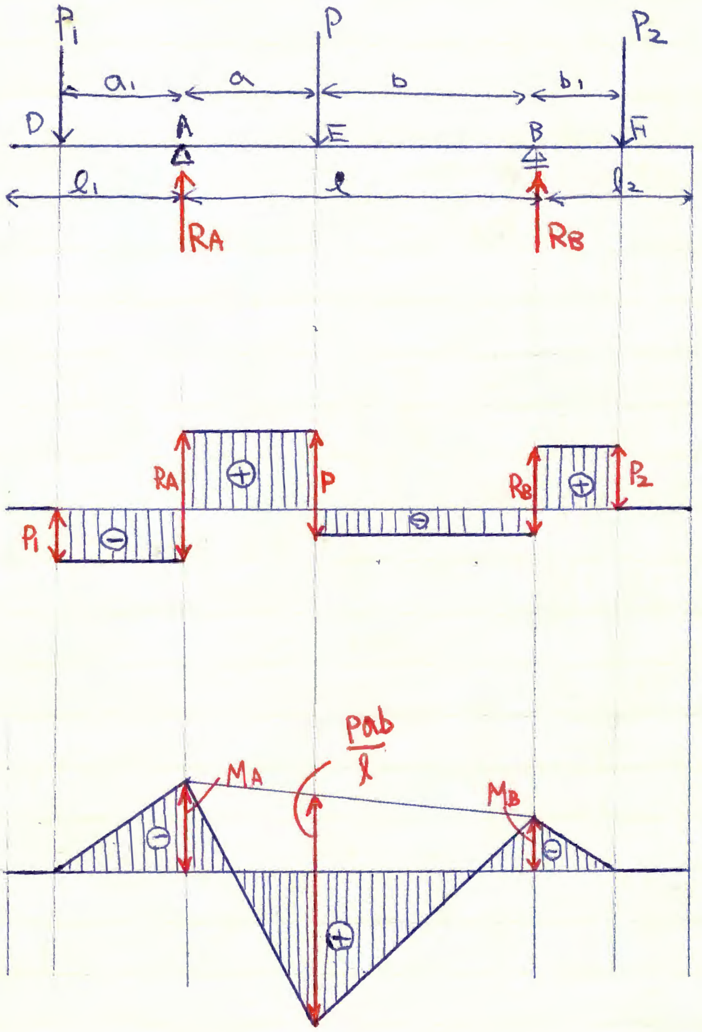
$M_x = R_B(l-x) - P_2(b_1+l-x)$

参) 単純バリABを考えると



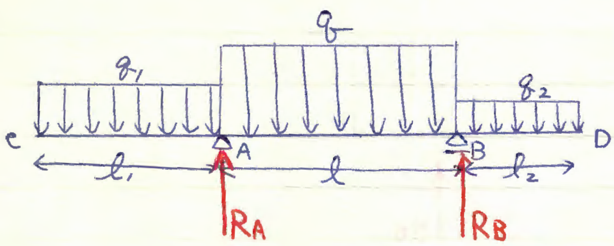
$$\begin{cases} \overline{AE} & M'_x = R'_A x \\ \overline{EB} & M'_x = R'_B(l-x) \end{cases}$$

故に $\boxed{M_x = M'x + \frac{M_B - M_A}{l}x + M_A}$



（集中荷重をうける張出梁のS図，M図）

等分布荷重（張出梁）



$$\boxed{\begin{aligned} R_A &= R_A' + P_1 + \frac{M_B - M_A}{l} \\ R_B &= R_B' + P_2 - \frac{M_B - M_A}{l} \\ S_x &= S_x' + \frac{M_B - M_A}{l} \\ M_x &= M_x' + \frac{M_B - M_A}{l}x + M_A \end{aligned}}$$

$$\begin{cases} P_1 = q_1 l_1 \\ P_2 = q_2 l_2 \end{cases}$$

を利用する。

〈支点反力 R_A, R_B〉

$$M_A = -\frac{q_1 l_1^2}{2} \quad , \quad M_B = -\frac{q_2 l_2^2}{2}$$

$$R_A' = R_B' = \frac{ql}{2}$$

$$S_x' = R_A' - qx = q\left(\frac{l}{2} - x\right)$$

$$M_x' = R_A' x - qx \cdot \frac{x}{2} = \frac{qx}{2}(l - x)$$

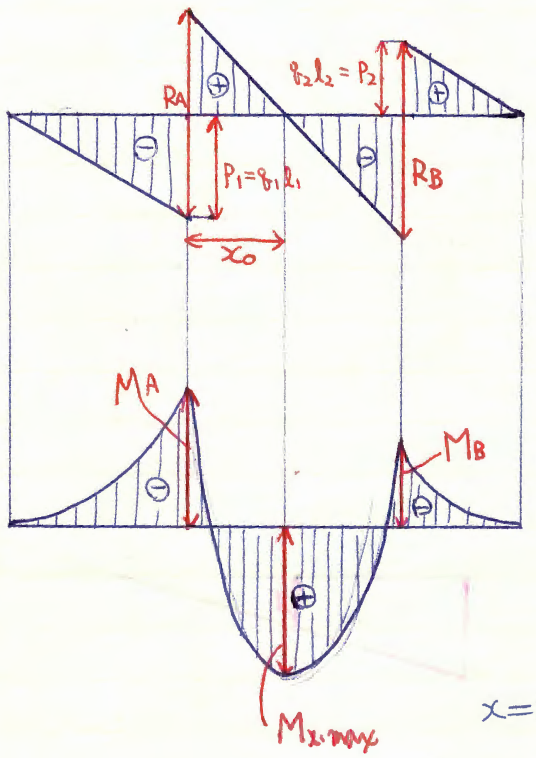
$$\boxed{\begin{aligned} R_A &= \frac{ql}{2} + q_1 l_1 + \frac{1}{2l}(q_1 l_1^2 - q_2 l_2^2) \\ R_B &= \frac{ql}{2} + q_2 l_2 - \frac{1}{2l}(q_1 l_1^2 - q_2 l_2^2) \end{aligned}}$$

〈せん断力〉 AB間

$$S_x = q\left(\frac{l}{2} - x\right) + \frac{1}{2l}\left(q_1 l_1^2 - q_2 l_2^2\right)$$

〈曲げモーメント〉 AB間

$$M_x = \frac{qx}{2}(l - x) + \frac{x}{2l}\left(q_1 l_1^2 - q_2 l_2^2\right) - \frac{q_1 l_1^2}{2}$$



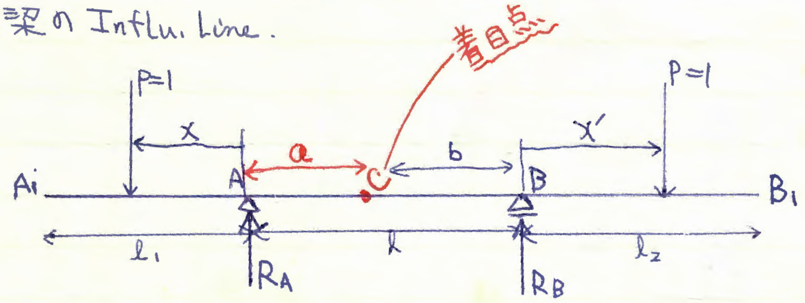
$S_{x_0} = 0$ より

$$x_0 = \frac{l}{2} + \frac{1}{2ql}\left(q_1 l_1^2 - q_2 l_2^2\right)$$

$x = x_0$

$$M_{x\,max} = \frac{1}{8}\left(ql^2 - 2q_1 l_1^2 - 2q_2 l_2^2\right) + \frac{1}{8ql^2}\left(q_1 l_1^2 - q_2 l_2^2\right)^2$$

張出梁の Influ. Line.



<R_A>

$\overline{A_1A}$: $R_A' = 0$, $P_1 = 1$, $M_B = 0$, $M_A = -x$　　　注) AB間は単純梁と同じ

$$R_A = R_A' + P_1 + \frac{M_B - M_A}{l} = \frac{x}{l} + 1$$

$\overline{BB_1}$: $R_A' = 0$, $P_1 = 0$, $M_B = -x'$, $M_A = 0$

$$R_A = -\frac{x'}{l}$$

<R_B>

$\overline{A_1A}$: $R_B' = 0$, $P_2 = 0$, $M_B = 0$, $M_A = -x$

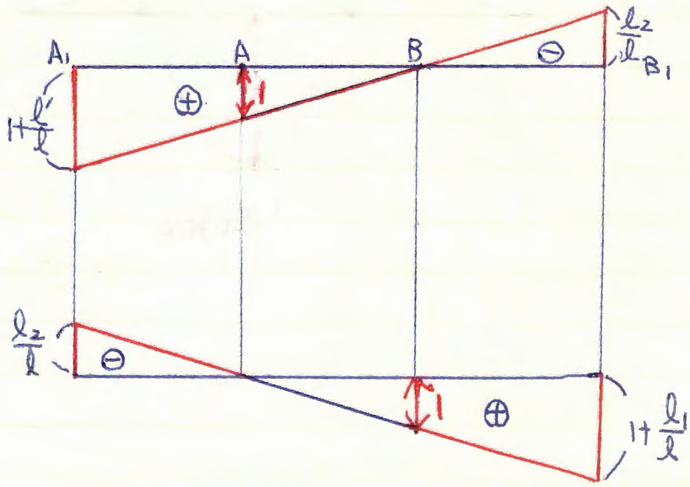
$$\therefore R_B = R_B' + P_2 - \frac{M_B - M_A}{l} = -\frac{x}{l}$$

$\overline{BB_1}$: $R_B' = 0$, $P_2 = 1$, $M_B = -x'$, $M_A = 0$

$$\therefore R_B = 1 + \frac{x'}{l}$$

R_AのI.L.

R_BのI.L.



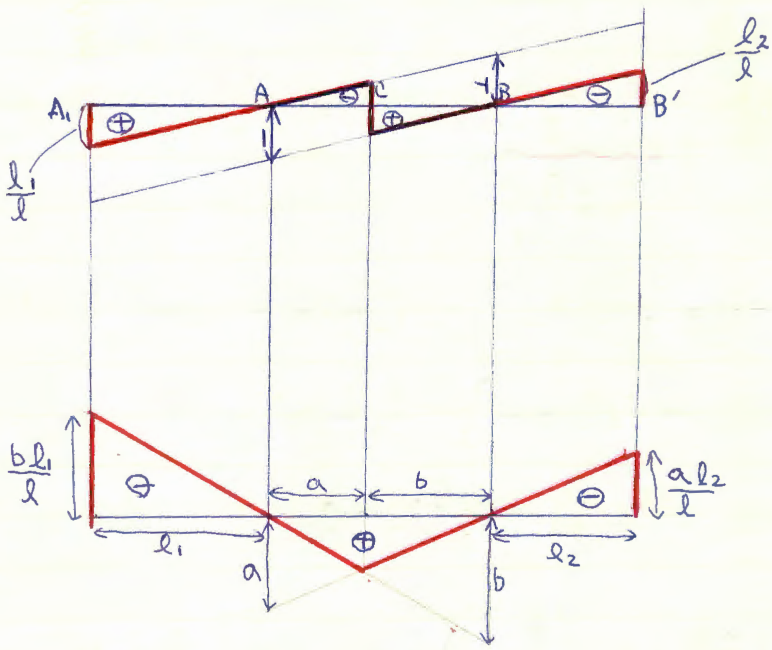
$A_1 A$ 間 $M_c = R_A a - P(x+a)$

$= \left(\frac{a}{l} - 1\right) x$

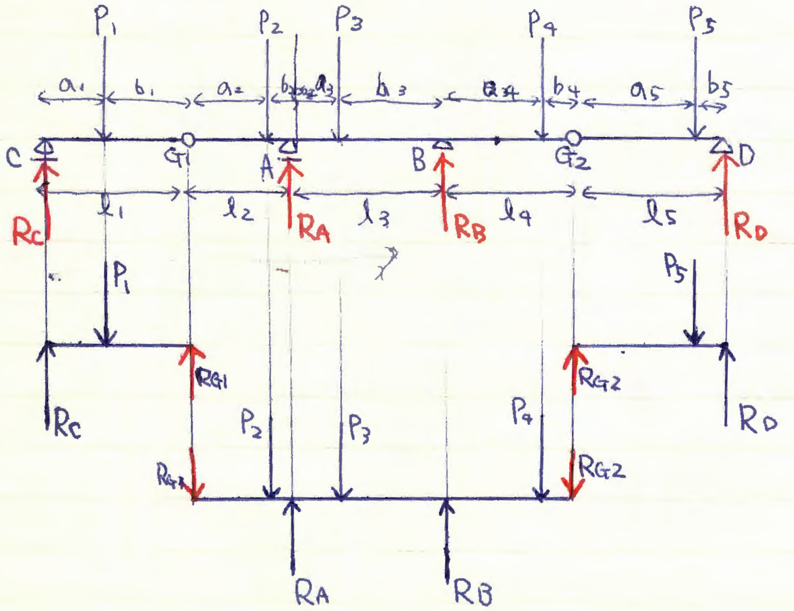
$S_c = R_A - P = \frac{x}{l}$

BB_1 間 $M_c = R_A a = -\frac{ax'}{l}$

$S_c = R_A = -\frac{x'}{l}$



(ゲルバー梁)



$$R_C=\frac{b_1P_1}{l_1},\quad R_D=\frac{a_5P_5}{l_5},\quad R_{G1}=\frac{a_1P_1}{l_1},\quad R_{G2}=\frac{b_5P_5}{l_5}$$

$$\begin{cases} R_A'=\dfrac{b_3P_3}{l_3}, \quad R_B'=\dfrac{a_3P_3}{l_3} \\ P_1=R_{G1}+P_2=\dfrac{a_1P_1}{l_1}+P_2, \quad P_2=R_{G2}+P_4=\dfrac{b_5P_5}{l_5}+P_4 \\ M_A=-l_2R_{G1}-b_2P_2=-\dfrac{a_1P_1}{l_1}l_2-b_2P_2 \\ M_B=-l_4R_{G2}-a_4P_4=-\dfrac{b_5P_5}{l_5}l_4-a_4P_4, \quad l=l_3 \end{cases}$$

故に

$$R_A=\frac{b_3P_3}{l_3}+\frac{a_1P_1}{l_1}+P_2+\frac{1}{l_3}\left(\frac{l_2}{l_1}a_1P_1+b_2P_2-\frac{l_4}{l_5}b_5P_5-a_4P_4\right)$$

$$R_B=\frac{a_3P_3}{l_3}+\frac{b_5P_5}{l_5}+P_4-\frac{1}{l_3}\left(\frac{l_2}{l_1}a_1P_1+b_2P_2-\frac{l_4}{l_5}b_5P_5-a_4P_4\right)$$

$R_c \quad \cancel{P_1(a_2+b_1)}$

C P_1 間 $\quad M_x = R_c x \quad , \quad S_x = R_c$

$P_1 P_2$ 間 $\quad M_x = R_c x - P_1(x-a_1) = (R_c - P_1)x + P_1 a_1$

$S_x = R_c - P_1$

P_2 A 間 $\quad M_x = (R_c - P_1)x + P_1 a_1 - P_2(x - l_1 - a_2)$

$= (R_c - P_1 - P_2)x + P_1 a_1 + P_2 l_1 + P_2 a_2$

$S_x = R_c - P_1 - P_2$

A P_3 間 $\quad M_x = (R_c - P_1 - P_2)x + P_1 a_1 + P_2 l_1 + P_2 a_2 + R_A(x - l_1 - l_2)$

$= (R_c - P_1 - P_2 + R_A)x + P_1 a_1 + P_2 l_1 + P_2 a_2 - R_A l_1 - R_A l_2$

$S_x = R_c - P_1 - P_2 + R_A$

P_3 B 間 $\quad M_x = R_D(l-x) - P_5(l-x-b_5) - P_4(l-x-l_5-b_4) + R_B(l-x-l_5-l_4)$

$= (-R_D + P_5 + P_4 - R_B)x + R_D l - P_5(l-b_5) - P_4(l-l_5-b_4) + R_B(l-l_5-l_4)$

$S_x = -R_D + P_5 + P_4 - R_B$

B P_4 間 $\quad M_x = R_D(l-x) - P_5(l-x-b_5) - P_4(l-x-l_5-b_4)$

$= (-R_D + P_5 + P_4)x + R_D l - P_5(l-b_5) - P_4(l-l_5-b_4)$

$S_x = -R_D + P_5 + P_4$.

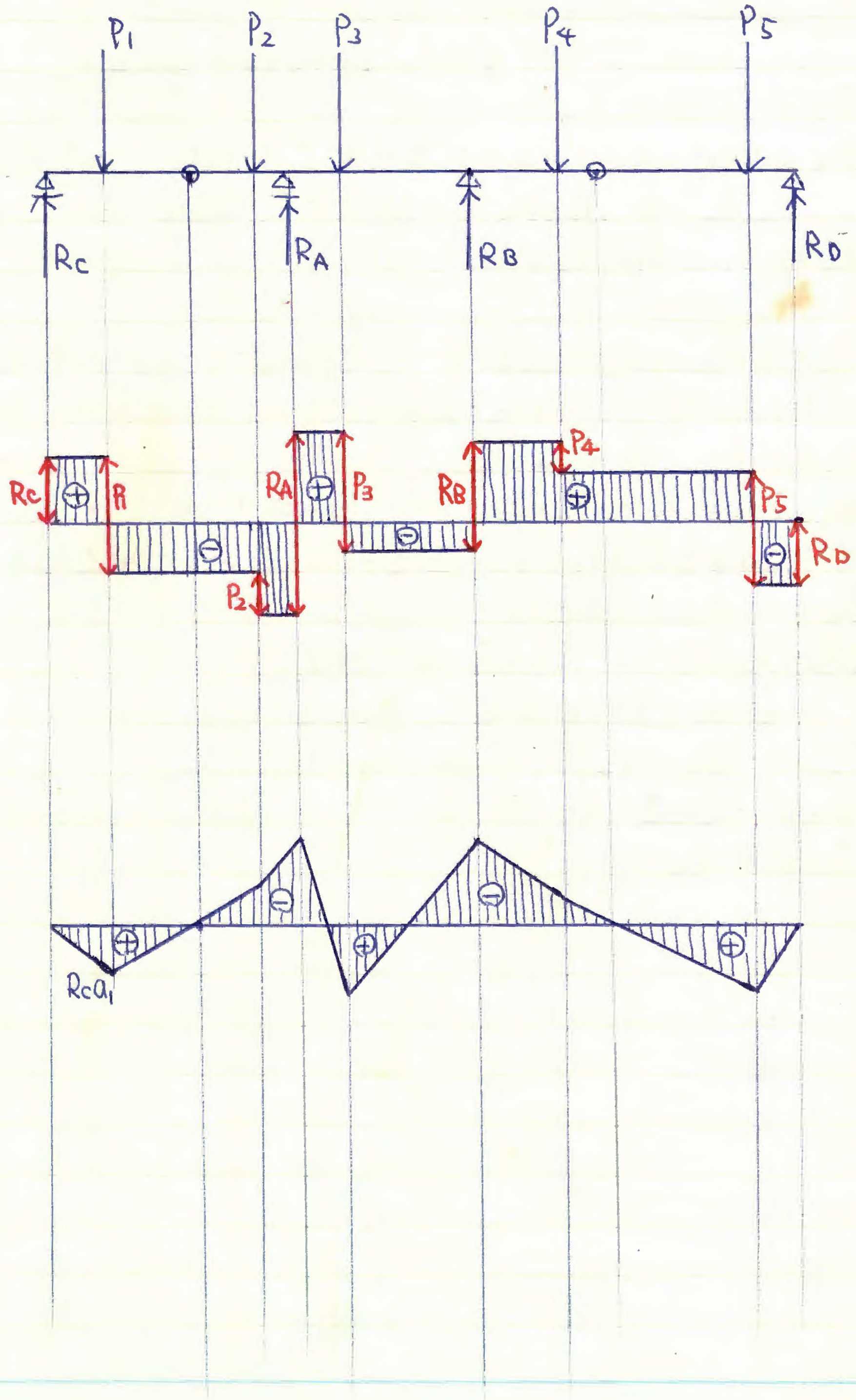
$P_4 P_5$ 間 $\quad M_x = R_D(l-x) - P_5(l-x-b_5)$

$= (-R_D + P_5)x + R_D l - P_5(l-b_5)$

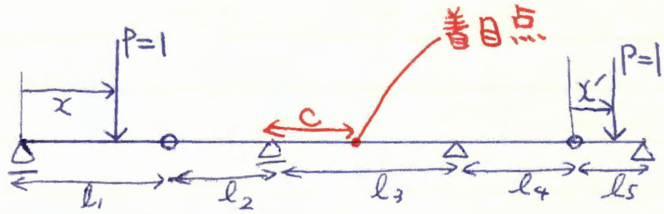
$S_x = -R_D + P_5$.

P_5 D 間 $\quad M_x = R_D(l-x)$

$S_x = -R_D$


P1
P2
P3
P4
P5
Rc
RA
RB
RD
Rc
P1
RA
P3
RB
P4
P5
P2
RD
Rca1

ゲルバー梁の Influ. line.



$0 \leqq x \leqq l_1$

$P_1 = P = 1, \quad P_2 = P_3 = P_4 = P_5 = 0, \quad a_1 = x$

$$R_A = \frac{x}{l_1} + \frac{l_2}{l_3 l_1} P_1 x = \frac{x}{l_1 l_3}(l_3 + l_2)$$

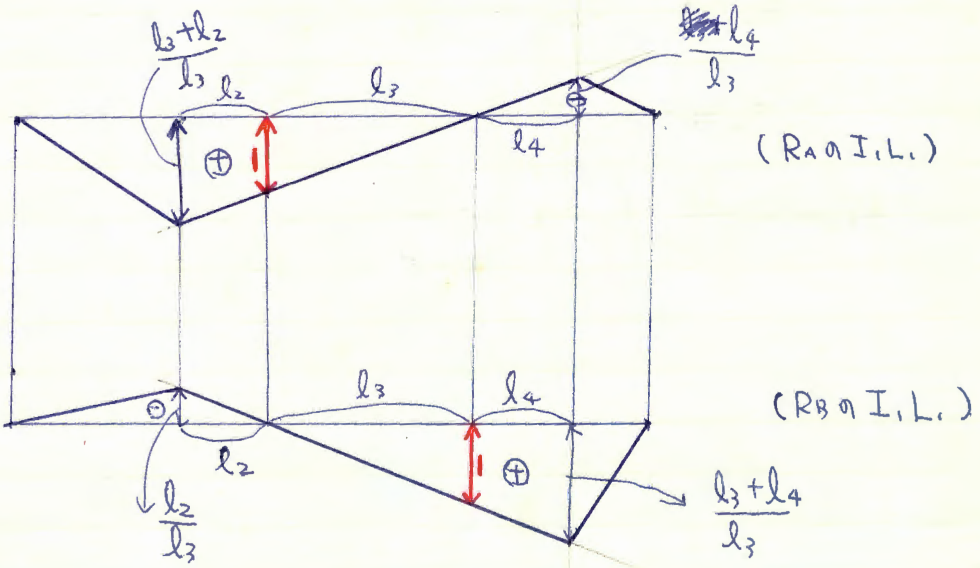
$$R_B = -\frac{l_2}{l_1 l_3} x$$

$0 \leqq x' \leqq l_5$

$P_5 = P = 1, \quad P_1 = P_2 = P_3 = P_4 = 0, \quad a_5 = x' \ (b_5 = l_5 - x')$

$$R_A = -\frac{l_4}{l_3 l_5}(l_5 - x')$$

$$R_B = \frac{P_5 b_5}{l_5} + \frac{l_4}{l_3 l_5} P_5 b_5 = \frac{l_3 + l_4}{l_3 l_5}(l_5 - x')$$



$0 \leqq x \leqq l_1$

$P_1 = P = 1$, $P_2 \sim P_5 = 0$, $a_1 = x$.

$S_x = -R_{G1} + R_A$.

$R_{G1} = \frac{x}{l_1}$, $R_A = \frac{l_2 + l_3}{l_1 l_3} x$

$\therefore S_x = \frac{l_2}{l_1 l_3} x$ //

$M_x = -R_{G1} \cdot (l_2 + c) + R_A \cdot c$

$= -\frac{l_2 + c}{l_1} x + \frac{l_2 + l_3}{l_1 l_3} c x$

$= \frac{x}{l_1 l_3} \{ c(l_2 + \cancel{l_3}) - l_3 (l_2 + \cancel{c}) \}$

$\frac{l_2(l_3 - c)}{l_3}$

$= \frac{l_2 (c - l_3)}{l_1 l_3} x = \frac{-l_2 (l_3 - c)}{l_1 l_3} x$ //

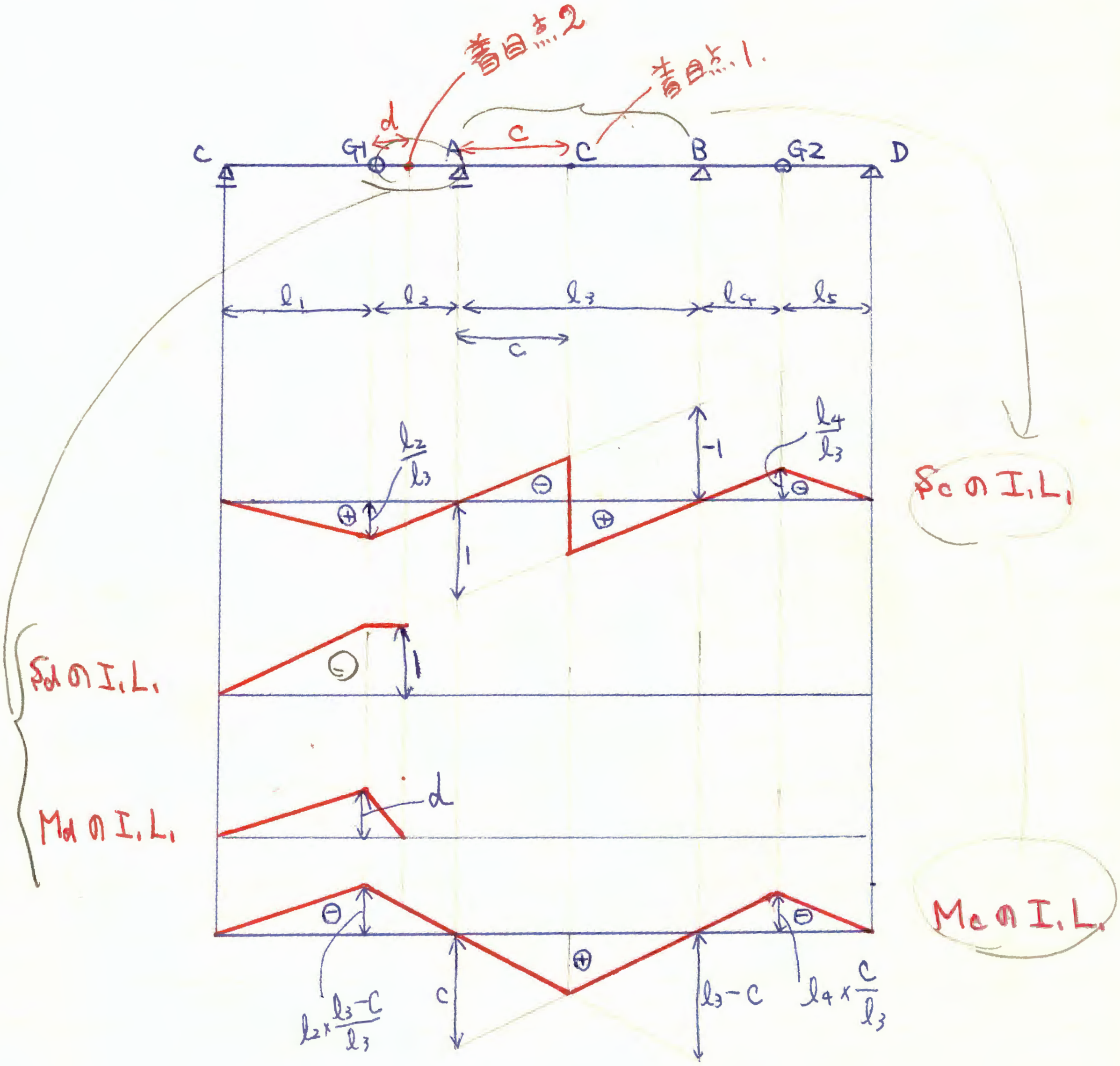
$0 \leqq x' \leqq l_5$ $P_5 = 1$, $P_1 \sim P_4 = 0$, $b_5 = l_5 - x'$.

$R_{G1} = 0$. $R_A = -\frac{l_4}{l_3 l_5}(l_5 - x')$

$\therefore S_x = -\frac{l_4}{l_3 l_5}(l_5 - x')$

$M_x = -\frac{l_4}{l_3 l_5}(l_5 - x') c$ //

$\frac{l + c}{l_3 f}$

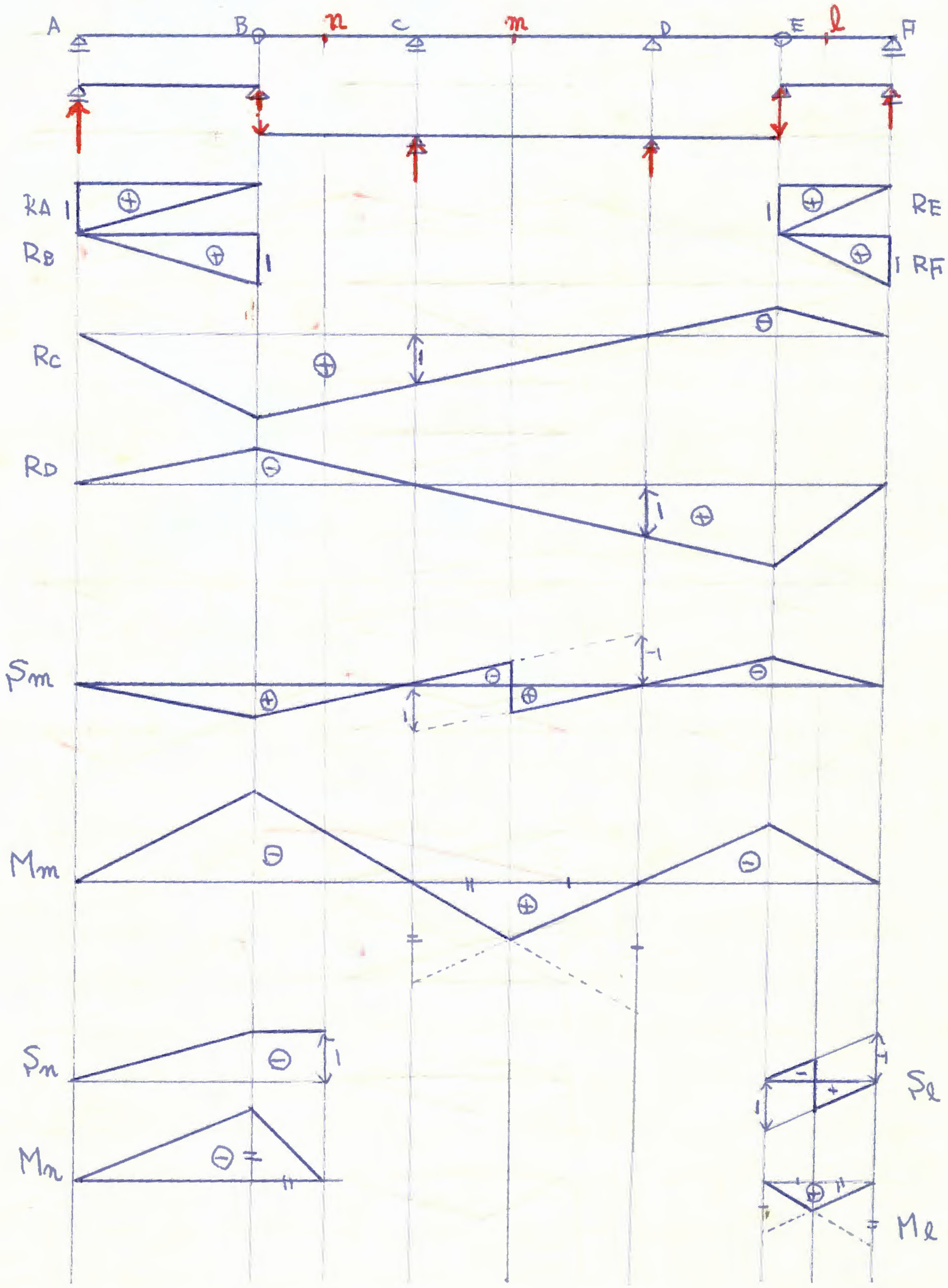


P=1が CG₁上: $S_d = -R_{G1} = -x/l_1$ P=1が G₁D上: $S_d = 1$

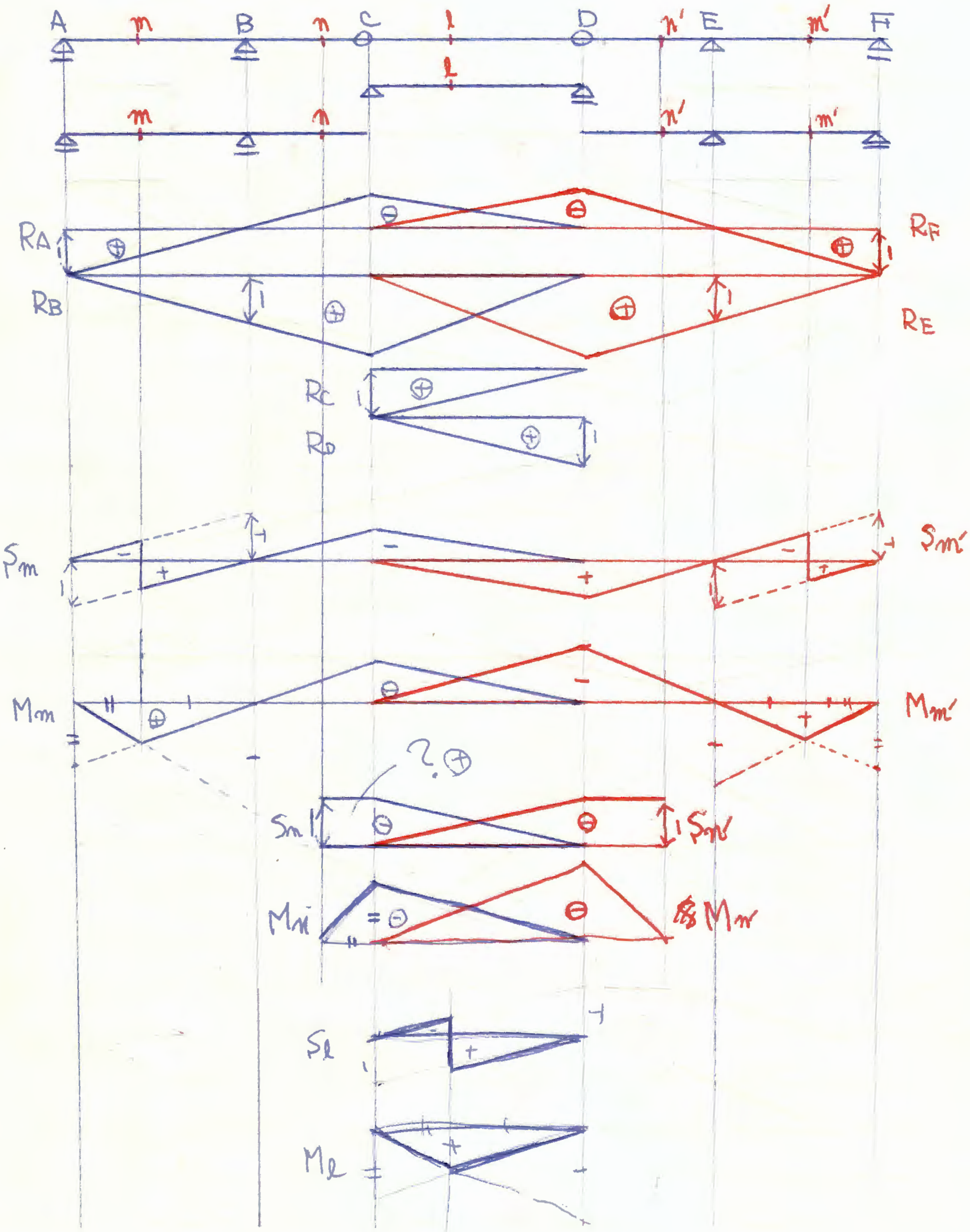
P=1が CG₁上: $M_d = -R_{G1}d = -\frac{d}{l_1}x$. P=1が G₁D上: ~~Md = d~~

$M_d = d - x$.

ゲルバー梁　Inf. line (1)



ゲルバー梁のInf.line (2)



4/13 講義

単純梁の絶対最大曲げモーメント.

(1) 梁の中央点を求める.

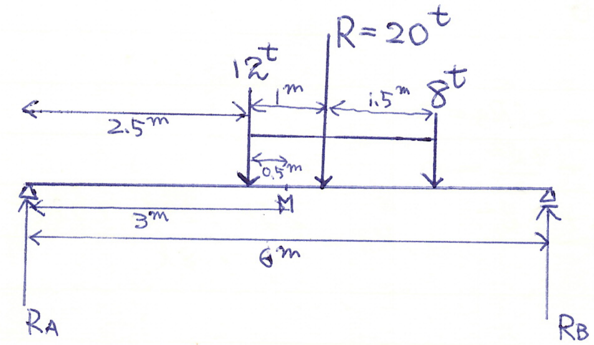
(2) 移動荷重の合力 R の位置を求める.

(3) 合力 R と最も近い荷重との距離の 2分の1 の点を求める

(4) 点(3) と 点(1) を一致せしめる.

(5) (4)の位置で, 中央点に最も近い荷重の載荷点の曲げモーメントを求める

(ex)



(1) $l = 6^m$ より $l/2 = 3^m$

(2) $z = 1^m$ より $z/2 = 0.5^m$

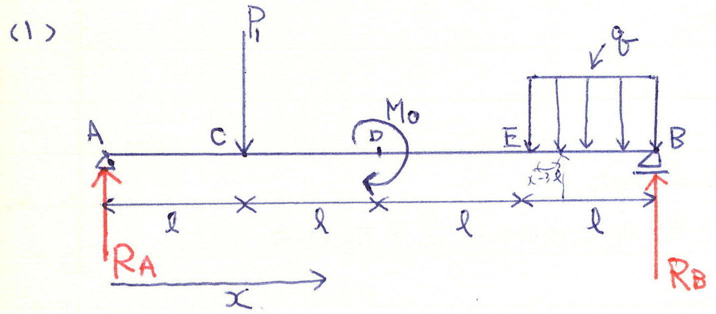
(3) $l/2 - z/2 = 2.5^m$

(4) $R_A = 20^t \times \dfrac{2.5^m}{6^m} = 8.3^t$

$M_{max} = 8.3^t \times 2.5^m = 20.75^{tm}$

4/17 演習.

(1)



S_x M_x を求める.

$$\Sigma V = P_1 + ql - R_A - R_B = 0$$

$$\Sigma M_A = P_1 l + M_0 + \frac{7}{2} q l^2 - 4l R_B = 0$$

$$\Sigma M_B = 4l R_A - 3l P_1 + M_0 - \frac{ql^2}{2} = 0$$

$$R_A = \frac{6lP_1 - 2M_0 + ql^2}{8l}, \quad R_B = \frac{2lP_1 + 2M_0 + 7ql^2}{8l}$$

(i) AC間 $M_x = R_A x$

$S_x = R_A$

(ii) CD間 $M_x = R_A x - P_1(x - l) = (R_A - P_1)x + P_1 l$

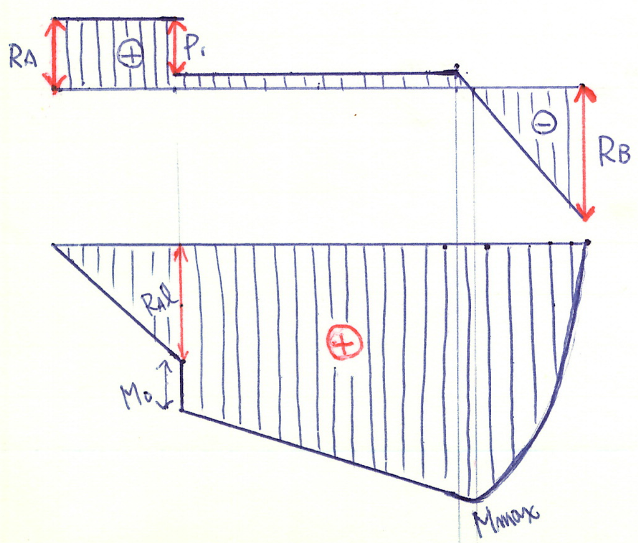
$S_x = R_A - P_1$

(iii) DE間 $M_x = (R_A - P_1)x + P_1 l + M_0$

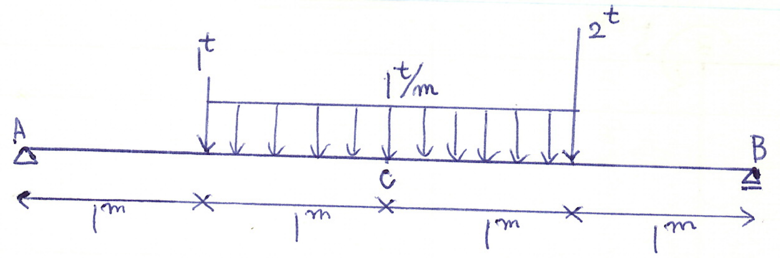
$S_x = R_A - P_1$

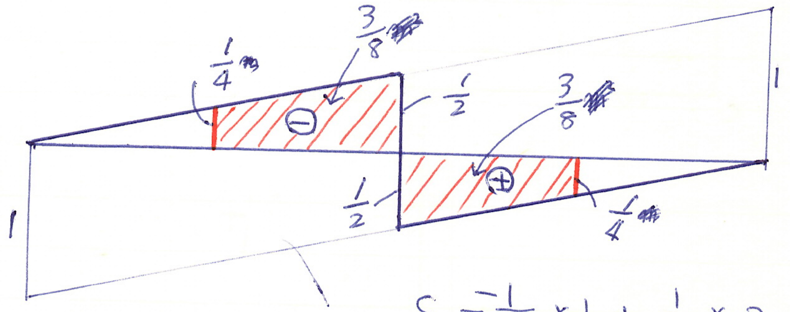
(iv) EB間 $M_x = (R_A - P_1)x + P_1 l + M_0 - \dfrac{(x-3l)^2 q}{2}$

$S_x = R_A - P_1 - (x - 3l)q$



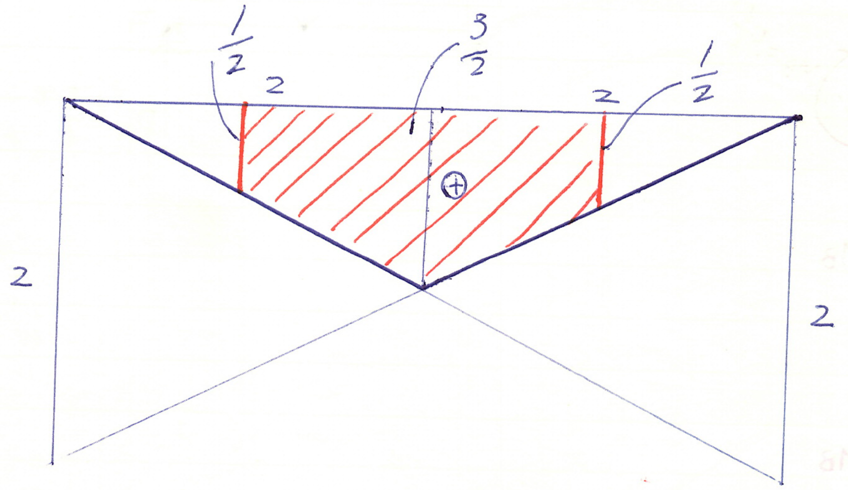
(2)





Sc の Inf. line

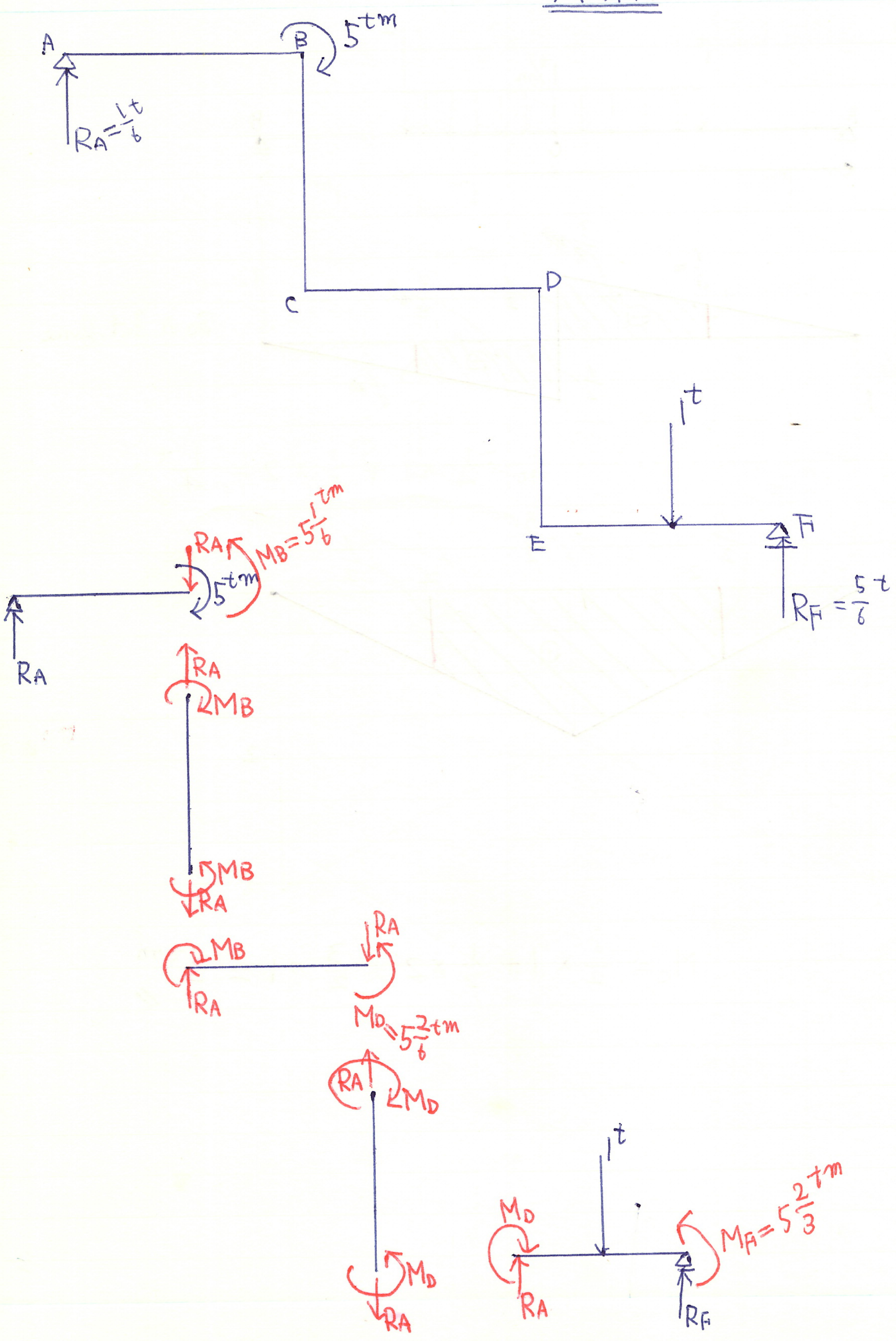
$$S_c = -\frac{1}{4} \times 1 + \frac{1}{4} \times 2 = \frac{1}{4}{}^{t}$$



$$M_c = \frac{1}{2} \times 1 + \frac{1}{2} \times 2 + \frac{3}{2} \times 1 = 3^{tm}$$

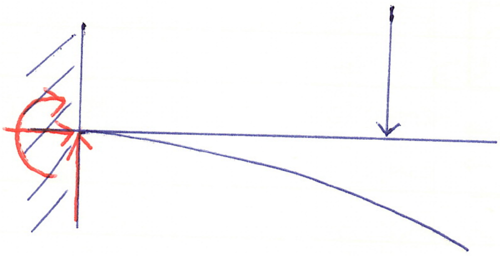
すべて1m

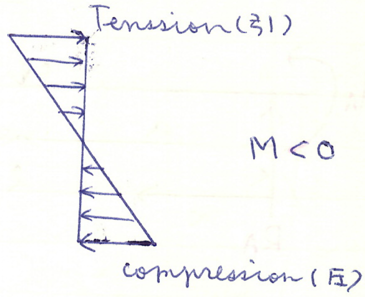
(3)


A
B
5tm
$R_A = \frac{1}{6}$ t
C
D
E
F
1t
$R_F = \frac{5}{6}$ t
R_A
5tm
$M_B = 5\frac{1}{6}$ tm
M_B
$M_D = 5\frac{2}{6}$ tm
M_D
$M_F = 5\frac{2}{3}$ tm
R_F

4/27 コーギ

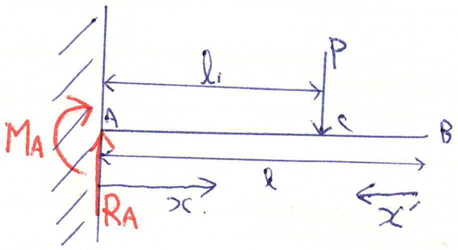
片持梁.





$M<0$

(ex)



$R_A = P$, $M_A = -Pl_1$

(i) AC 間

$M_x = R_A x + M_A$

$= P(x - l_1)$

$S_x = P$

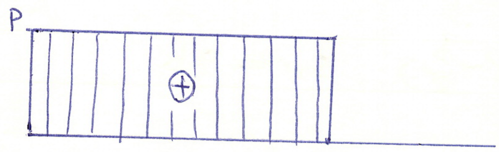
(ii) CB 間 $M_x = P(x-l_1) - P(x-l_1) = 0$

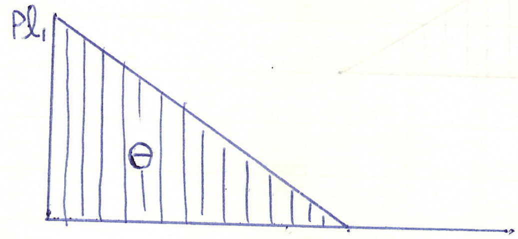
$S_x = 0$

CB 間 $M_{x'} = 0$ $S_{x'} = 0$

AC 間 $M_{x'} = -P(x' - l + l_1)$

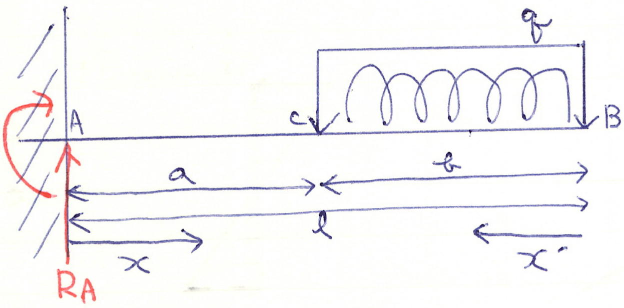
$S_{x'} = P$





2

(ex)



$R_A = qb$, $M_A = -qb(a+\frac{b}{2})$

AC間 $M_x = R_A x + M_A = qbx - qb(a+\frac{b}{2})$

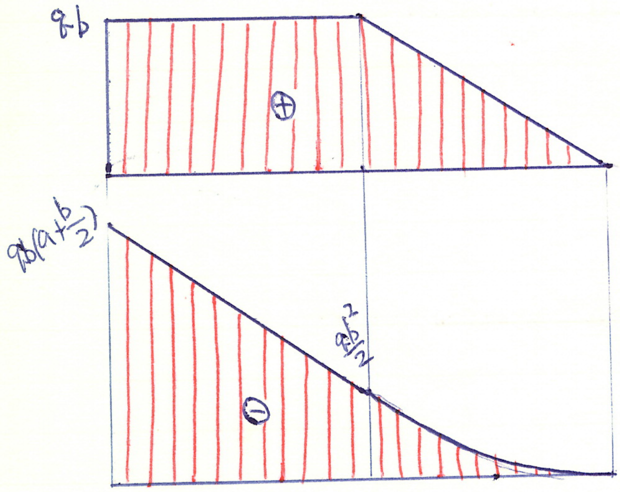
$S_x = qb$

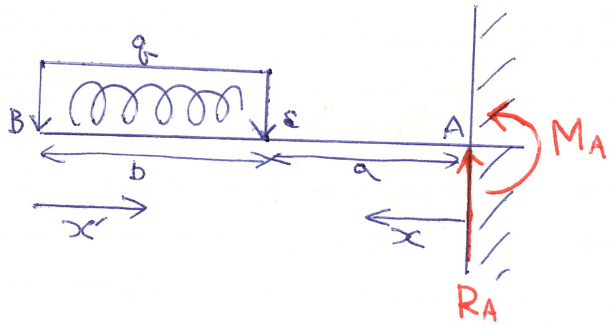
CB間 $M_x = qbx - qb(a+\frac{b}{2}) - \frac{q(x-a)^2}{2}$

$S_x = qb - q(x-a)$

~~AC~~ CB間 $M_{x'} = -\frac{qx'^2}{2}$ $S_{x'} = qx'$

AC間 $M_{x'} = -qb\cdot(x'-\frac{b}{2})$ $S_{x'} = qb$





$R_A = qb$

$M_A = -qb\left(a+\frac{b}{2}\right)$

x' →

BC間 $M_{x'} = -\frac{qx'^2}{2}$

$S_{x'} = -qx'$

CA間 $M_{x'} = -qb\left(x'-\frac{b}{2}\right)$

$S_{x'} = -qb$

← x

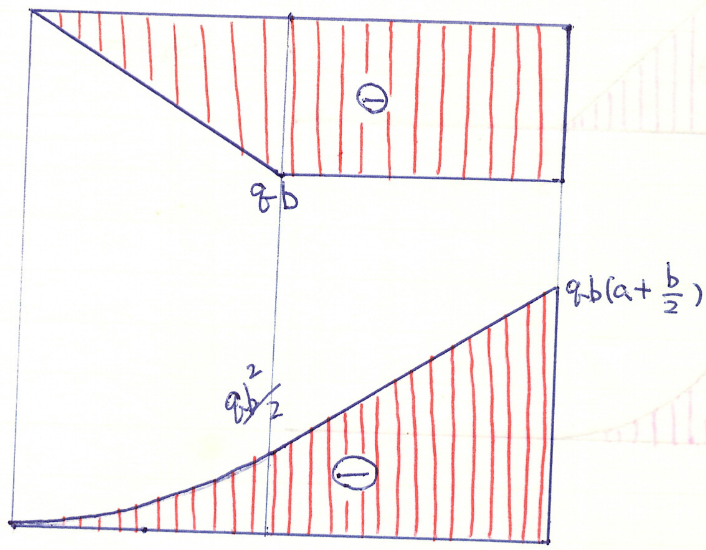
CA間 $M_x = +R_A x + M_A$

$= +qbx - qb\left(a+\frac{b}{2}\right)$

$S_x = -qb$

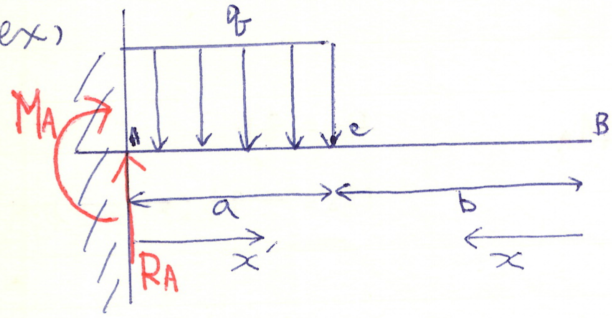
BC間 $M_x = qbx - qb\left(a+\frac{b}{2}\right) - \frac{q(x-a)^2}{2}$

$S_x = -qb + q(x-a)$



4

(ex)



$$\begin{cases} R_A = qa \\ M_A = -\dfrac{qa^2}{2} \end{cases}$$

$x \longleftarrow$

BC間 $M_x = 0$

$S_x = 0$

CA間 $M_x = -\dfrac{q(x-b)^2}{2}$

$S_x = q(x-b)$

$\longrightarrow x'$

CA間 $M_{x'} = R_A x' + M_A - \dfrac{qx'^2}{2}$

$= qax' - \dfrac{qa^2}{2} - \dfrac{qx'^2}{2}$

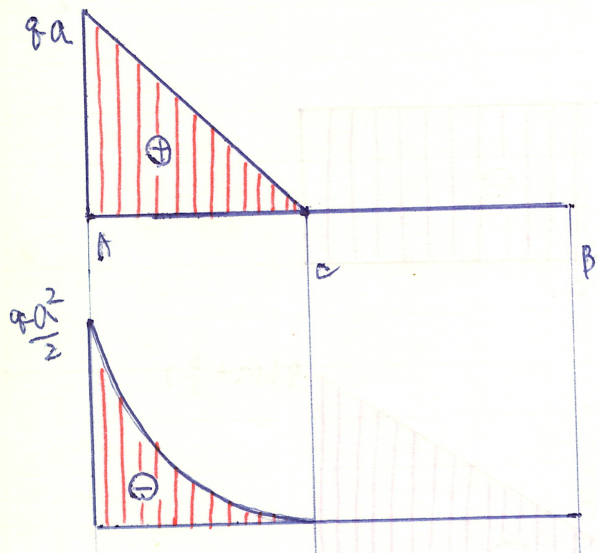
$S_{x'} = R_A - qx' = q(a - x')$

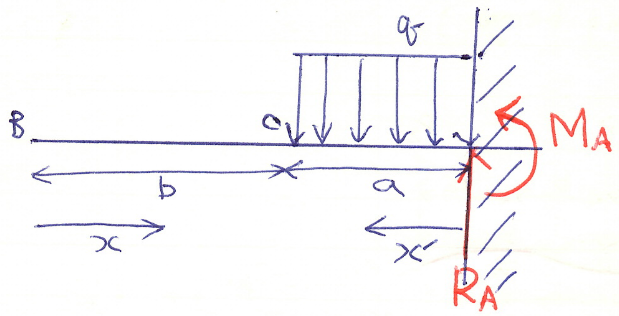
BC間 ~~$M_{x'} = qax' - \dfrac{qa^2}{2} - \dfrac{qx'^2}{2}$~~

$M_{x'} = R_A x' + M_A - qa\left(x' - \dfrac{a}{2}\right)$

$= qax' - \dfrac{qa^2}{2} - qa\left(x' - \dfrac{a}{2}\right) = 0$

$S_{x'} = 0$





$$\begin{cases} R_A = qa \\ M_A = -\dfrac{qa^2}{2} \end{cases}$$

$\longrightarrow x$ $\qquad$ $\longleftarrow x'$

BC 間 $M_x = 0$

$S_x = 0$

CA 間 $M_x = \dfrac{-q(x-b)^2}{2}$

$S_x = -q(x-b)$

$$\begin{cases} S_{x=b} = 0 \quad M_{x=b} = 0. \\ S_{x=a+b} = -qa, \\ M_{x=a+b} = \dfrac{-qa^2}{2} \end{cases}$$

CA 間 $M_{x'} = R_A x' + M_A - \dfrac{qx'^2}{2}$

$= qax' - \dfrac{qa^2}{2} - \dfrac{qx'^2}{2}$

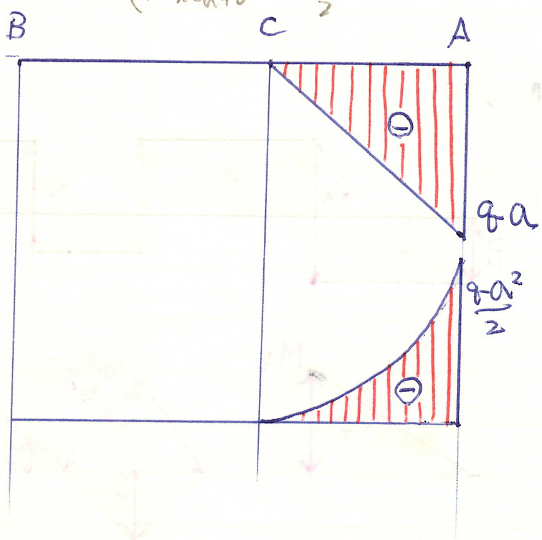
$S_{x'} = -qa + qx'$

$= q(x'-a)$

BC 間 $M_{x'} = qax' - \dfrac{qa^2}{2} - qa(x'-\dfrac{a}{2})$

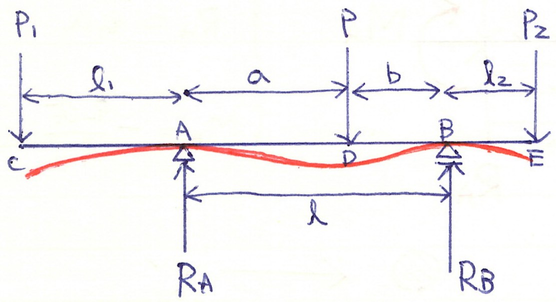
$= 0$

$S_{x'} = 0$



張出し梁 (Overhanging beam)

(ex)



$$\Sigma M_A = -P_1 l_1 + Pa - R_B l + P_2(a+b+l_2) = 0$$

$$R_B = \frac{Pa - P_1 l_1 + P_2(a+b+l_2)}{l}$$

$$\Sigma M_B = -P_1(l_1+a+b) + R_A \cancel{(a)} \cdot l - Pb + P_2 l_2 = 0$$

$$R_A = \frac{Pb \cancel{+} + P_1(l_1+a+b) - P_2 l_2}{l}$$

$\boxed{AC}$ $M_x = -P_1 x$

$\cancel{P}S_x = -P_1$

$\boxed{AD}$ $M_x = -P_1 x + R_A(x-l_1)$

$S_x = -P_1 + R_A$

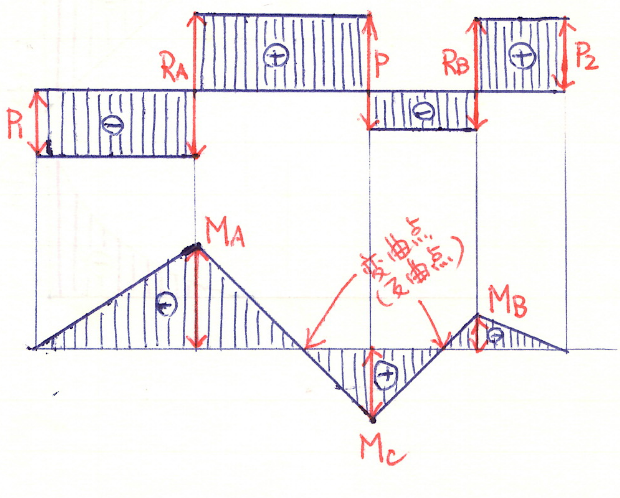
$\boxed{DB}$ $M_x = -P_1 x + R_A(x-l_1) - P(x-l_1-a)$

$S_x = -P_1 + R_A - P$

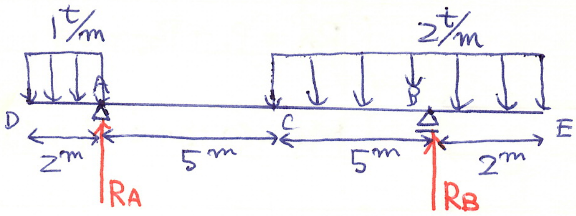
$\boxed{BE}$ $M_x = -P_2(a+b+l_1+l_2-x)$

$S_x = P_2$

$$\begin{cases} M_A = -P_1 l_1 \\ M_C = -P_1(a+l_1) + R_A a \\ M_B = -P_2 l_2 \end{cases}$$



(ex)



$$\begin{cases} R_A + R_B = 2 + 14 = 16^t \\ \Sigma M_B = -2\times 11 + R_A \times 10 - 10\times 2.5 + 4\times 1 = 0 \\ \quad R_A = \frac{1}{10}(22+25-4) = 4.3^t \qquad \therefore R_B = 11.7^t \end{cases}$$

D−A $M_x = -\frac{x^2}{2}$

$S_x = -x$

A−C $M_x = -2(x-1) + R_A(x-2)$

$= 2.3x - 6.6$ 16.1 − 1.1

$S_x = 2.3$

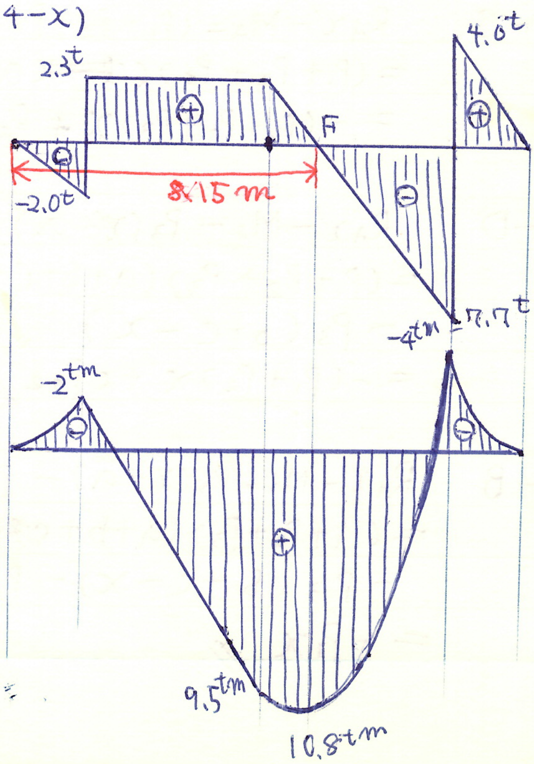
C−B $M_x = \cancel{2(x-1)+}\ 2.3x - 6.6 - (x-7)^2$

$S_x = 2.3 - 2(x-7) = -2x + 16.3$ 24

B−E $M_x = \cancel{-\frac{1}{2}\cdot 2(x-12)^2 = -(x-12)^2} = -(14-x)^2$

$S_x = \cancel{2(x-14)} = 2(14-x)$

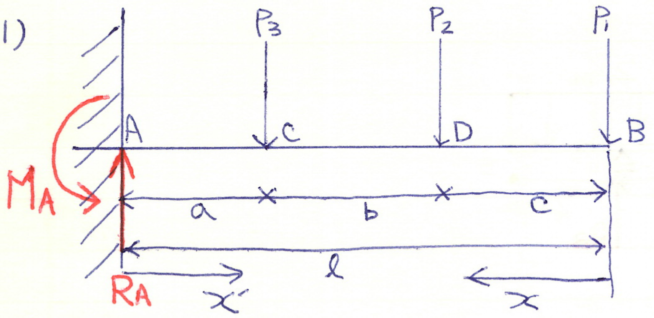
$$\begin{cases} M_A = -2^{tm} \\ M_C = 9.5^{tm} \\ M_F = M_{max} = 2.3\times 8.15 - 6.6 - 1.15^2 \\ \quad = 10.8^{tm} \\ M_B = -4.0^{tm} \end{cases}$$



8

5/1 演習

(1)



$$\begin{cases} R_A = P_1 + P_2 + P_3 \\ M_A = (a+b+c)P_1 + (a+b)P_2 + aP_3 \end{cases}$$

B→D $M_x = -P_1 x$
$S_x = P_1$

D→C $M_x = -P_1 x - P_2(x-c)$
$S_x = P_1 + P_2$

C→A $M_x = -P_1 x - P_2(x-c) - P_3(x-b-c)$
$S_x = P_1 + P_2 + P_3$

A→C $M_{x'} = R_A x' - M_A$
$S_{x'} = R_A = P_1 + P_2 + P_3$

C→D $M_{x'} = R_A x' - M_A - P_3(x'-a)$
$S_{x'} = R_A - P_3 = P_1 + P_2$

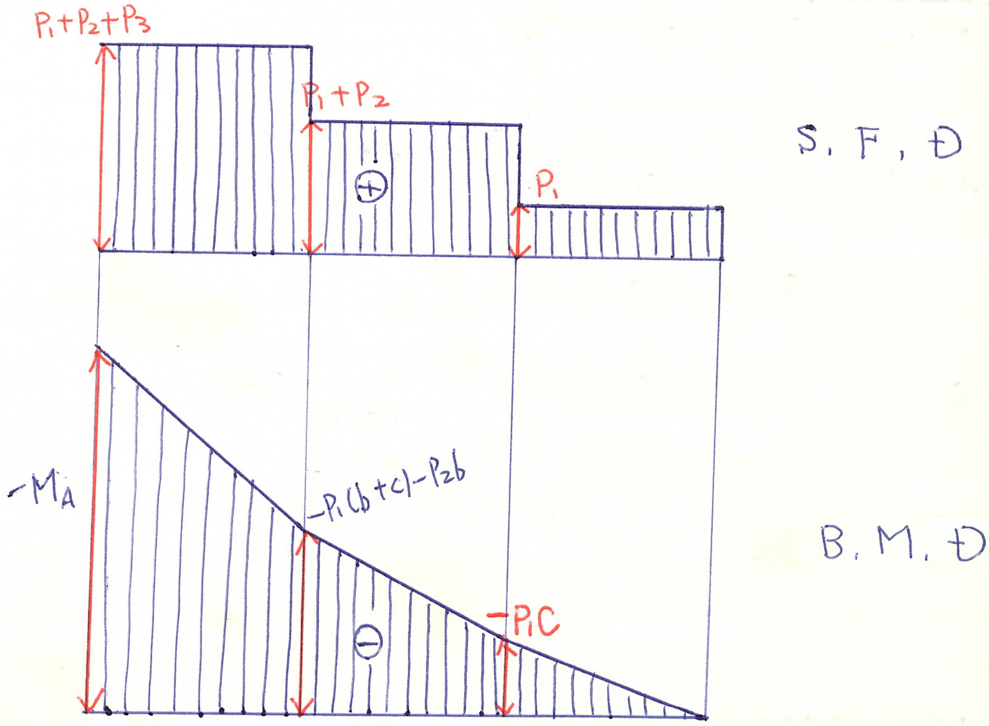
D→B $M_{x'} = R_A x' - M_A - P_3(x'-a) - P_2(x'-a-b)$
$S_{x'} = R_A - P_3 - P_2 = P_1$

(参)

A−C $R_A x' - M_A = (P_1+P_2+P_3)x' - (a+b+c)P_1 - (a+b)P_2 - aP_3$
$= (P_1+P_2+P_3)(a+b+c-x) - (a+b+c)P_1 - (a+b)P_2 - aP_3$
$= -(P_1+P_2+P_3)x + P_2 c + P_3(b+c)$ //

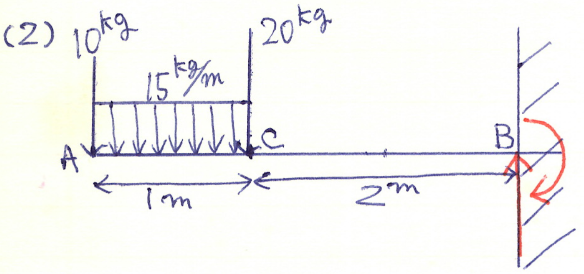
C−D $R_A x' - M_A - P_3(x'-a)$
$= (P_1+P_2+P_3)(a+b+c-x) - (a+b+c)P_1 - (a+b)P_2 - aP_3 - P_3(b+c-x)$
$= -(P_1+P_2)x + cP_2$ //

D−B $R_A x' - M_A - P_3(x'-a) - P_2(x'-a-b)$
$= (P_1+P_2+P_3)(a+b+c-x) - (a+b+c)P_1 - (a+b)P_2 - aP_3 - P_3(b+c-x) - P_2(c-x)$
$= -P_1 x$ //


P1+P2+P3
P1+P2
P1
-MA
-P1(b+c)-P2b
-P1C
S.F.D
B.M.D

10

(2)



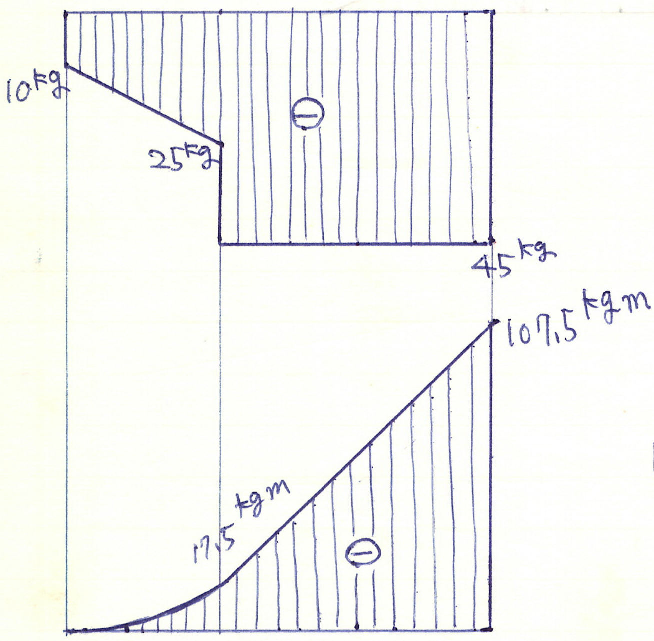
S, F, D　B, M, D を求めよ。

A−C　$M_x = -10x - \frac{15x^2}{2}$ kg·m

$S_x = -10 - 15x$ kg

C−B　$M_x = -10x - 15(x - \frac{1}{2}) - 20(x-1)$
$= -45x + 27.5$ kg·m

$S_x = -45$ kg

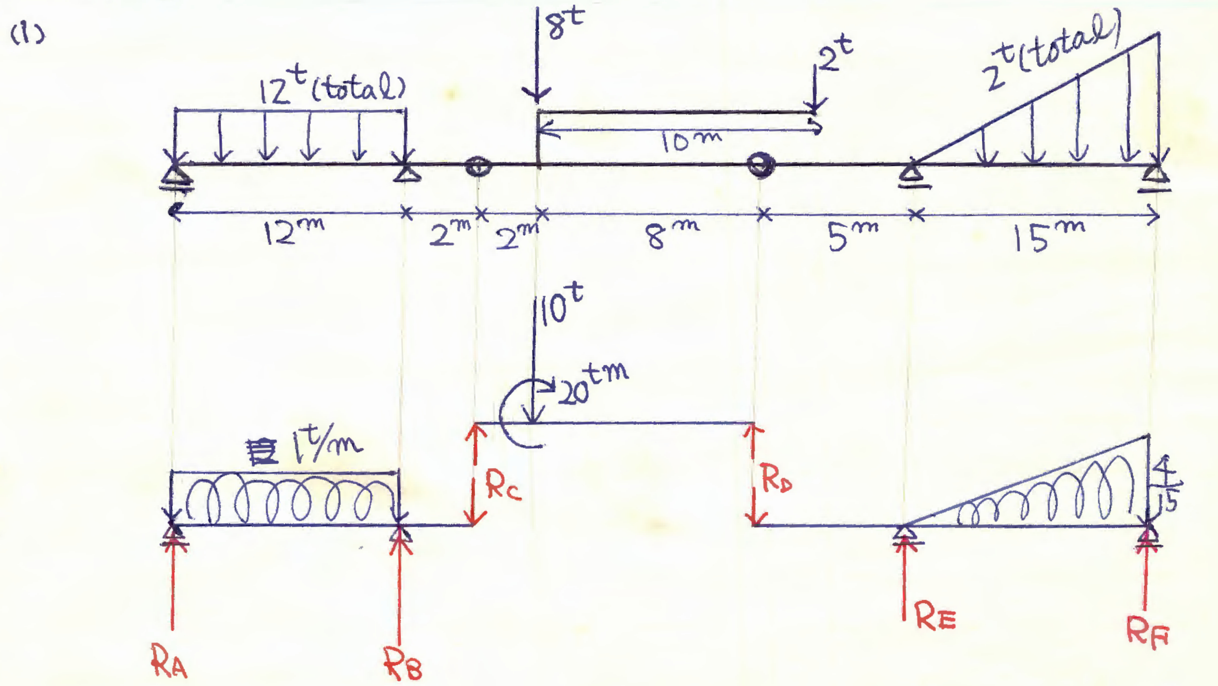


S, F, D

B, M, D

5/8 演習.

(1)



$$\begin{cases} R_C + R_D = 10^t \\ \Sigma M_C = 20 + 20 - 10R_D = 0 \end{cases} \quad \therefore R_D = 4^t \quad \therefore R_C = 10-4=6^t$$

$$\begin{cases} R_A + R_B = 12+6 = 18^t \\ \Sigma M_A = 6 \times 12 - 12 \times R_B + 14 \times 6 = 0 \end{cases} \quad \therefore R_B = 13^t \quad \therefore R_A = 5^t$$

$$\begin{cases} R_E + R_F = 6^t \\ \Sigma M_F = 5 \times 2 - 15 \times R_E + 20 \times 4 = 0 \end{cases} \quad \therefore R_E = 6^t \quad \therefore R_F = 0^t$$

AB間 $S_x = 5 - x^t$ $M_x = 5x - \frac{x^2}{2}{}^{tm}$

BC間 $S_x = 5 + 13 - 12 = 6^t$ $M_x = 5x - (x-6)\cdot 12 + 13(x-12)$
$= 6x - 84^{tm}$

CG間 $S_x = 6^t$ $M_x = 6(x-14)^{tm}$

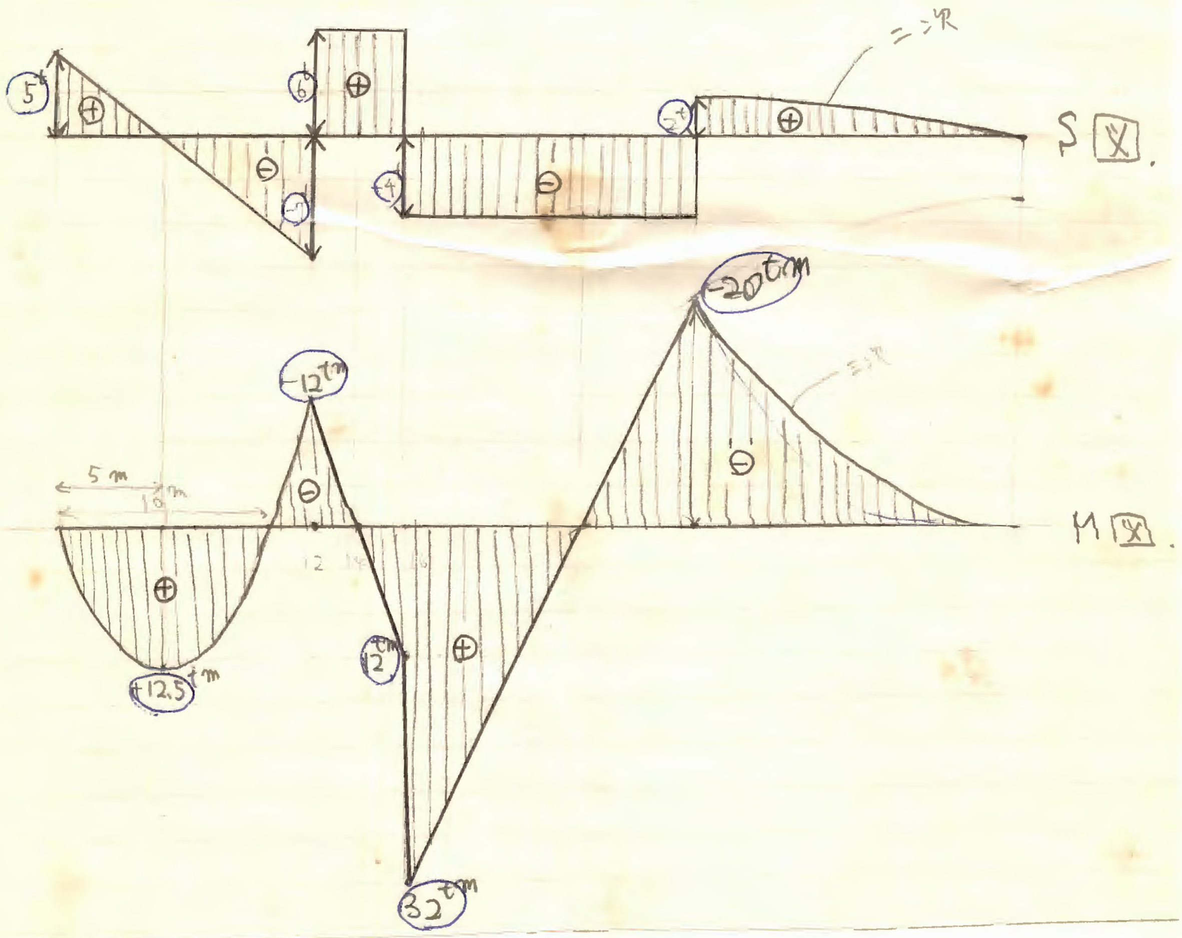
GD間 $S_x = 6 - 10 = -4^t$ $M_x = 6(x-14) - 10(x-16) + 20$
$= -4x + 96$

間. $S_x = -4^t \qquad M_x = -4(x-24)^{tm}$

EF間. $S_x = -4 + 6 - \frac{\cancel{4}^{2}}{15} \times \frac{x-29}{15} \times \frac{x-29}{\cancel{2}} = 2 - \frac{2}{225}(x-29)^2 \; {}^t$

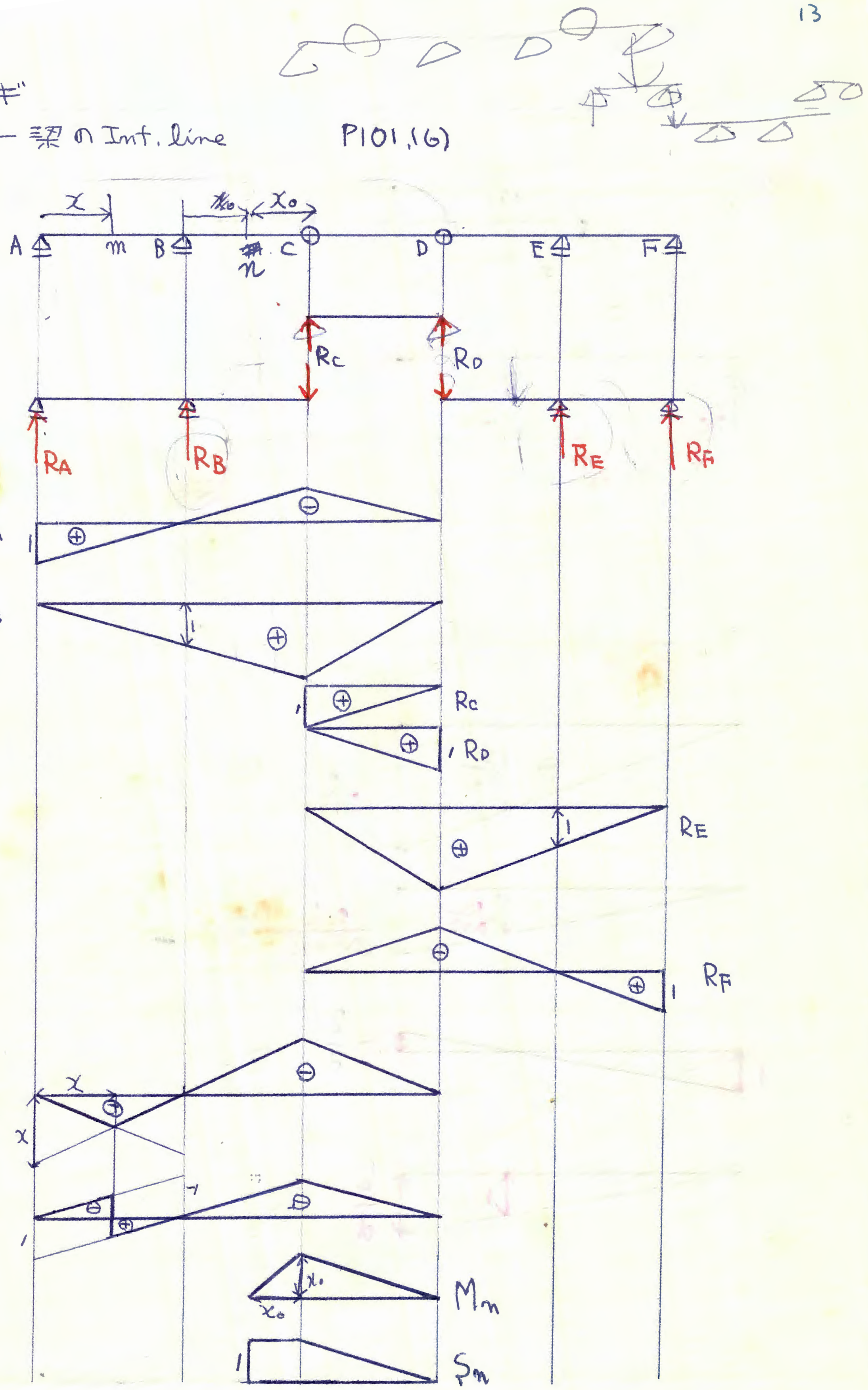
$$M_x = -4(x-24) + 6(x-29) - \frac{2}{225}(x-29)^2 \cdot \frac{x-29}{3}$$

$$= 2x - 78 - \frac{2}{675}(x-29)^3 \; {}^{tm}$$



5/11 講ギ

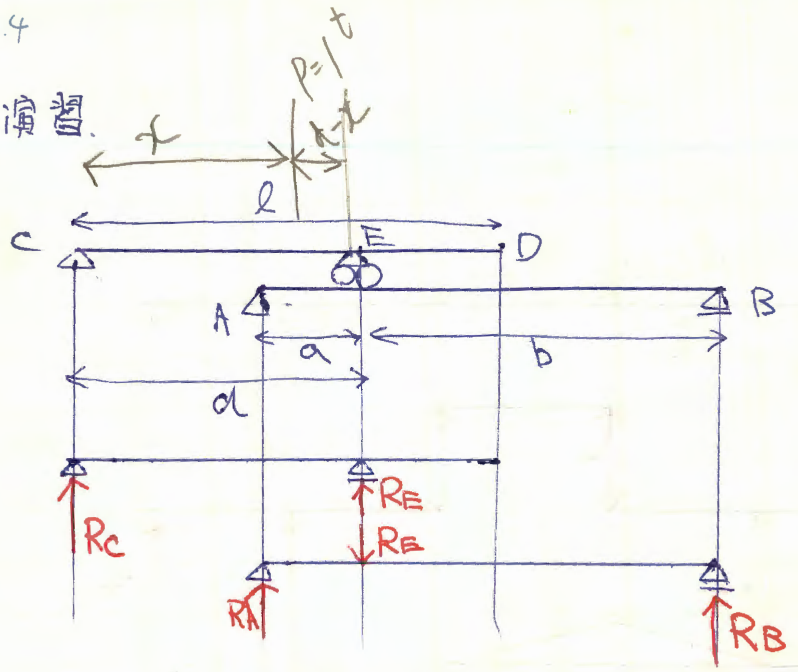
ゲルバー梁の Inf. line　　P101,(6)



1.4

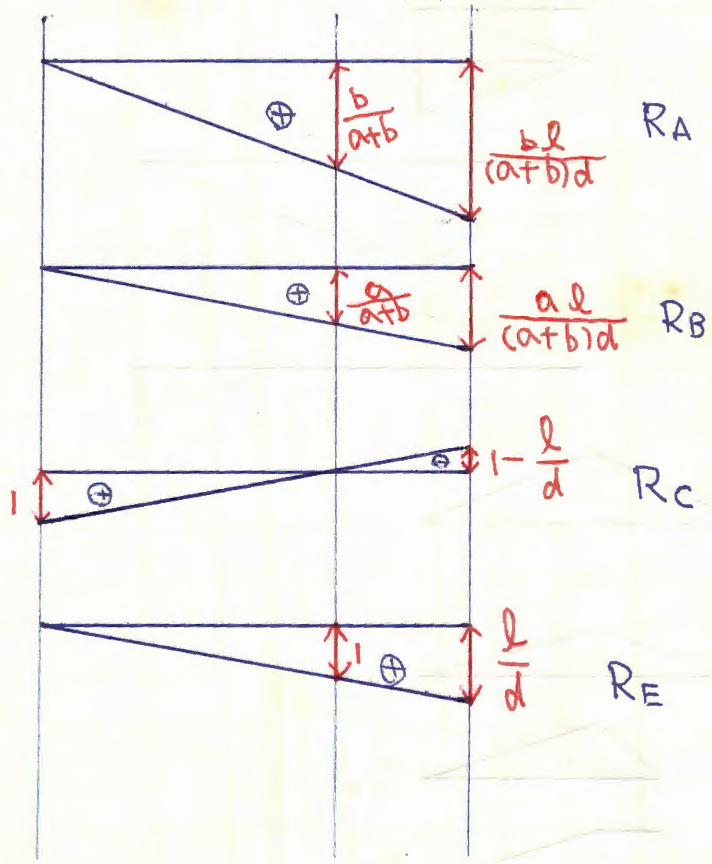
5/15 演習

(1)

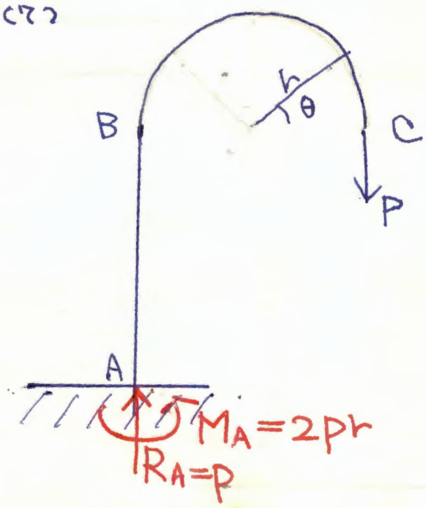


$$R_C = \frac{d-x}{d} \quad , \quad R_E = \frac{x}{d}$$

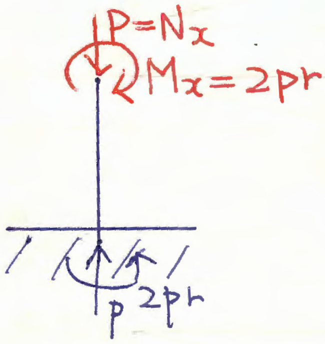
$$R_A = \frac{b}{a+b} R_E = \frac{bx}{(a+b)d} \quad , \quad R_B = \frac{a}{a+b} R_E = \frac{ax}{(a+b)d}$$



(2) P101(7)

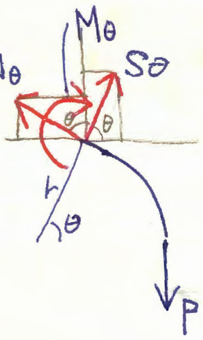


(i) AB間



$$\begin{cases} N_x = -R_A = -P \ (\text{圧縮}) \\ S_x = 0 \\ M_x = -2Pr \end{cases}$$

(ii) BC間



$$\begin{cases} \Sigma V = P - S_\theta \sin\theta - N_\theta \cos\theta = 0 \quad \text{①} \\ \Sigma H = S_\theta \cos\theta - N_\theta \sin\theta = 0 \quad \text{②} \end{cases}$$

① × $\cos\theta$ + ② × $\sin\theta$

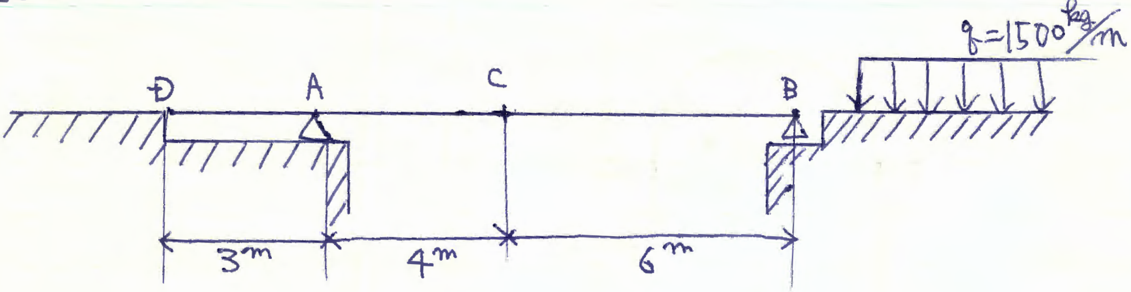
$$P\cos\theta - N_\theta \cos^2\theta - N_\theta \sin^2\theta = 0$$

$\therefore N_\theta = P\cos\theta$ (引張)

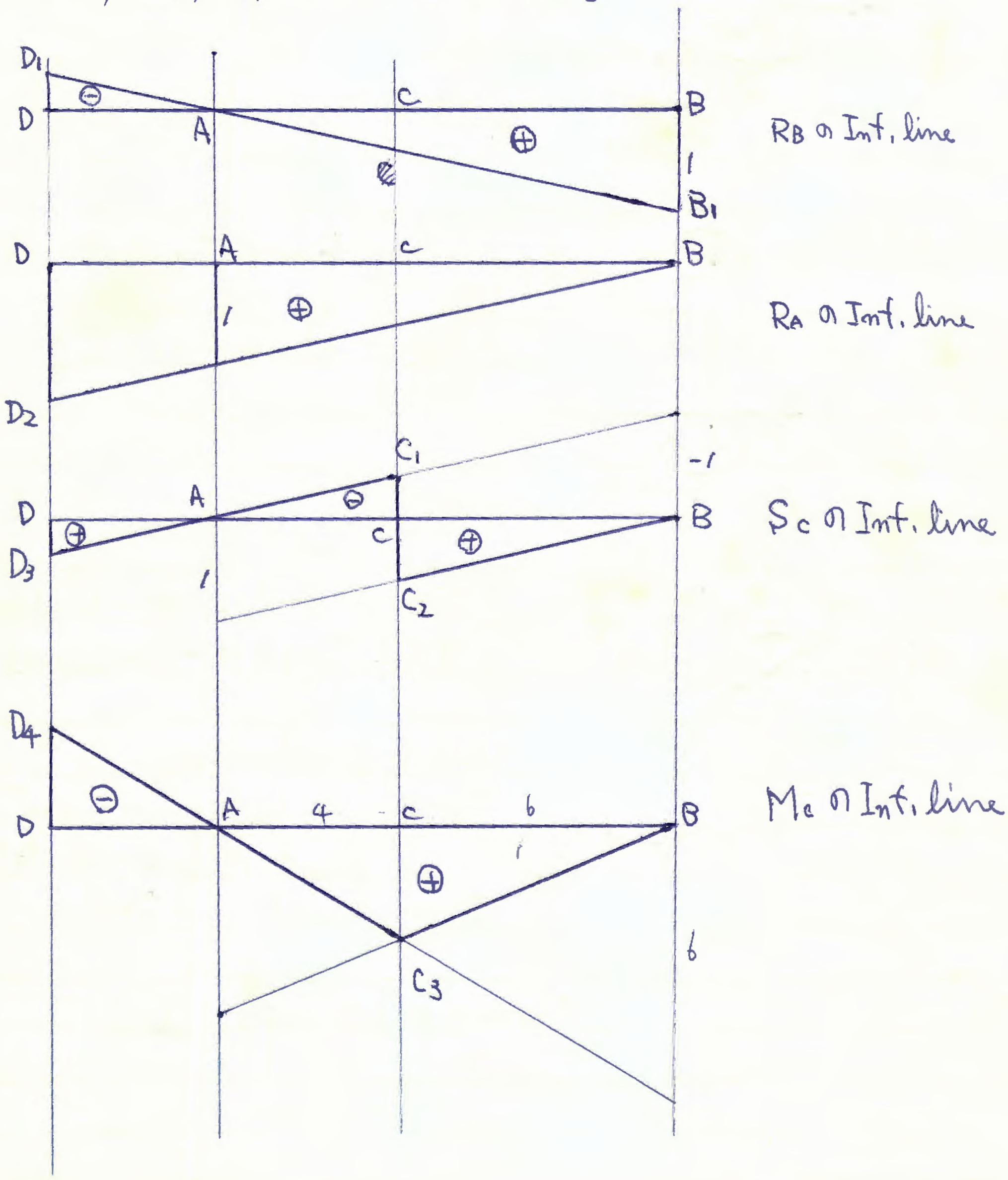
$S_\theta = P\sin\theta$ (↑↓)

$M_\theta = -Pr(1-\cos\theta)$

5/22 演習.



最も危険な長さ不定の等分布荷重がかかるときの M_c, S_c, R_B の最大を求める。



(1) AB間に載荷 —— R_B pos max
DA間 〃 —— R_B neg max

$$R_B \text{ pos max} = \triangle ABB_1 \times q = (1 \times 10 \times \tfrac{1}{2}) \times 1500 = 7500^{kg}$$
$$R_A \text{ neg max} = -\triangle ADD_1 \times q = 3 \times \tfrac{3}{10} \times \tfrac{1}{2} \times 1500 = -675^{kg}$$

(2) DB間の載荷 —— R_A max

$$R_A \text{ max} = 13 \times \tfrac{13}{10} \times \tfrac{1}{2} \times 1500 = 12475^{kg}$$

(3) BC間に載荷 —— S_C pos max
AC間 〃 —— S_C neg max.

$$S_C \text{ pos max} = \triangle BCC_2 \times q = 6 \times \tfrac{6}{10} \times \tfrac{1}{2} \times 1500 = 2700^{kg}$$
$$S_C \text{ neg max} = -\triangle ACC_2 \times q = -4 \times \tfrac{4}{10} \times \tfrac{1}{2} \times 1500 = -1200^{kg}$$

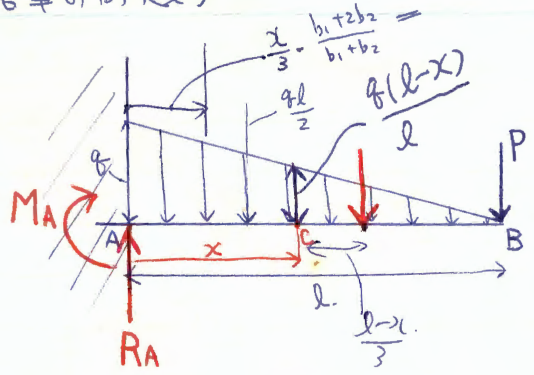
(4) AB間に載荷 —— M_C pos max
DA間 〃 —— M_C neg max.

$$M_C \text{ pos max} = \triangle ABC_3 \times q = 10 \times \overset{6}{\cancel{4}} \times \tfrac{4}{10} \times \tfrac{1}{2} \times 1500 = 18000^{kgm}$$

$$M_C \text{ neg max} = -\triangle ADD_4 \times q = -3 \times 6 \times \tfrac{3}{10} \times \tfrac{1}{2} \times 1500 = -4050^{kgm}$$

（6章の問題）

(1)

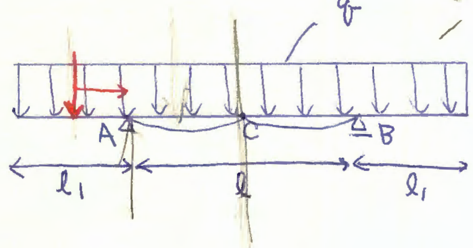


$$\begin{cases} R_A = \dfrac{q\ell}{2} + P \\ M_A = -P\ell - \dfrac{q\ell^2}{6} \end{cases}$$

$$\begin{cases} S_{\cancel{x}\,cx} = P + (\ell - x)\cdot\dfrac{q(\ell-x)}{\ell}\cdot\dfrac{1}{2} = P + \dfrac{q(\ell-x)^2}{2\ell} \\ M_{\cancel{x}\,cx} = -P(\ell-x) - \dfrac{q(\ell-x)^2}{2\ell}\left\{\dfrac{\ell-x}{3}\right\} = -P(\ell-x) - \dfrac{q(\ell-x)^3}{6\ell} \end{cases}$$

19

(2)



$$|M_A| = |M_B| = \frac{q l_1^2}{2}$$

また

~~$M_c = \frac{q}{2}$~~

$$M_x = \frac{qx}{2}(l - x) + \frac{x}{2l}(q_1 l_1^2 - q_2 l_2^2) - \frac{q_1 l_1^2}{2}$$

$q_1 = q_2 = q$ 　$x = l/2$ とおくと　$l_2 = l_1$

$$|M_c| = \frac{ql}{4} \cdot \frac{l}{2} + \frac{q}{4}(l_1^2 - l_2^2) - \frac{q l_1^2}{2}$$

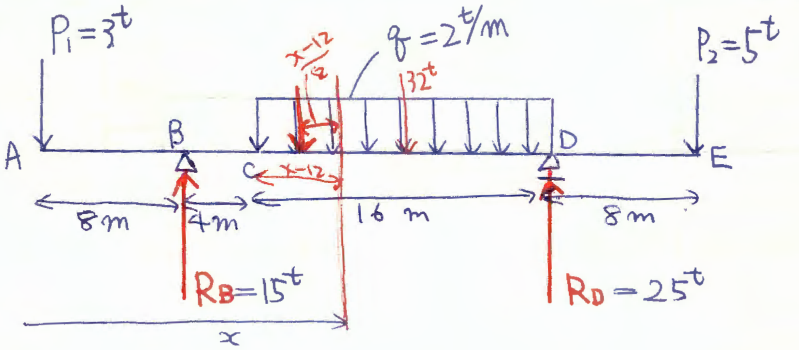
$$= \frac{ql^2}{8} + \frac{q l_1^2}{4} - \frac{q l_1^2}{4} = \frac{ql^2}{8} - \frac{q l_1^2}{2}$$

故に　$\frac{q l_1^2}{2} = \frac{q l^2}{8} - \frac{q l_1^2}{2}$

$\therefore q l_1^2 = \frac{q l^2}{8}$

$\therefore 8 l_1^2 = l^2$ 　　$\therefore l = 2\sqrt{2}\, l_1$

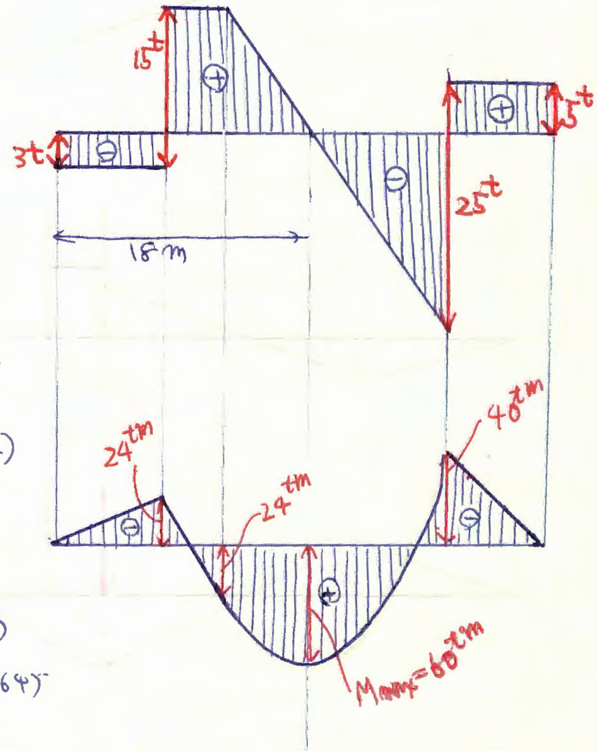
(3)



$R_B + R_D = 40^t$

$\Sigma M_D = 8\cdot 5 + 20R_B - 32\cdot 8 - 3\cdot 28 = 0$

$\therefore R_B = \frac{-40+256+84}{20} = 15^t$ $\therefore R_D = 25^t$



i) $0 \leqq x \leqq 8$ $M_x = -3x^{tm}$

$S_x = -3^t$

ii) $8 \leqq x \leqq 12$ $M_x = -3x + R_B(x-8)$

$= 12x - 120^{tm}$

$S_x = 12^t$

iii) $12 \leqq x \leqq 28$ $M_x = 12x - 120 - \frac{(x-12)^2}{\cancel{2}}\cancel{2}$

$\cancel{= \frac{1}{2}(x^2 - 48x + 384)}$

$\cancel{\frac{1}{2}(x-24)^2}$

$= -(x^2 - 36x + 264)$

$= -\{(x-18)^2 - 324 + 264\}$

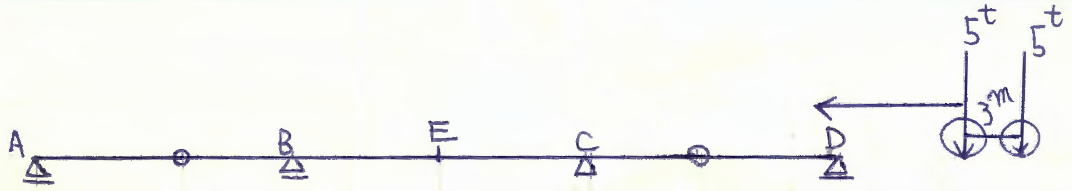
$= -(x-18)^2 + 60^{tm}$

$S_x = -2(x-18)^t$

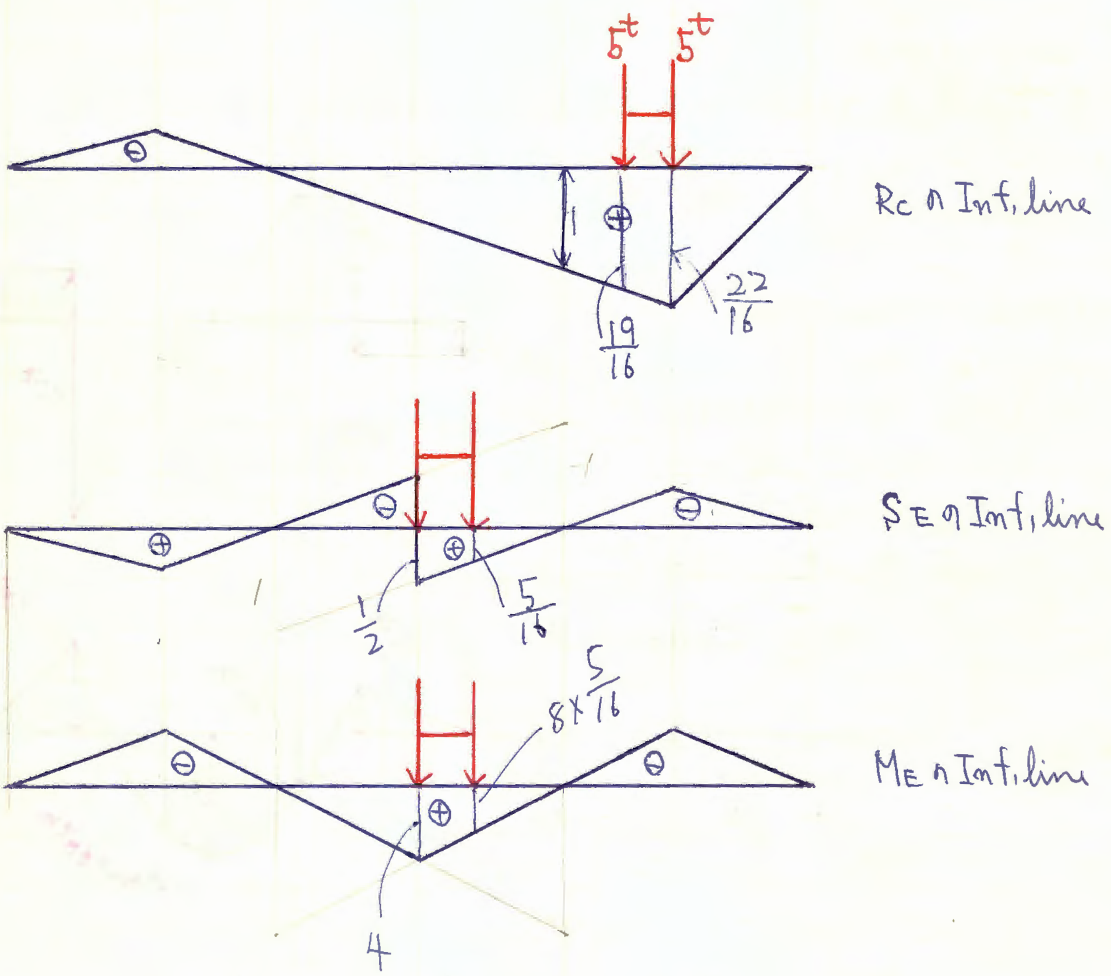
iv) $28 \leqq x \leqq 36$. $M_x = -5(36-x)^{tm}$

$S_x = 5^t$

(4)



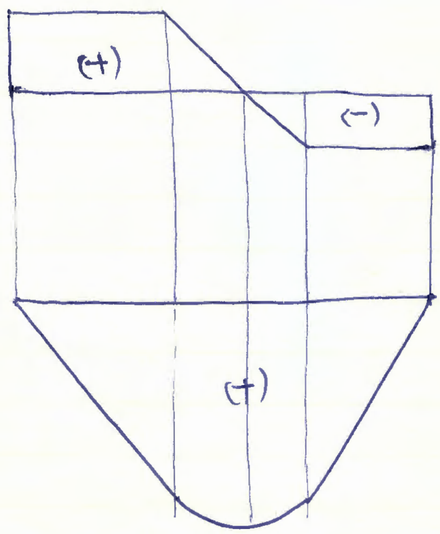
R_C, M_E, S_E の max.



$$R_{C\,max} = 5\left(\frac{19}{16}+\frac{22}{16}\right) = \frac{5}{16}\times 41 = 12.8^{t}$$

$$S_{E\,max} = 5\left(\frac{8}{16}+\frac{5}{16}\right) = \frac{5}{16}\times 13 = 4.06^{t}$$

$$M_{E\,max} = \frac{5}{16}(64+40) = 32.5^{tm}$$



$a \leqq x \leqq a+c$ において

$S_x = R_A x - (x-a)q = 0$

とおくと

$x = \dfrac{c}{2l}(2b+c) + a$

このとき

$M_x = \left\{\dfrac{cq}{2l}(2b+c)\right\}\left\{\dfrac{c}{2l}(2b+c)+a\right\}$

$\quad - \dfrac{q}{2}\left\{\dfrac{c}{2l}(2b+c)\right\}^2$

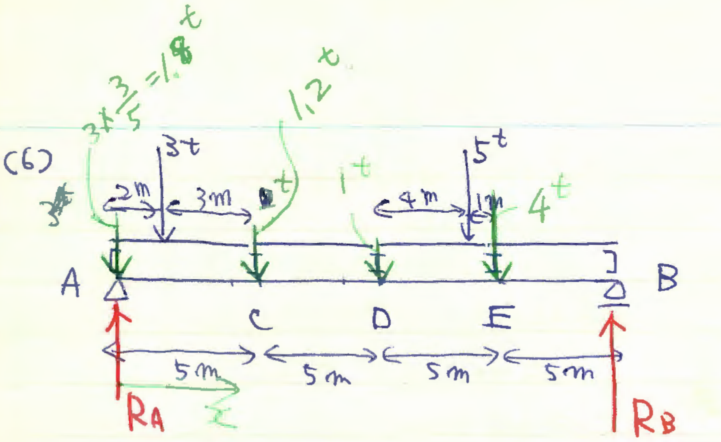
$\cancel{= q\dfrac{1}{4l^2}}$

$= \dfrac{q}{2}\left\{\dfrac{c}{2l}(2b+c)\right\}^2 + \dfrac{cqa}{2l}(2b+c)$

$= \dfrac{cq}{8l^2}\left\{\cancel{2bc+c^2}\ c(2b+c)^2 + 4la(2b+c)\right\}$

$= \dfrac{cq}{8l^2}(2b+c)(4la+2bc+c^2)$

(6)



$$\begin{cases} R_A = 3\cdot\frac{18}{20} + 5\cdot\frac{6}{20} = \frac{84}{20} = 4.2^t \\ R_B = 8 - R_A = 3.8^t \end{cases}$$

i) $0 \leqq x \leqq 5$ $\quad S_x = 2.4^t$

$$M_x = (R_A - 1.8)x = 2.4x^{t\cdot m}$$

ii) $5 \leqq x \leqq 10$ $\quad S_x = 1.2^t$

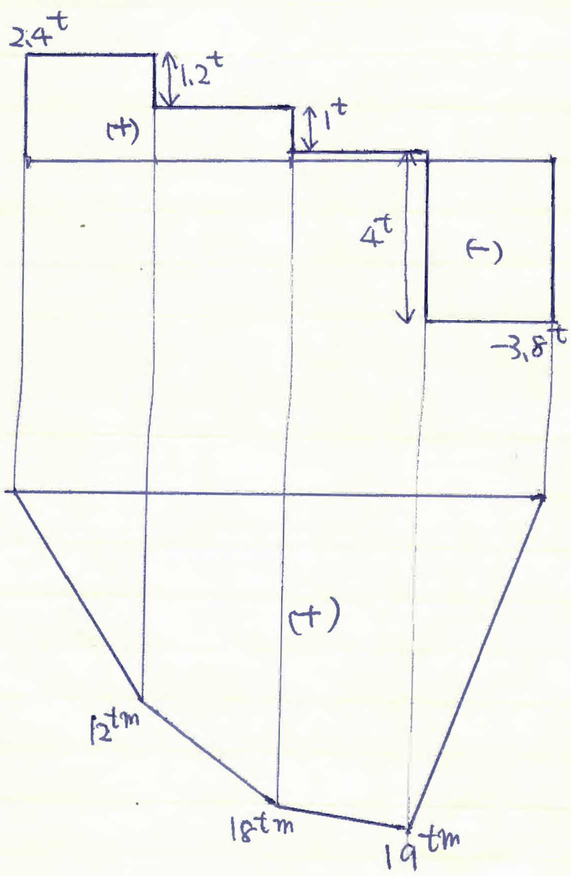
$$M_x = 2.4x - 1.2(x-5)$$
$$= 1.2x + 6^{\ t\cdot m}$$

iii) $10 \leqq x \leqq 15$ $\quad S_x = 0.2^t$

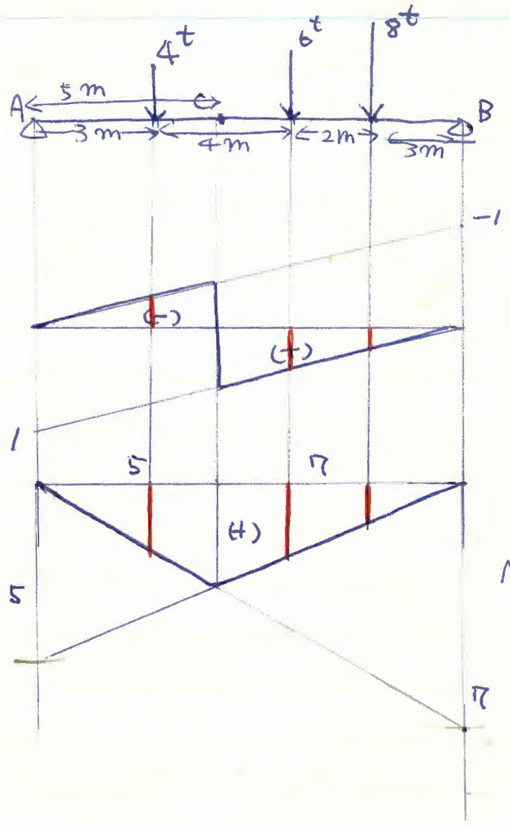
$$M_x = 1.2x + 6 - 1(x-10)$$
$$= 0.2x + 16^{\ t\cdot m}$$

iv) $15 \leqq x \leqq 20$ $\quad S_x = -3.8^t$

$$M_x = 0.2x + 16 - 4(x-15)$$
$$= -3.8x + 76^{\ t\cdot m}$$



(8)



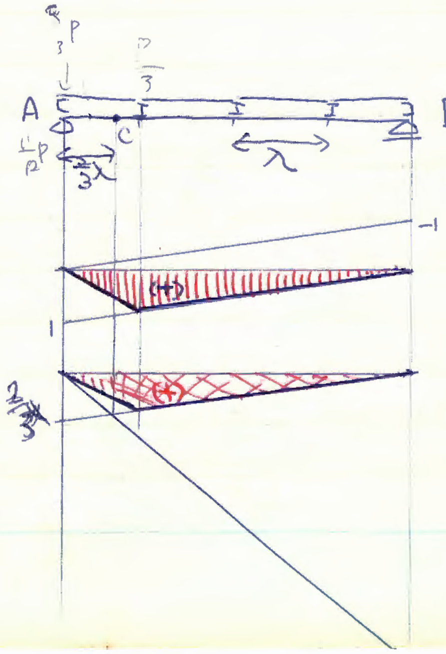
S_c の Influ Line

M_c の Influe Line

$$S_c = 4 \times \frac{-3}{12} + 6 \times \frac{5}{12} + 8 \times \frac{3}{12} = -1 + 2.5 + 2 = 3.5^{t}$$

$$M_c = 4 \times \left(7 \times \frac{3}{12}\right) + 6 \times \left(5 \times \frac{5}{12}\right) + 8 \times \left(5 \times \frac{3}{12}\right)$$
$$= 7 + 12.5 + 10 = 29.5^{t \cdot m}$$

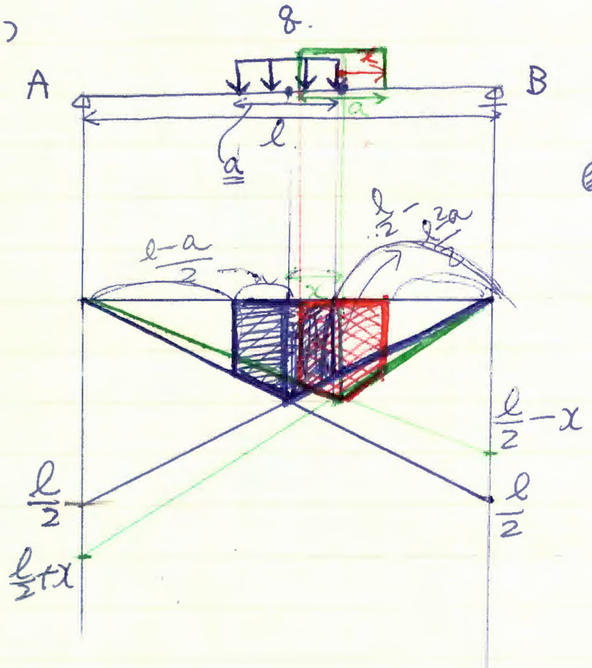
(9)



$$S_{max} = P \cdot \frac{3}{4} = \frac{3P}{4}\ ^{t.}$$

$$M_{max} = P \cdot \frac{2}{3}\lambda \times \frac{3}{4} = \frac{P\lambda}{2}\ ^{t \cdot m}$$

(10)



曲げモーメントは荷重の真下で最大になる。

$$M_c=\left(\frac{l}{4}+\frac{\cancel{l}}{2}\cdot\frac{l-a}{2\cancel{l}}\right)\times\frac{\frac{a}{2}}{\cancel{2}}\times\cancel{2}\times q=\frac{2l-a}{4}\cdot\frac{aq}{2}=\frac{aq(2l-a)}{8}$$

$$M_x=\left\{\frac{\frac{l}{2}+x}{l}\left(\frac{l}{2}-x\right)+\frac{\frac{l}{2}+x-\frac{a}{2}}{l}\left(\frac{l}{2}-x\right)\right\}\times\frac{\frac{a}{2}}{2}$$

$$+\left\{\frac{\frac{l}{2}+x}{l}\left(\frac{l}{2}-x\right)+\frac{\frac{l-a}{2}-x}{l}\cdot\frac{l}{2}\right\}\times\frac{\frac{a}{2}}{2}$$

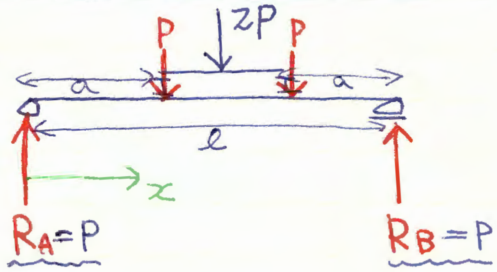
$$=\frac{a}{4}\cdot\frac{\frac{l}{2}+x}{l}\left(\frac{l}{2}-x\right)\left\{\frac{\frac{l-a}{2}+x}{l}\left(\frac{l}{2}-x\right)+\frac{\frac{l-a}{2}-x}{l}\cdot\frac{l}{2}\right\}$$

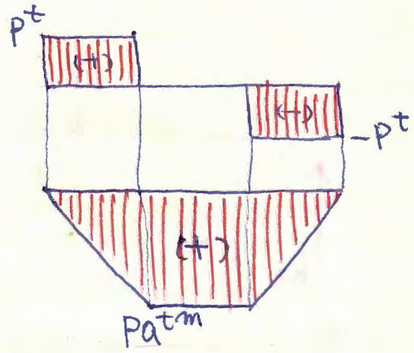
$$=(d+x)(e-x)\left\{(a+x)(b-x)+(c-x)\right\}$$

$$\frac{a(l+2x)}{8}\cdot\frac{l-2x}{2}\left\{\frac{l-a+2x}{2l}\,\frac{l-2x}{2}+\frac{l-a-2x}{2l}\cdot\frac{l}{2}\right\}\ ?$$

$M'_x<0$ //

(演1)





i) $0 \leqq x \leqq a$

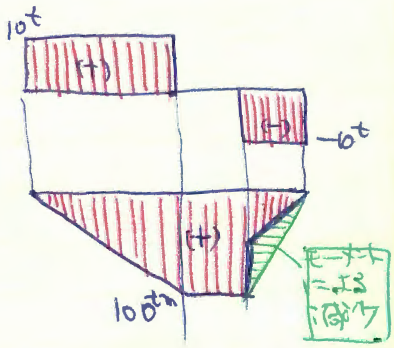
$$M_x = R_A x = Px^{t\cdot m}$$
$$S_x = P^t$$

ii) $a \leqq x \leqq l-a$

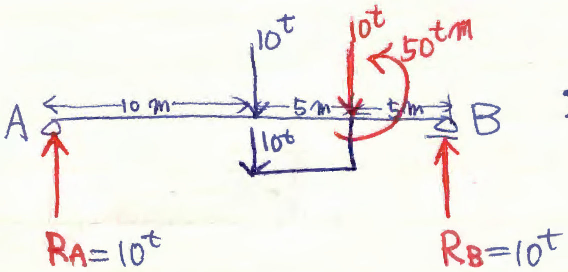
$$M_x = Px - P(x-a) = Pa^{t\cdot m}$$
$$S_x = 0^t$$

iii) $l-a \leqq x \leqq l$

$$M_x = Pa - P(x-l+a)$$
$$= -Px + Pl^{\,t\cdot m}$$
$$S_x = -P^t$$



(演2)



$$\Sigma M_A = 10\cdot 10 + 10\cdot 15 - 20R_B - 50 = 0$$
$$\therefore R_B = 10^t$$
$$R_A = 20 - R_B = 10^t$$

i) $0 \leqq x \leqq 10$ $\quad M_x = R_A x = 10x^{tm}$

$$S_x = 10^t$$

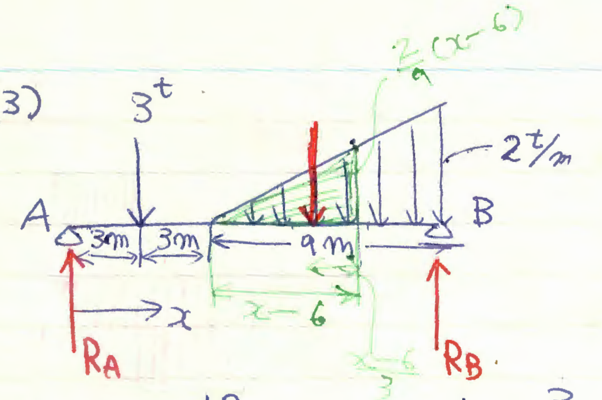
ii) $10 \leqq x \leqq 15$ $\quad M_x = 10x - 10(x-10)$

$$= 100^{tm}$$
$$S_x = 0$$

iii) $15 \leqq x \leqq 20$

$$M_x = 100 - 10(x-15) - 50$$
$$= -10x + 200^{tm}$$
$$S_x = -10^{t}$$

(演3)



$$\begin{cases} R_A = 3\times\frac{12}{15} + \cancel{2}\times 9\times\frac{1}{\cancel{2}}\times\frac{3}{15} = \frac{63}{15} = 4.2^t \\ R_B = 12 - 4.2 = 7.8^t \end{cases}$$

i) $0 \leqq x \leqq 3$ $M_x = R_A x = 4.2x^{tm}$

$S_x = 4.2^t$

ii) $3 \leqq x \leqq 6$ $M_x = 4.2x - 3(x-3)$

$= 1.2x + 9^{\ tm}$

$S_x = 1.2^t$

iii) $6 \leqq x \leqq 15$ $M_x = 1.2x + 9 - \frac{x-6}{3}\cdot\frac{1}{2}(x-6)\cdot\frac{2}{9}(x-6)$

$= 1.2x + 9 - \frac{1}{27}(x-6)^3$

$S_x = 1.2 - \frac{1}{9}(x-6)^2$

$$\begin{cases} M_{x=3} = 12.6^{tm} \\ M_{x=6} = 16.2^{tm} \end{cases}$$

~~$M_{x=15}$~~

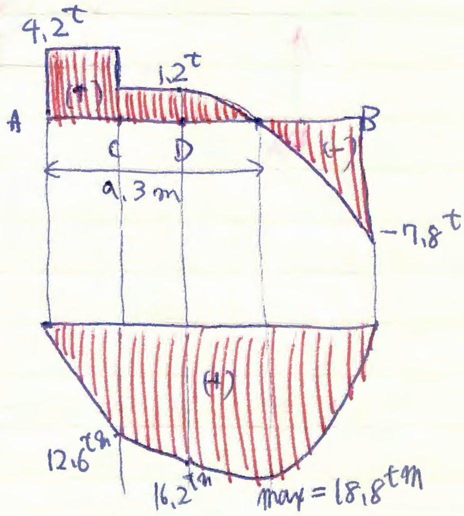
iii)において

$S_x = 0$ とおくと

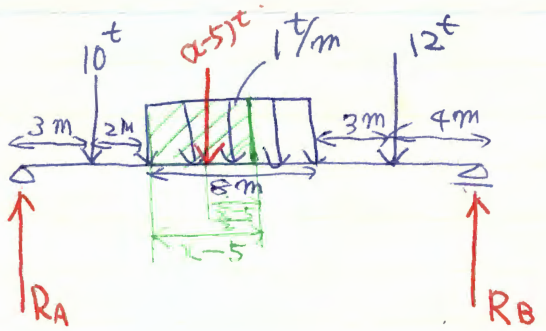
$x = 9.3$.

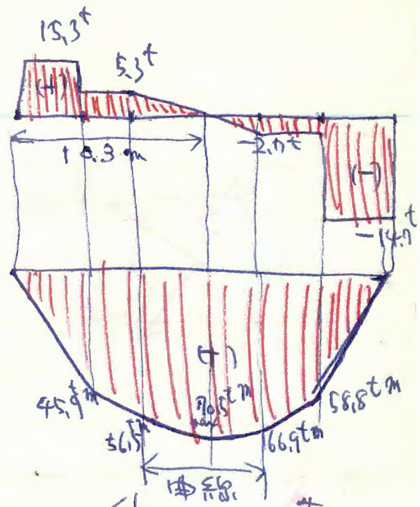
$\therefore M_{max} = 1.2\times 9.3 + 9 - \frac{1}{27}\times 3.3^3$

$= 18.8^{tm}$



(演4)





$$\begin{cases} R_A = 10 \times \frac{17}{20} + 1 \times \frac{8}{2}\left(\frac{7}{20} + \frac{15}{20}\right) + 12 \times \frac{4}{20} = 15.3^t \\ R_B = 20 - R_A = 14.7^t \end{cases}$$

i) $0 \leqq x \leqq 3$

$M_x = R_A x = 15.3x^{tm}$

$S_x = 15.3^t$

ii) $3 \leqq x \leqq 5$

$M_x = 15.3x - 10(x-3)$

$= 5.3x + 30^{tm}$

$S_x = 5.3^{tm}$

iii) $5 \leqq x \leqq 13$

$M_x = 5.3x + 30 - \frac{x-5}{2} \cdot (x-5)$

$= 5.3x + 30 - \frac{1}{2}(x-5)^2 \ ^{tm}$

$S_x = 5.3 - (x-5)$

$= -x + 10.3^t$

iv) $13 \leqq x \leqq 16$

$M_x = 5.3x + 30 - \frac{1}{2}(x-5)^2 - 12(x-13)$

$= -6.7x - \frac{1}{2}(x-5)^2 + 186$

$S_x = -6.7 - (x-5)$

$= -x + 1.7$

iv) $13 \leqq x \leqq 16$.

$M_x = 5.3x + 30 - 8(x-9)$

$= -2.7x + 102.^{tm}$

$S_x = -2.7^t$

v) $16 \leqq x \leqq 20$

$M_x = -2.7x + 102 - 12(x-16)$

$= -14.7x + 294^{tm}$

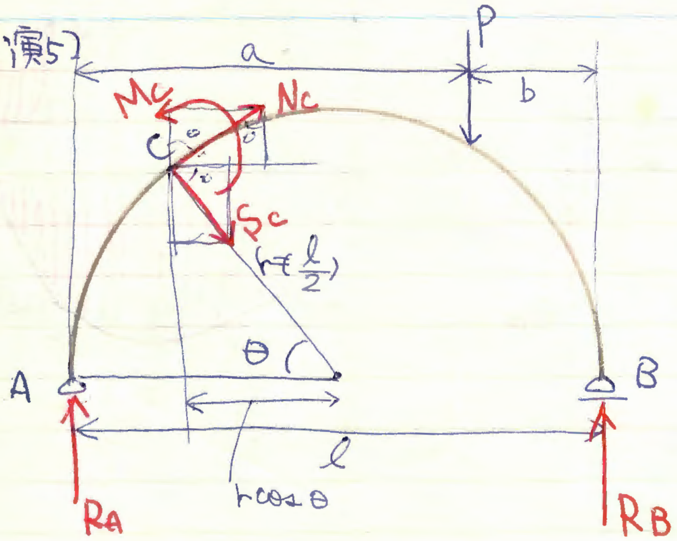
$S_x = -14.7^t$

iii) で、$x = 10.3$ のとき.

$M_x = 70.5$; max.

$$\begin{cases} M_{x=0} = 0 \\ M_{x=3} = 45.9^{tm} \\ M_{x=5} = 56.5^{tm} \\ M_{x=13} = 66.9^{tm} \\ M_{x=16} = 58.8^{tm} \\ M_{x=20} = 0. \end{cases}$$

(演5)



$$\left(R_A = \frac{bP}{l},\ R_B = \frac{aP}{l} \right)$$

$$M_c = R_A(r - r\cos\theta) = \frac{bP}{l} \cdot r(1-\cos\theta) = \frac{bP}{2}(1-\cos\theta)$$

$$\begin{cases} \Sigma V = R_A + N_c\cos\theta - S_c\sin\theta = 0 \\ \Sigma H = N_c\sin\theta + S_c\cos\theta = 0 \end{cases}$$

$$\therefore\ S_c = -\tan\theta\, N_c$$

$\Sigma V = 0$ に代入. $\quad (\cancel{\cos\theta + \tan\theta})N_c + R_A = 0$

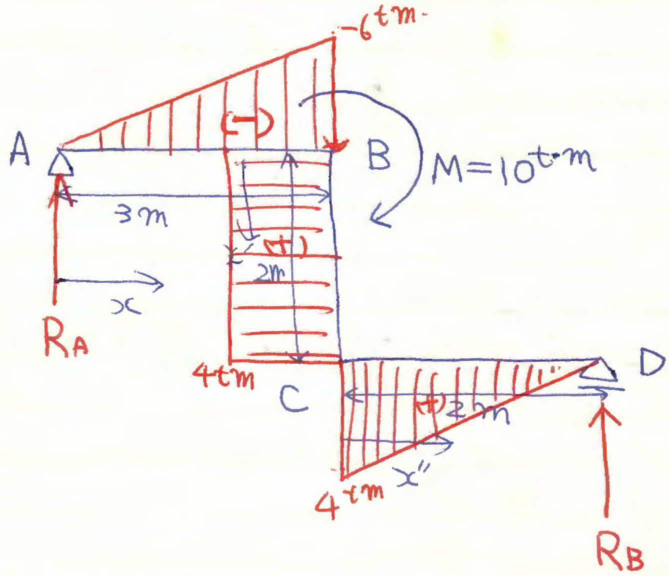
$$(\cos\theta + \tan\theta\sin\theta)N_c + R_A = 0$$

$$\therefore\ N_c = -\frac{R_A}{\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}} = -\cos\theta\, R_A$$

$$\therefore\ S_c = +\sin\theta\, R_A$$

$$\therefore\ N_c = -\frac{bP}{l}\cos\theta,\quad S_c = \frac{bP}{l}\sin\theta$$

P80 (15)



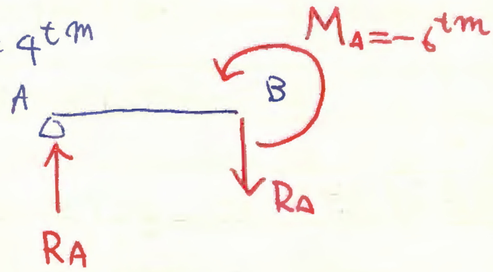
$\Sigma M_D = R_A * 5 + 10 = 0$

$\begin{cases} R_A = -2^t \\ \therefore R_B = 2^t \end{cases}$

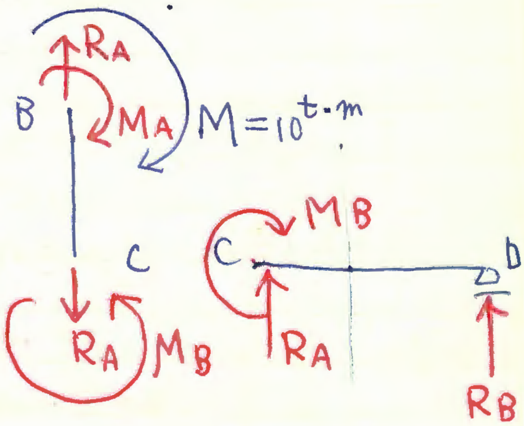
i) AB間 $\begin{cases} M_x = R_A x = -2x^{tm} \\ S_x = -2^t \\ N_x = 0 \end{cases}$

B点に於て $\begin{cases} M_x = -6^{tm} \\ S_x = -2^t \end{cases}$

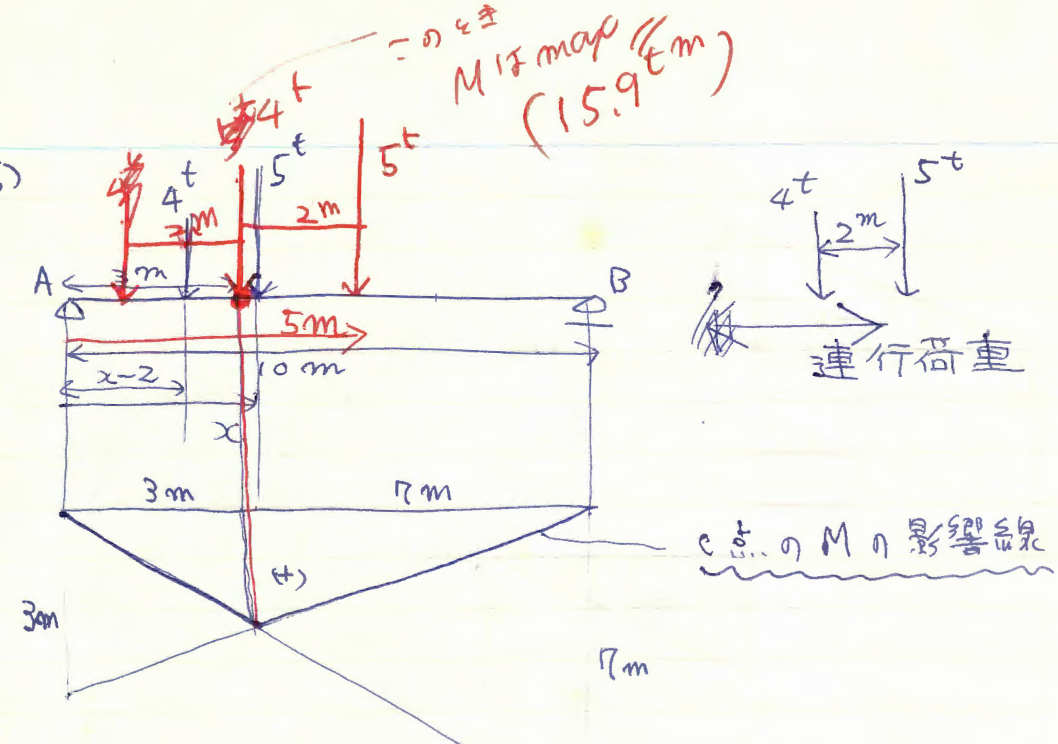
ii) BC間 $\begin{cases} M_x' = M_A + M = 10 - 6 = 4^{tm} \\ S_x' = 0 \\ N_x' = R_A = -2^t \end{cases}$



iii) CD間 $\begin{cases} M_x'' = R_A x + M_B \\ \quad = -2x + 4^{tm} \\ S_x'' = -2^t \\ N_x'' = 0 \end{cases}$



(演6)



0) $0 \leqq x \leqq 2$ $\quad M_x = 5\cdot 7\cdot\frac{x}{10} = 3.5x \qquad x=2 \longrightarrow M_x = 7^{tm}$

i) ~~$0 \leqq x \leqq 3$~~ $2 \leqq x \leqq 3$ $\quad M_x = 5\cdot\left(7\cdot\frac{x}{10}\right) + 4\cdot\left(7\cdot\frac{x-2}{10}\right)$

$$= 6.3x - 5.6$$

$x=3 \longrightarrow M_x = 13.3^{tm}$

ii) $3 \leqq x \leqq 5$ $\quad M_x = 5\cdot\left(3\cdot\frac{10-x}{10}\right) + 4\left(7\cdot\frac{x-2}{10}\right)$

$$= 1.3x + 9.4$$

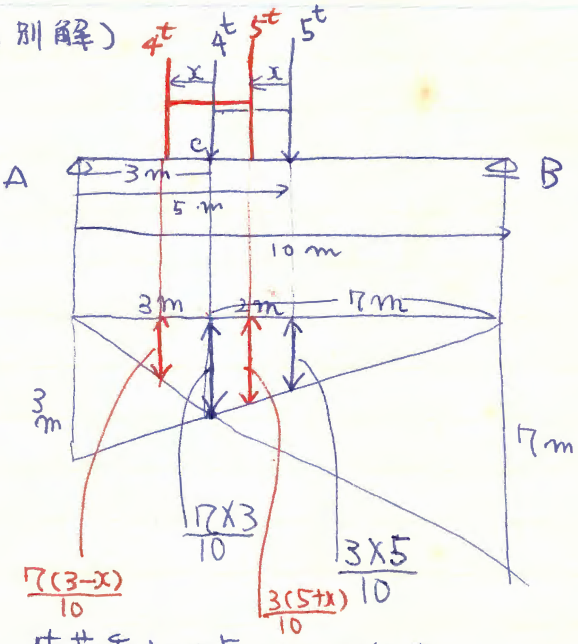
$x=5 \longrightarrow 15.9^{tm}$

iii) $5 \leqq x \leqq 10$ $\quad M_x = 5\cdot\left(3\cdot\frac{10-x}{10}\right) + 4\cdot\left(3\cdot\frac{12-x}{10}\right)$

$$= -2.7x + 29.4$$

$x=5 \longrightarrow 15.9^{tm}$

(演6 別解)



4t荷重がC点にのったとき

$$M = 4\times\frac{7\times3}{10} + 5\times\frac{3\times5}{10} = 15.9^{t\cdot m}$$

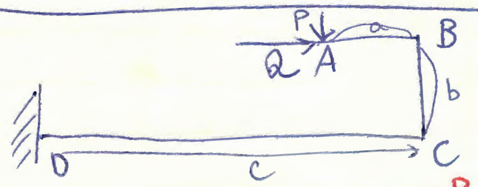
そこから左へx (0≦x≦2) うごいた時.

$$M_x = 4\times\frac{7(3-x)}{10} + 5\times\frac{3(5+x)}{10} = 15.9 - 1.3x.$$

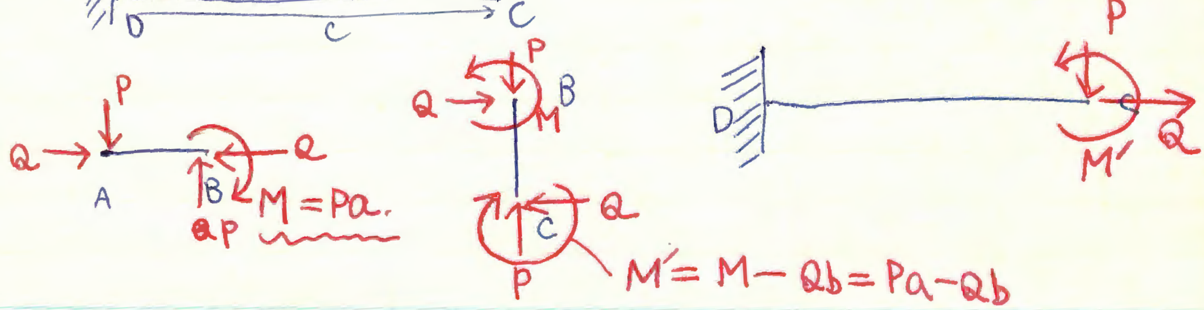
x=0 で M_x は最大

$$M_{max} = 15.9^{t\cdot m}$$

(参)



S, M, N を求める。



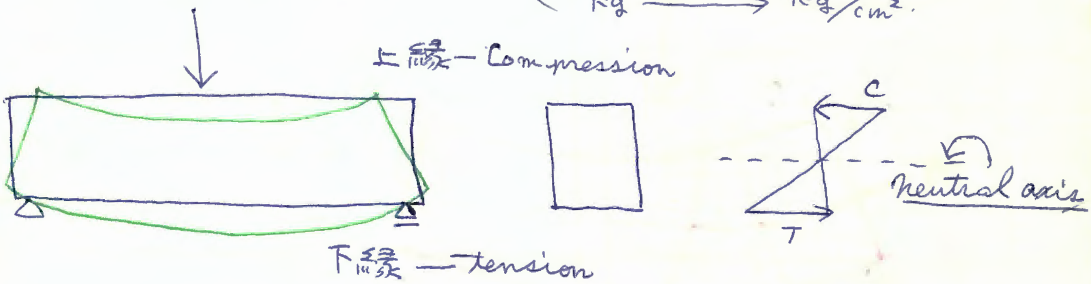
5/18 講ギ

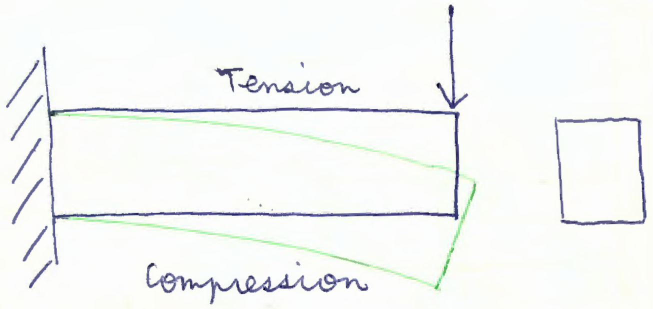
7章. はりの応力.

曲げモーメント ⟶ 曲げ応力

せん断力 ⟶ せん断応力

(t·m ⟶ t/m², kg·cm ⟶ kg/cm², t ⟶ t/cm², kg ⟶ kg/cm²)





~~5/18 講ギ~~

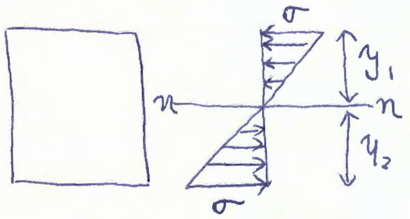
曲げ応力.

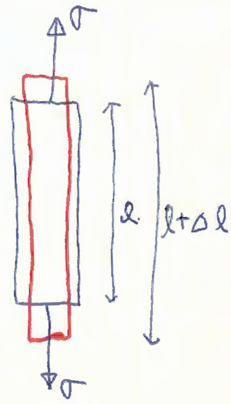
$$\sigma_b = \frac{M}{I} \cdot y$$

M 曲げモーメント

I 断面二次モーメント

y 上縁下縁の距離.



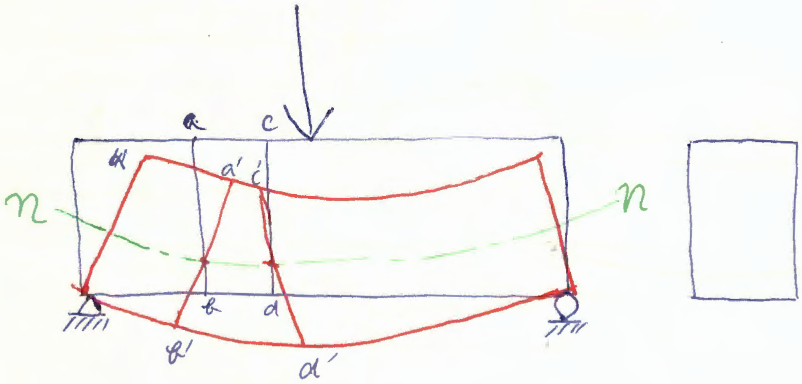


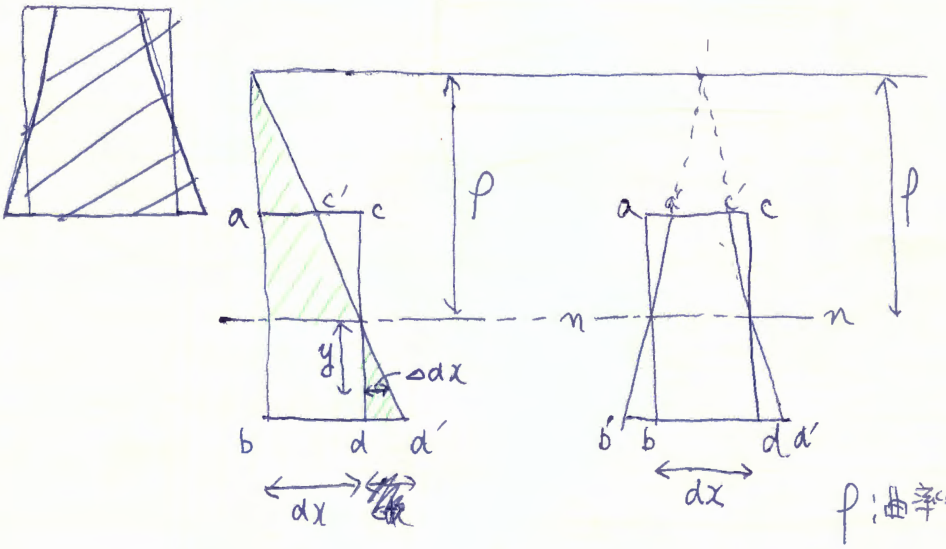
$$\varepsilon = \frac{\Delta l}{l}$$

$$\sigma = \varepsilon E$$

(E ヤング率)

ベルヌイの平面保持の法則；変形前に於て平面であった断面は変形後も平面を保つ。





ρ：曲率半径

$$\frac{\Delta dx}{dx} = \frac{y}{\rho} = \varepsilon_x$$

$$\sigma_x = \varepsilon_x \cdot E$$

$$\therefore \sigma_x = \frac{Ey}{\rho}$$

$$\Sigma H = 0$$

$$\Sigma \sigma_x dA = 0$$

$$\frac{E}{\rho}\underbrace{\Sigma y dA}_{\text{中立軸に関する断面一次モーメント}} = 0$$

∴ 中立軸＝図心軸

$$M = \sum y\sigma_x\, dA = \frac{E}{\rho}\underbrace{\sum y^2 dA}_{\downarrow}$$

中立軸に関する断面二次モーメント（Iとする）

$$\frac{1}{\rho} = \frac{M}{EI}$$

参 P129 (6)式

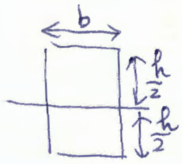
曲げ剛さ

$$\sigma_x = \frac{M}{I}y$$

断面係数 $(長さ)^3$

$$W = \frac{I}{y}$$

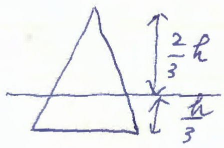
（長方形）



$$I = \frac{bh^3}{12}, \quad y = \frac{h}{2}$$

$$W = \frac{bh^3}{12} \Big/ \frac{h}{2} = \frac{bh^2}{6}$$

（三角形）



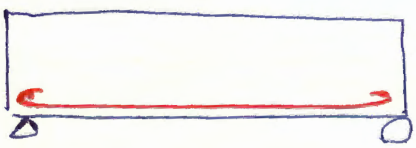
$$I = \frac{bh^3}{36}, \quad y_1 = \frac{2}{3}h \quad y_2 = \frac{h}{3}$$

$$W_1 = \frac{bh^2}{24}, \quad W_2 = \frac{bh^2}{12}$$

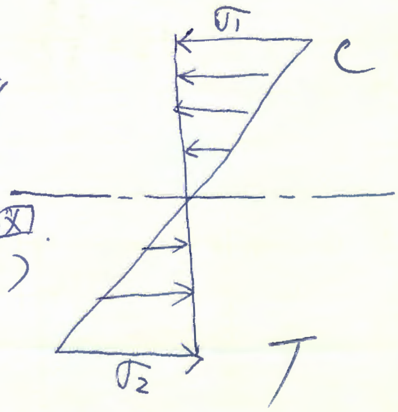
$$\sigma_x = \frac{M}{I}y = \frac{M}{W} \leftarrow W_1, W_2$$

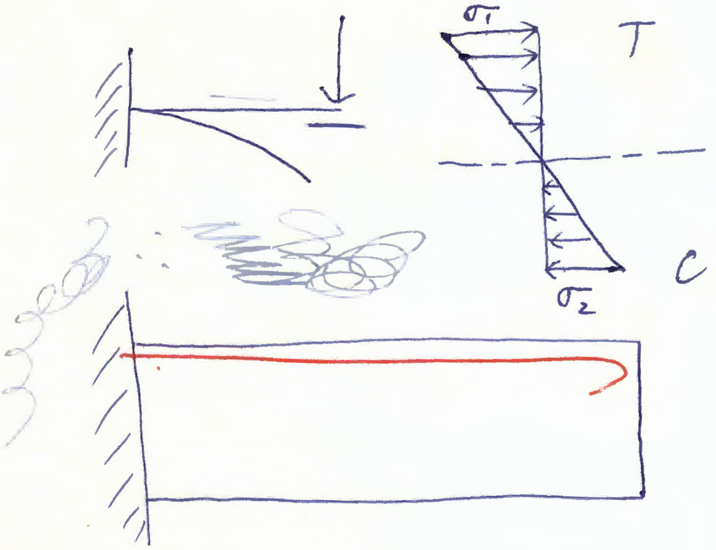
$$\boxed{\sigma_1 = \frac{M}{W_1}, \quad \sigma_2 = \frac{M}{W_2}}$$

縁応力 //



曲げ応力分布図（単純梁）

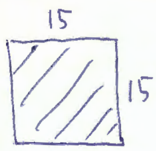


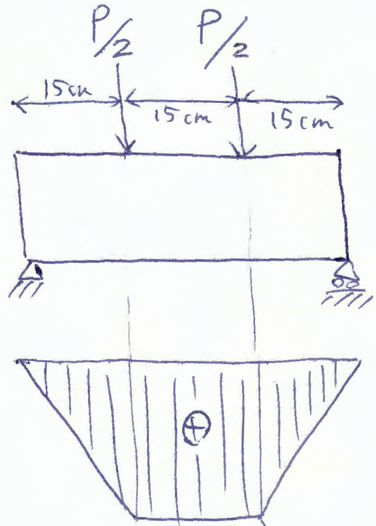


配筋

（コンクリート梁の曲げ試験①
モルタルの曲げ試験②

①





M図

$$\sigma_b = \frac{M}{W}$$

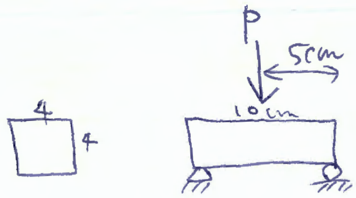
$$M = \frac{Pl}{6},\quad W = \frac{bh^2}{6}$$

$$\sigma_b = \frac{Pl}{bh^2} \quad //$$

$$M = \frac{P}{2}\cdot 15 = \frac{15P}{2}$$

$$= \frac{P}{2}\cdot\frac{l}{3} = \frac{Pl}{6}.$$

②

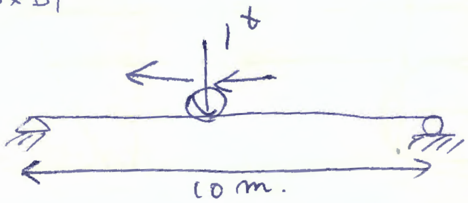


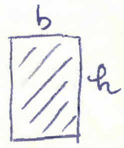
$$W = \frac{bh^2}{6} = \frac{4^3}{6}$$

$$M = \frac{P}{2} \times 5$$

$$\sigma_b = \frac{M}{W} = \frac{5P}{2} \Big/ \frac{4^3}{6} = 0.234P$$

梁の設計



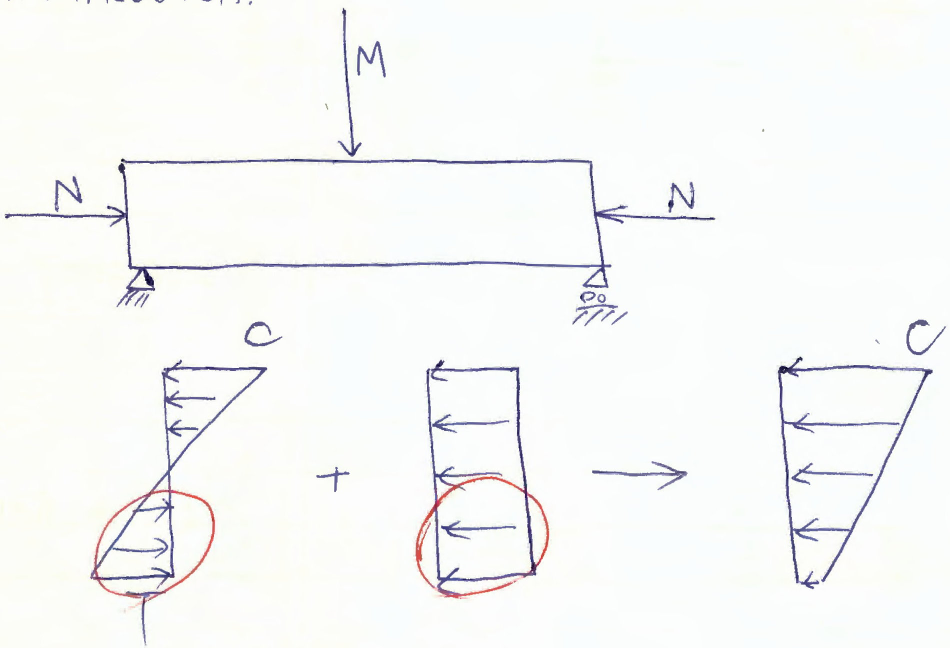


$b : h = 1 : 2$

$$\sigma_a = 40\ \mathrm{kg/cm^2}$$

$$\frac{bh^2}{6} = W = \frac{M_{max}}{\sigma_a}$$

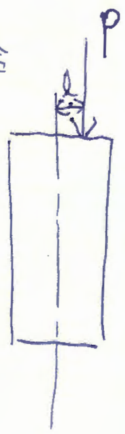
軸方向力による応力.



Prestressed concrete.

$$\pm \frac{M}{I} y + \frac{N}{A} \rightarrow \oplus$$

柱の場合

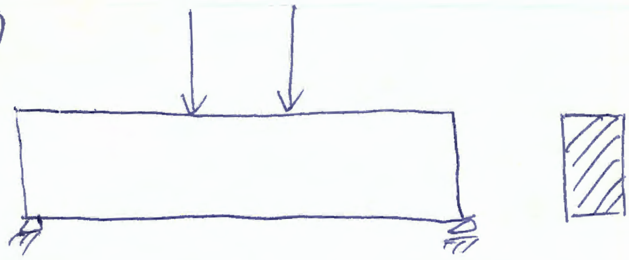


$$\sigma_c = \frac{P}{A}$$

$M = Pl.$ (p177 参)

$$\sigma = \frac{P}{A} \pm \frac{Pl}{W}$$ //

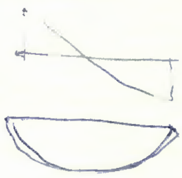
ex)



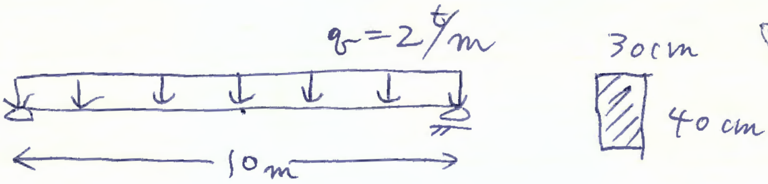
安全や否や？

$$\left(\begin{array}{ll} \sigma_a & \text{許容曲げ応力} \\ \tau_a & 〃 \ \text{せん断} \ 〃 \end{array} \right)$$

$$\left. \begin{array}{l} \sigma_b = \dfrac{M}{W} < \sigma_a \\ \tau = \dfrac{3S}{2bh} < \tau_a \end{array} \right\} \to$$ このときのみ安全 //



ex)



$$M = 10^t \times 5^m - 5^t \cdot 2.5^m = 5t \times 7.5 = 37.5$$

$$I = \frac{bh^3}{12} = \frac{30 \times 40^3}{12}, \quad y = 20 \qquad W = \frac{30 \times 40^2}{12}$$

$$\therefore \sigma_b = \frac{M}{W} = \frac{37.5 \cancel{\times 12} \ ^4}{\cancel{30}_{10} \times 40^2 \ 400} = \frac{37.5}{400} \times 10^4 = \frac{0.0125}{\cancel{80})100} \ \frac{80}{200}$$

$$= 9.375 \times 10^4 = 9375\ t/m^2$$

$$\tau = \frac{3S}{2bh} = 10 \times \frac{3}{2} \times \frac{10^4}{\cancel{30} \times 80} = \frac{1}{80} = \cancel{0.0125}$$

$$= 0.0125 \times 10^4$$

$$= 125\ t/m^2 \ //$$

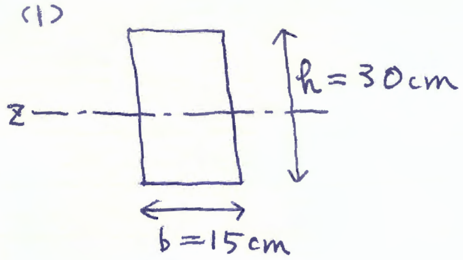
$$\frac{10^3 \ \cancel{kg}}{\cancel{10^4}}$$

$$\frac{\cancel{g}\ell^2}{8} = \frac{2 \times 10^2}{8} = 2,500,000 \ \mathrm{kg \cdot cm.}$$

$$W = \frac{bh^2}{6} = \frac{30 \times 40^2}{6} = 8000 \ \mathrm{cm^3.}$$

$$\sigma = \frac{M}{W} = 312,5 \ \mathrm{kg/cm^2}$$

(1)



$$I_z = \frac{bh^3}{12}$$

縁応力は $y = 15\,\mathrm{cm}$ として

$$\sigma = \pm \frac{M_x}{I_z} y = \pm \frac{1.8 \times 10^5 \times 15}{15 \times 30^3 \times \frac{1}{12}} = \pm 80\ \mathrm{kg/cm^2}$$

最大せん断応力

$$G_z = \int_y^{\frac{h}{2}} y\,b\,dy = \frac{b}{2}\left[\frac{h^2}{4} - y^2\right]$$

$$\therefore \tau = \frac{SG}{Ib} = \frac{S\frac{b}{2}\left[\frac{h^2}{4} - y^2\right]}{\frac{bh^3}{12} \times b} = \frac{3S(h^2 - 4y^2)}{2bh^3}$$

$$y = 0 \longrightarrow \tau_{max} = \frac{3S}{2bh}$$

$$\therefore \tau_{max} = \frac{3 \times 900}{2 \times 15 \times 30} = 3\ \mathrm{kg/cm^2}$$

せん断応力

$$\tau = \frac{SG}{Ib}$$

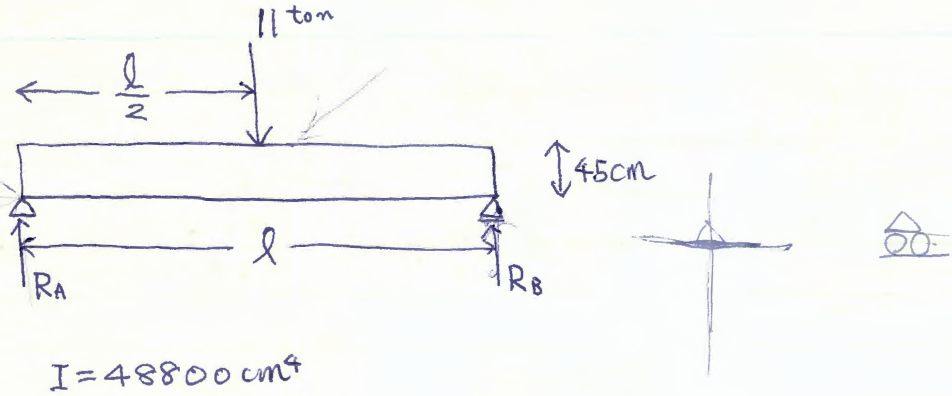
せん断力

断面一次モーメント

given

$$G = \int_{y_0}^{y} y\,b\,dy$$

(3)



$I = 48800\,cm^4$

$\sigma_c = 1000\ kg/cm^2$

$\gamma = 115\ kg/m = 1.15\ kg/cm$.

$\Sigma M_B = R_A l - 11 \times 10^3 \times \frac{l}{2} - 1.15 \times l \times \frac{l}{2}$

$\therefore R_A = 5.5 \times 10^3 + \frac{1.15}{2} l = R_B$.

$M_{x=\frac{l}{2}} = (5.5 \times 10^3 + \frac{1.15}{2} l) \times \frac{l}{2} - 1.15 \cdot \frac{l}{2} \cdot \frac{l}{4}$

$= \frac{5.5 \times 10^3}{2} l + \frac{1.15}{8} l^2$

$\therefore \sigma_{max} = \frac{M}{I} y_{max} = \frac{22.5}{48800} \left(\frac{5.5 \times 10^3}{2} l + \frac{1.15}{8} l^2 \right)$

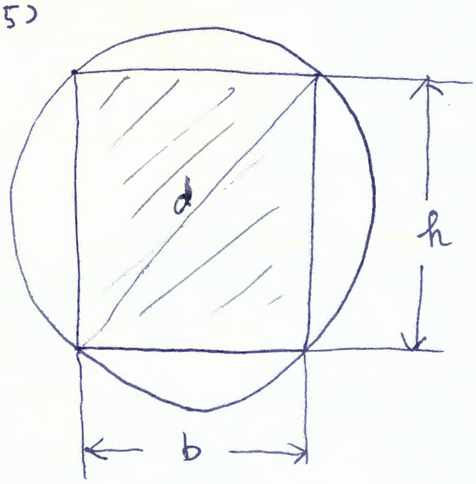
許容曲げ圧縮応力 $\sigma_c = 1000\ kg/cm^2$

$\therefore 1000 \times 48800 = 22.5 \left(\frac{5.5 \times 10^3}{2} l + \frac{1.15}{8} l^2 \right)$

$\therefore \frac{1.15}{8} l^2 + \frac{5.5 \times 10^3}{2} l - \frac{2 \times 4.88 \times 10^7}{45} = 0$

$\therefore l = 759\ cm$ //

(5)



$\sigma = \frac{M}{I} y \quad (y = \frac{h}{2})$

σを最小にするには $\frac{y}{I}$ を最小にすればよい。

すなわち

$\frac{y}{I} = \frac{h/2}{bh^3/12} = \frac{6}{bh^2}$ を最小にする

or

断面係数 $W = \frac{bh^2}{6}$ を最大にする。

$h^2 = d^2 - b^2$ より

$\frac{6}{bh^2} = \frac{6}{b(d^2-b^2)}$

$\frac{1}{x} = \frac{y}{I}$ とおけば

$\boxed{\frac{dW}{dh} = 0}$

$x = -\frac{1}{6}b^3 + \frac{d^2}{6}b$

$x' = -\frac{b^2}{2} + \frac{d^2}{6}$

$x' = 0$ より $\quad b^2 = \frac{d^2}{3} \qquad b = \pm\frac{d}{\sqrt{3}}$

$b = \frac{d}{\sqrt{3}}$ のとき、最大となる //

このとき h は $\quad h^2 = d^2 - b^2 = \frac{2}{3}d^2 \qquad \therefore h = \sqrt{\frac{2}{3}}d$ //

6/1 講ギ

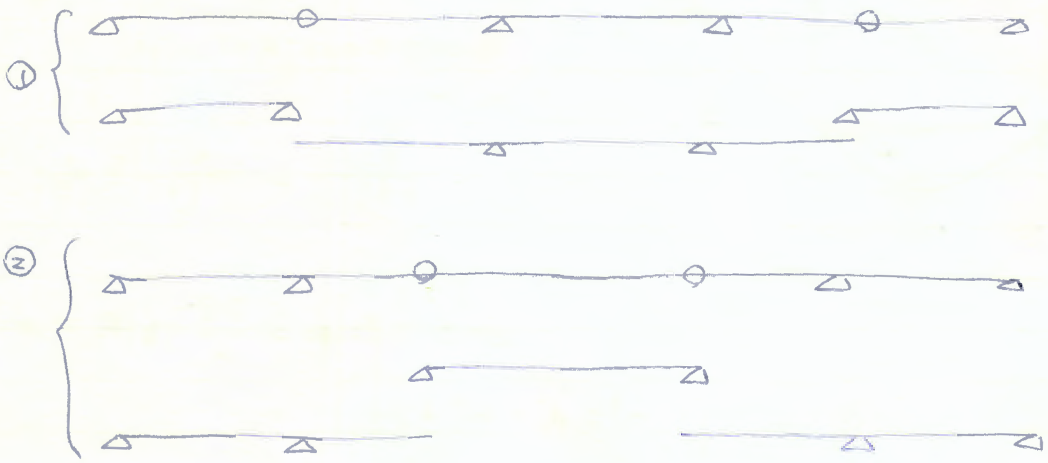
6章 片持梁.

張出し梁 = 片持梁 + 単純梁

ゲルバー梁 = 張出し梁 + 単純梁

①

②



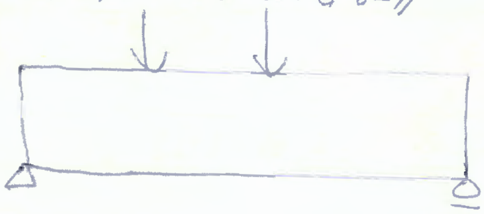
Inf. line をかけ。

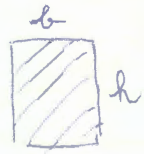
について考える。

7章. 縁応力
せん断応力

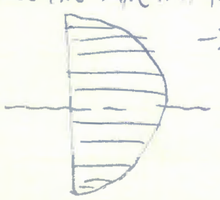
$\sigma = \frac{M}{I} y = \frac{M}{W}$

矩形については暗記



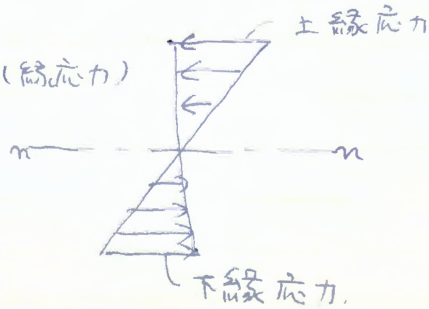


せん断応力 応力分布.



一次モーメント Q

応力分布図. (縁応力)



$\sigma = \frac{M}{I} y$

6章2節、7章1節

梁の設計.

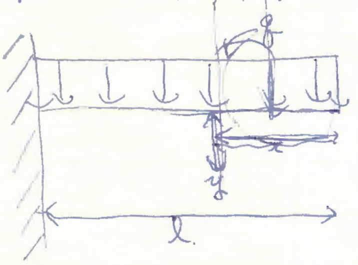
許容曲げ応力
許容せん断応力 } ⇒ かかる曲げモーメント、せん断力より を決める

最大応力 (P111) σ′, τ′ の求め方

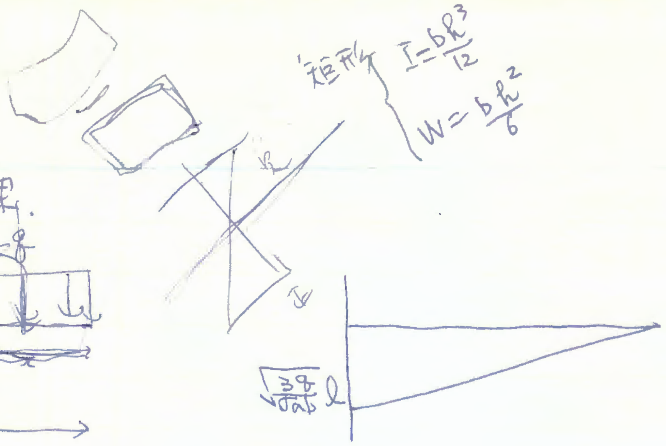
平等強さの梁 (P118)

6/2 演習（補）

(8) P124.　等強梁.



矩形 $\begin{cases} I = \dfrac{bh^3}{12} \\ W = \dfrac{bh^2}{6} \end{cases}$



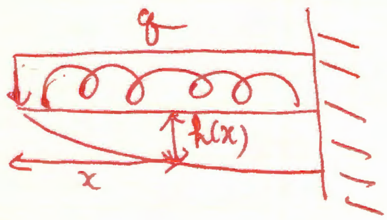
曲げモーメント $M_x = -\frac{1}{2} q x^2$

断面係数 $W = \dfrac{b y^2}{6}$

許容曲げ応力を σ_a とすると

$$\sigma_a = \left|\frac{M_x}{W}\right| = \left|\frac{-\frac{qx^2}{2}}{by^2/6}\right| = \left|\frac{-\cancel{6}^{3} q x^2}{2 b y^2}\right| \quad \therefore y^2 = +\frac{3q}{\sigma_a b} x^2$$

$$\therefore y = \sqrt{\frac{3q}{\sigma_a b}}\, x$$



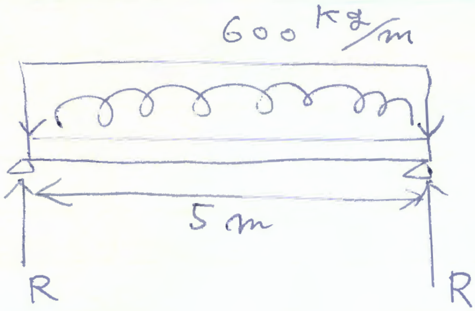
$$\sigma = \frac{M_x}{I_z} y = \frac{M_x}{\frac{bh^3}{12}} \cdot \frac{h(x)}{2}$$

$$= \frac{6M_x}{bh^2}$$

$$\therefore h^2 = \frac{6M_x}{b\sigma} = \frac{3q}{b\sigma} x^2$$

$$\therefore h = \sqrt{\frac{3q}{b\sigma}}\, x$$

(9)



鉄筋コンクリート梁の単位長当りの重 $\rho_{R,C} \doteqdot 2400 \times 0.3 \times 0.4 = 288$ kg/m.

$R = (600+288) \times \frac{5}{2} = 2200$ kg

中央点での曲げモーメント

$M_{x=2.5} = 2.5R - (600+288) \times 2.5 \times \frac{5}{4}$

$= 2725$ kg·m.

鉄筋の断面積 As

$As = 3 \times \frac{\pi \phi^2}{4} = \frac{3}{4} \times 3.14 \times 0.019^2 = 8.5 \times 10^{-4}$ m^2

中立軸の位置 x

$x = \frac{nAs}{b}\left(\sqrt{1+\frac{2bd}{nAs}} - 1\right)$

$\doteqdot \frac{15 \times 8.5 \times 10^{-4}}{0.3}\left(\sqrt{1+\frac{2 \times 0.3 \times 0.36}{15 \times 8.5 \times 10^{-4}}} - 1\right)$

$= 0.137$ m

鉄筋の引張応力 σ_s

$\sigma_s = \frac{M}{As(d - \frac{x}{3})}$

$= \frac{2725}{8.5 \times 10^{-4}(0.36 - \frac{0.137}{3})}$

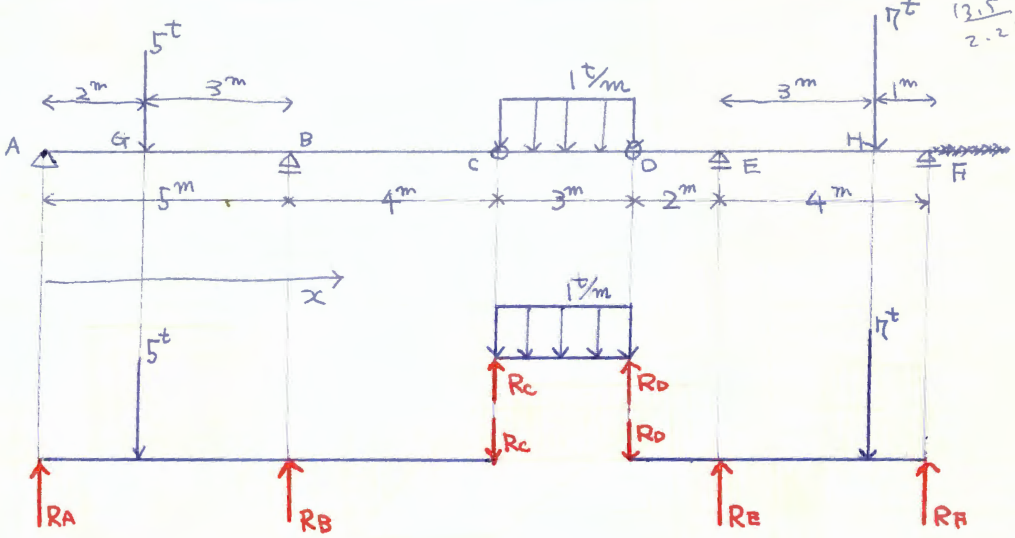
$= 1020 \times 10^4$ kg/m^2

$= 1020$ kg/cm^2

コンクリートの圧縮縁応力 σ_c

$\sigma_c = \frac{2M}{bx(d - \frac{x}{3})} = \frac{2 \times 2725}{0.3 \times 0.137 \times (0.36 - \frac{0.137}{3})}$

$\doteqdot 4.22 \times 10^5$ kg/m^2 $= 42.2$ kg/cm^2



$R_C = R_D = 1.5^t$

$R_A + R_B = 6.5^t$, $\Sigma M_A = 5^t \times 2^m - R_B \times 5^m + 1.5^t \times 9^m = 0$

$\therefore R_B = 4.7^t$, $R_A = 1.8^t$

$R_E + R_F = 8.5^t$, $\Sigma M_F = 7^t \times 1^m - R_E \times 4^m + 1.5^t \times 6^m = 0$

$\therefore R_E = 4.0^t$, $R_F = 4.5^t$

A−G. $M_x = R_A x = 1.8x^{tm}$

$S_x = 1.8^t$

G−B. $M_x = 1.8x - 5(x-2) = -3.2x + 10^{tm}$

$S_x = -3.2^t$

B−C. $M_x = -3.2x + 10 + 4.7(x-5) = 1.5x - 13.5^{tm}$

$S_x = 1.5^t$

C−D. $M_x = 1.5x - 13.5 - \frac{1}{2}(x-9)^2 \,{}^{tm}$ $\quad x = 10.5$ $\quad M_{max}$

$S_x = 1.5 - (x-9) = 10.5 - x^t$

D−E. $M_x = -R_D(x-12) = -1.5x + 18^{tm}$

$S_x = -1.5^t$

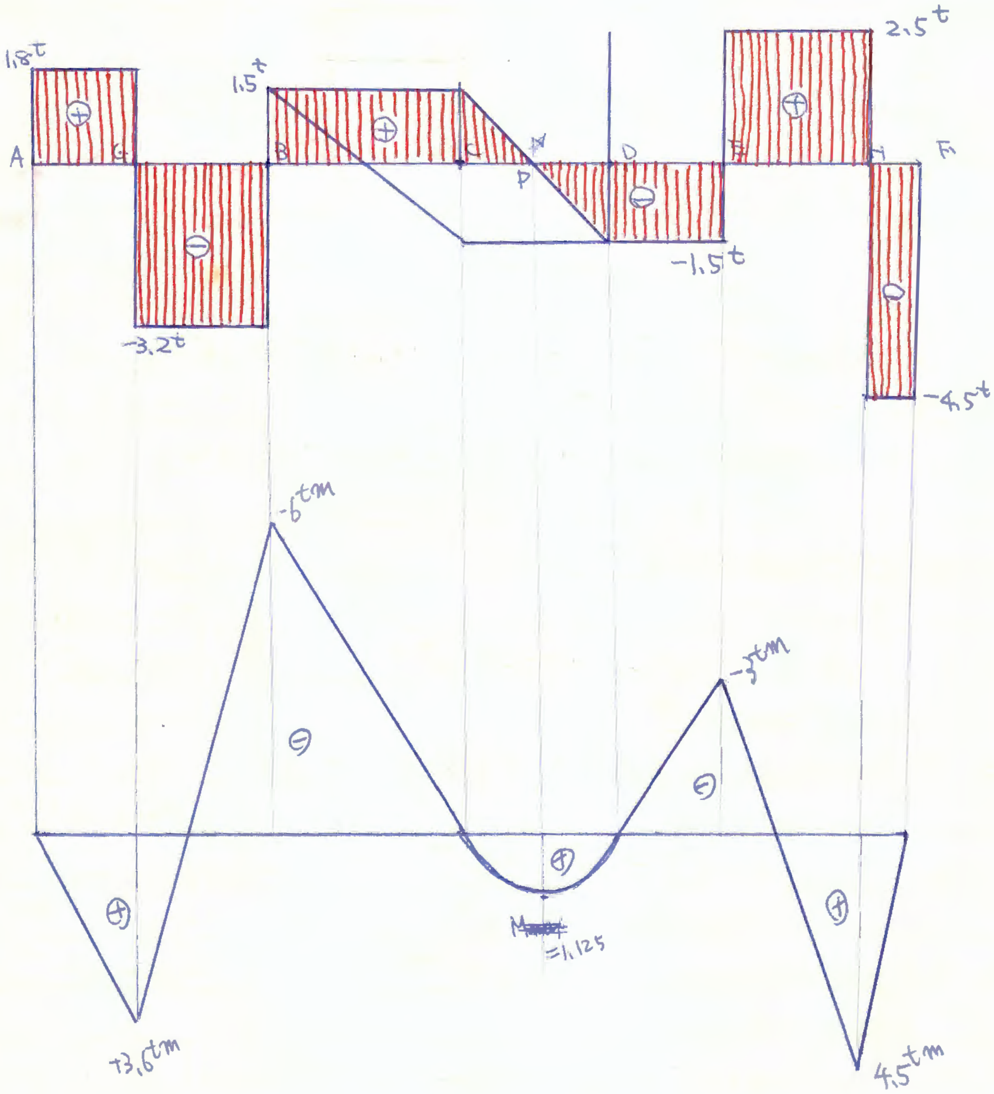
E−H. $M_x = -1.5x + 18 + R_E(x-14) = -1.5x + 18 + 4.0(x-14) = 2.5x - 38^{tm}$

$S_x = 2.5^t$

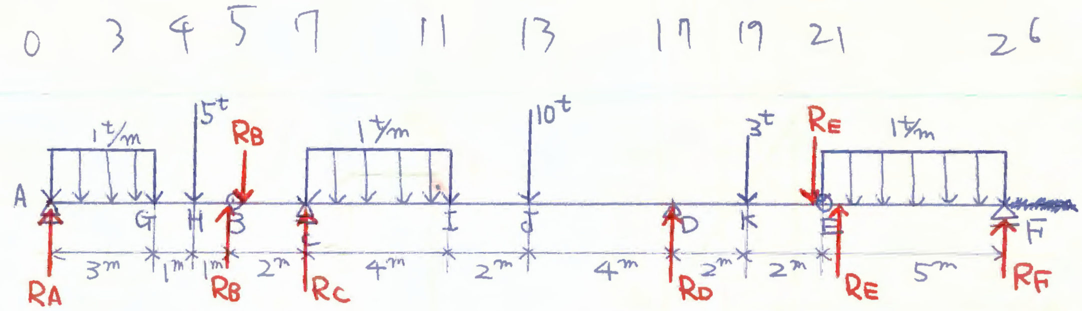
H−F. $M_x = 2.5x - 38 - 7(x-18) = -4.5x + 81$

$S_x = -4.5^t$

$M_A = 0$
$M_G = 3.6^{tm}$
$M_B = -6^{tm}$
$M_C = 0$
$M_D = 0$
$M_E = -3^{tm}$
$M_H = 4.5^{tm}$
$M_F = 0$



1.8t
1.5t
2.5t
A
G
B
C
P
D
E
F
-3.2t
-1.5t
-4.5t
-6tm
-3tm
M=1.125
+3.6tm
4.5tm



(3 ñ) $x = 23.5$

$R_A + R_B = 3 + 5 = 8^t$ $\Sigma M_A = 3^t \times 1.5^m + 5^t \times 4^m - R_B \times 5^m = 0$

∴ $R_B = 4.9^t$, $R_A = 3.1^t$

$R_E + R_F = 5^t$ ∴ $R_E = R_F = 2.5^t$

$R_C + R_D = 4.9^t + 4^t + 10^t \cancel{10.9^t} + 3^t + 2.5^t = 24.4^t$

$\Sigma M_C = -4.9 \times 2 + 4 \times 2 + 10 \times 6 - 10 \times R_D + 12 \times 3 + 2.5 \times 14 = 0$

∴ $R_D = 12.92^t$ $R_C = 11.48^t$

A–G $M_x = R_A x - \frac{x^2}{2} = 3.1 x - \frac{x^2}{2}$ ∴ $S_x = 3.1 - x$ $M_A = 0$

G–H $M_x = R_A x - 3(x - 1.5) = 0.1x + 4.5$ ∴ $S_x = 0.1$ $M_G = 4.8^{tm}$

H–B $M_x = R_B(5 - x) = 4.9(5 - x)$ ∴ $S_x = -4.9$ $M_H = 4.9^{tm}$

B–C $M_x = -4.9(x - 5)$ ∴ $S_x = -4.9$ $M_B = 0$

C–I $M_x = -4.9(x-5) + 11.48(x-7) - \frac{1}{2}(x-7)^2$ $S_C = 6.58$ $M_C = -9.8$

$S_x = 6.58 - (x - 7) = -x + 13.58$ $S_I = 2.58$ $M_I = 8.52$

I–J $M_x = -4.9(x-5) + 11.48(x-7) - 4(x-9)$ $M_J = 13.68$

$M_D = -16$

$= 2.58x - 19.86$ $M_K = -5$

$S_x = 2.58$ $M_E = 0$

J–D $M_x = 2.58x - 19.86 - 10(x - 13)$ $M_F = 0$

$= -7.42x + 110.14$

$S_x = -7.42$

D–K $M_x = -2.5(21 - x) - 3(19 - x) = 5.5x - 109.5$

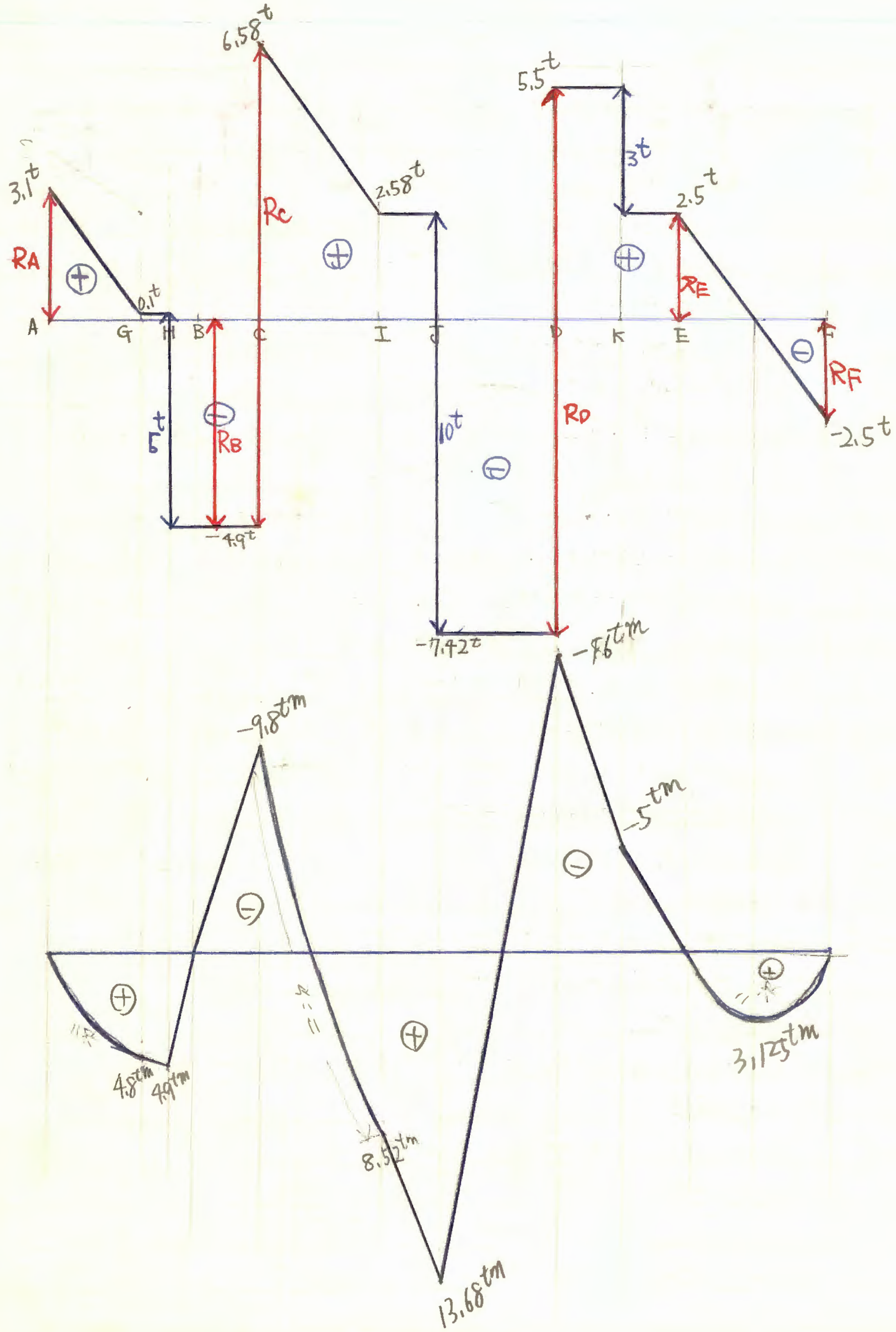
$S_x = 5.5$

K–E $M_x = -2.5(21 - x) = 2.5x - 52.5$ $x = 23.5$ $M_{max} =$

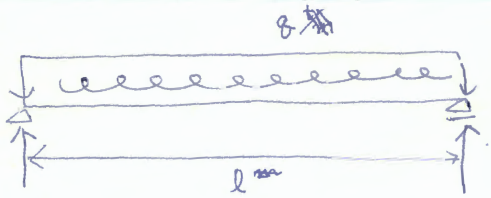
$S_x = 2.5$

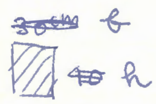
E–F $M_x = R_E(x - 21) - \frac{1}{2}(x-21)^2$ $S_E = 2.5$

$S_x = 2.5 - (x - 21) = -x + 23.5$ $S_F = -2.5$



6.58t
5.5t
3t
3.1t
2.58t
2.5t
RA
RC
RE
RF
RB
RD
0.1t
A
G
H
B
C
I
J
D
K
E
F
5t
10t
-4.9t
-7.42t
-2.5t
-9.8tm
-5tm
4.8tm
4.9tm
8.52tm
13.68tm
3.125tm





$R_A = R_B = ql/2$

$$M = \frac{l}{2}\cdot\frac{ql}{2} - \frac{qx^2}{2} = \frac{ql^2}{4} - \frac{qx^2}{2}$$

$$M_{max} = M_{x=l/2} = \frac{ql^2}{8}$$

$$I = \frac{bh^3}{12}$$

より $$\sigma_{max} = \frac{M_{max}}{I}\cdot y = \frac{\frac{ql^2}{8}}{\frac{bh^3}{12}}\cdot\frac{h}{2} = \frac{6ql^2}{8bh^2} = \frac{3ql^2}{4bh^2}$$ //

$$S_{max} = \frac{ql}{2}$$ //

$$\cancel{G = \int_0^{\frac{h}{2}} y\,dA = \int_0^{\frac{h}{2}} y\,b\,dy = \frac{b}{2}\frac{h^2}{4} = \frac{bh^2}{8}}$$ //

$$\tau_{max} = \frac{3S}{2bh} = \frac{3\cdot\frac{ql}{2}}{2bh} = \frac{3ql}{4bh}$$ //

◎ 垂直応力．

$$\varepsilon = \frac{\Delta dx}{dx} = Cy$$

$$\therefore \sigma_M = \varepsilon E = ECy$$

中立軸と図心軸は一致

また $\Sigma H = \int_A \sigma_M dA = \int_A ECy\,dA = EC\int_A y\,dA = 0$

次に $M_z = \int_A y\sigma_M dA = \int_A ECy^2 dA = EC\int_A y^2 dA = EcI_z \quad (\because \Sigma M = 0)$

$$\therefore M_x = ECI_z = \frac{I_z}{y}\sigma_M \qquad \therefore \sigma_M = \frac{M_x}{I_z}y$$

（縁応力）$\sigma_{M_1} = \frac{M_x}{I_z}y_1 = \frac{M_x}{W_1}$ ， $\sigma_{M_2} = \frac{M_x}{I_z}y_2 = \frac{M_x}{W_2}$

矩形の場合

$$\left.\begin{aligned} y_1 = y_2 &= \frac{h}{2} \\ I_z &= \frac{bh^3}{12} \end{aligned}\right\} \rightarrow W = \frac{bh^2}{6}$$

$$\therefore \sigma_{M_1} = -\sigma_{M_2} = \frac{6M_x}{bh^2} \qquad \frac{M}{\frac{bh^2}{6}} = \frac{6M}{bh^2}$$

◎ せん断応力.

$$\sigma = \frac{M}{I} y$$

$$\Sigma H = \int_{y_0}^{y_1} \sigma \, dA = \int_{y_0}^{y_1} \sigma z \, dy = \frac{M}{I} \int_{y_0}^{y_1} y z \, dy$$

$$\sigma' = \frac{M+dM}{I} y$$

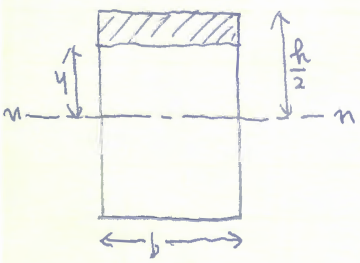
$$\Sigma H' = \int_{y_0}^{y_1} \sigma' \, dA = \frac{M+dM}{I} \int_{y_0}^{y_1} y z \, dy$$

$$T = \tau b \, dx = \Sigma H' - \Sigma H = \frac{dM}{I} \int_{y_0}^{y_1} y z \, dy$$

$$\therefore \tau = \frac{1}{Ib} \cdot \frac{dM}{dx} \cdot \int_{y_0}^{y_1} y z \, dy$$

$$\therefore \tau = \frac{S \cdot G_z}{Ib} \quad //$$

矩形の場合



$$I = \frac{bh^3}{12}$$

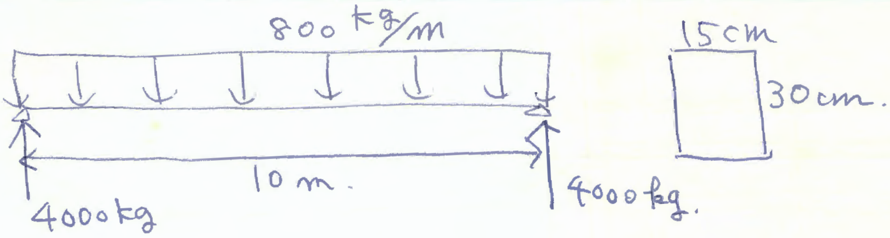
$$G_z = \int_{y_0}^{y_1} y z \, dy = \int_{y}^{\frac{h}{2}} b y \, dy$$

$$= \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

$$\therefore \tau = \frac{S \cdot \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)}{\frac{bh^3}{12} \cdot b} = \frac{3S(h^2 - 4y^2)}{2bh^3} \quad //$$

$$\tau_{max} = \tau_{y=0} = \frac{3S}{2bh} \quad //$$

(4)



$$I = \frac{bh^3}{12} = \frac{0.15 \times 0.3^3}{12} \quad , \quad y = \frac{h}{2} = 0.15 .$$

$$M_{max} = 4000^{kg} \times 5^{m} - \frac{800 \times 5^2}{2}$$

$$= \cancel{4000}\ 20000 - 10000 = 10000^{kgm} .$$

$$\sigma_{max} = \frac{M_{max}}{I} \cdot y = \frac{10000}{\frac{\cancel{0.15} \times 0.3^3}{12}} \times \cancel{0.15} = \frac{10000 \times 12}{0.3^3} = 4.44 \times 10^6 \ kg/m^2$$

$$= 444 \ kg/cm^2$$

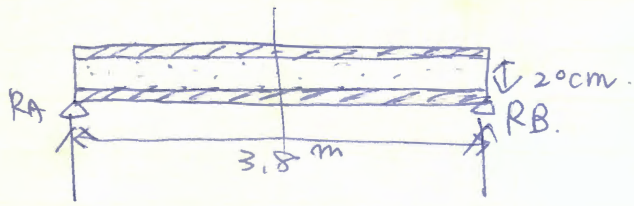
許容曲げ応力 $90^{kg}/cm^2$

$$4 < \frac{444\ kg/cm^2}{90\ kg/cm^2} < 5$$

より. 5本必要とする//

$\sigma = \frac{M}{I} y$

(2)



$\cancel{M_{max} =}$

$\cancel{R_A = R_B = \frac{1}{2} 3.8 \times (7250 + 1000)}$

$\cancel{= 8250 \times 1.9 = 15675} \ kg$

$\cancel{M_{max} = 15675 \times 1.9 - \frac{1.9^2}{2} \times 8250}$

$\cancel{= 14891.25 \ kg\,m}$

$\pi \times 0.1^2 \times 1 = 0.0314 \times 1 \ m^2 \quad = 0.0314 \ m^3 = 31.4 \ kg/m$

$R_A = R_B = \frac{1}{2} \times 3.8 \times (7250 + 31.4)$

$= 1.9 \times 7281.4 = 13835 \ kg$

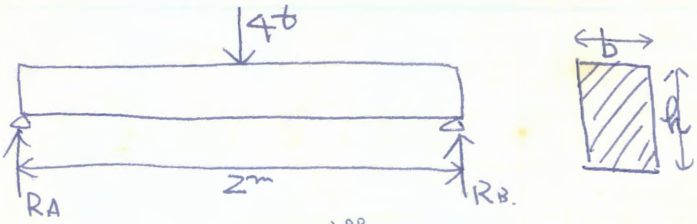
$M_{max} = 13835 \times 1.9 - \frac{1.9^2}{2} \times 7281.4$

$= 13835 \times 1.9 \times \frac{1}{2} = 13143 \ kg\,m$

$I = \frac{\pi d^4}{64}, \quad y = \frac{d}{2} \quad \therefore \quad W = \frac{\pi d^3}{32}$

$\therefore \sigma = \frac{32 \times 13143}{\pi \times 0.22^3} =$

(7)



$\sigma_a = 90\,kg/cm^2$ $\cancel{900\ t/m}$

$M_{max} = 2^t \times 1^m = 2^{tm}$

$\sigma_{max} = \frac{M_{max}}{I} \times y = \frac{12}{bh^2}\ t/m^2$ $\cancel{= 90\,kg/cm^2 \times 10^{-1} = 90}$.

$\therefore \frac{12}{bh^2} = 900.$

$\frac{12}{10bh^2} = 90$ $\quad h=2b \rightarrow \frac{12^{3}}{4b^3 \times 10} = \cancel{90}\ 3^3$

$\frac{2\times10^7 \times 6}{90}$

$\frac{M}{\frac{bh^3}{12}} \cdot \frac{h}{2} = \frac{6M_{max}}{bh^2} \leqq \sigma_a \qquad bh^2 \geqq \frac{6M_{max}}{\sigma_a}$

$= \frac{6 \times 2\times10^7\ kg\cdot cm}{\cancel{90}\ kg/cm^2\ 15} = \frac{2\times10^7}{15}\ cm^3 = \frac{20}{15}\times10^6$

$bh^2 \geqq \frac{4}{3}\times10^6\ cm^3$

$h=2b$ とすると

$4b^3 \geqq \frac{4}{3}\times10^6$ $\quad b^3 = \frac{10^6}{3}$

$\frac{1000}{3}\times10^3$

$\cancel{h = \frac{4\times10^6}{300}}$ 10^4

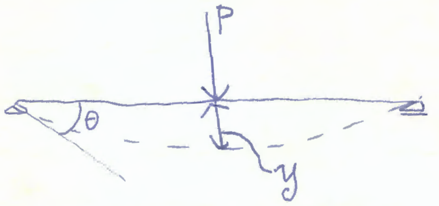
$\cancel{1.33\times10^4}$

333

15
15
75
15
225
15
1125
225
3375

6/8 講義

8章　梁の撓(たわ)み



θ：撓み角
y：撓み

▨ たわみ曲線の微分方程式

P128 (a) ~ (e)

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

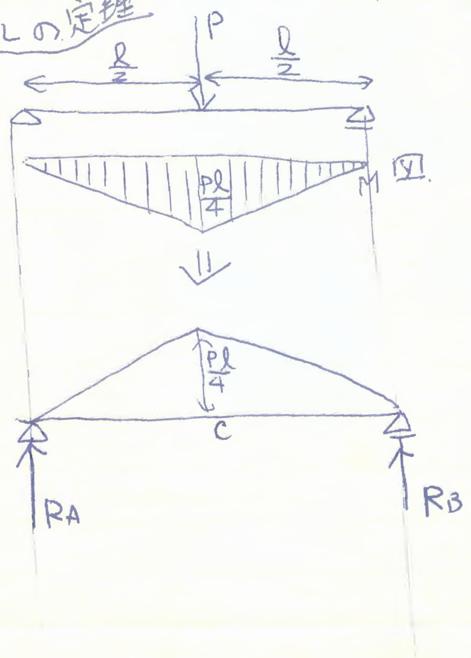
EI — 曲げ剛(こわ)さ

たわみ角 $\frac{dy}{dx} = -\int \frac{M}{EI}dx + C_1$

たわみ $y = -\iint \frac{M}{EI}dx\,dx + C_1x + C_2$

境
(支点) $x=0,\ x=l \longrightarrow$ (たわみ) $y=0$
$x=\frac{l}{2} \longrightarrow \frac{dy}{dx}=0$
(載荷点)　(たわみ角)

▨ モールの定理



弾性荷重法

$$R_A = \frac{Pl}{4}\times\frac{l}{2}\times\frac{1}{2} = \frac{Pl^2}{16}$$

$$M_c = R_A\times\frac{l}{2} - \frac{Pl}{4}\times\frac{l}{2}\times\frac{1}{2}\times\frac{l}{6}$$

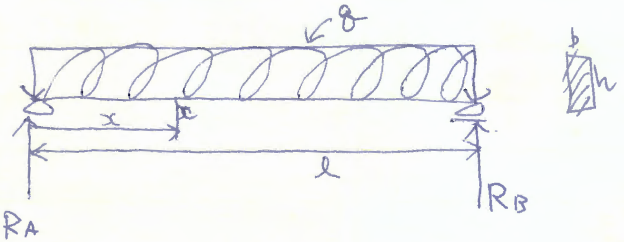
$$= \frac{Pl^3}{32} - \frac{Pl^3}{96} = \frac{2Pl^3}{96} = \frac{Pl^3}{48}$$

$$S_c = R_A - \frac{Pl}{4}\times\frac{l}{2}\times\frac{1}{2} = 0$$

$$S_A = \frac{Pl^2}{16}$$

$$\begin{cases} \theta_A = \dfrac{S_A}{EI} & \text{たわみ角} = \dfrac{pl^2}{16EI} \quad (\text{定理}3) \\ y_c = \dfrac{M_c}{EI} & \text{たわみ} = \dfrac{pl^3}{48EI} \quad (\text{定理}4) \end{cases}$$

(ex)



b, h, E は一定

$$M_x = R_A x - \frac{qx^2}{2} = \frac{ql}{2}x - \frac{qx^2}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

$$EI\frac{d^2y}{dx^2} = -M = -\frac{ql}{2}x + \frac{qx^2}{2}$$

$$EI\frac{dy}{dx} = -\frac{ql}{4}x^2 + \frac{qx^3}{6} + C_1$$

$$EIy = -\frac{ql}{12}x^3 + \frac{qx^4}{24} + C_1x + C_2$$

$x=0,\ x=l \longrightarrow y=0$

$C_2 = 0$

$$0 = -\frac{ql^4}{12} + \frac{qx^4}{24} + C_1 l$$

$$\therefore C_1 = \frac{ql^3}{24}$$

$$EI\frac{dy}{dx} = -\frac{qlx^2}{4} + \frac{qx^3}{6}$$

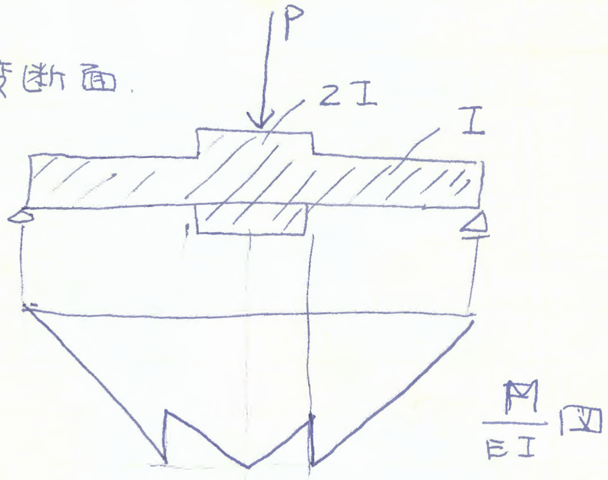
$$EIy = -\frac{qlx^3}{12} + \frac{qx^4}{24} + \frac{ql^3}{24}x$$

最大たわみ $x \to \frac{l}{2}$

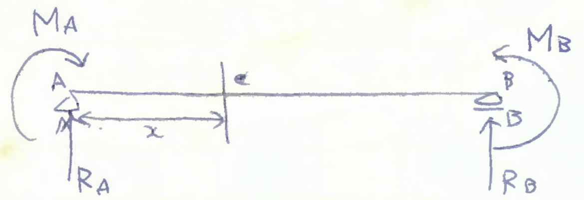
$$EIy_{(x=\frac{l}{2})} = \frac{5ql^4}{384EI}$$

$$\theta_A = \frac{ql^3}{24EI}$$

(ex) 変断面

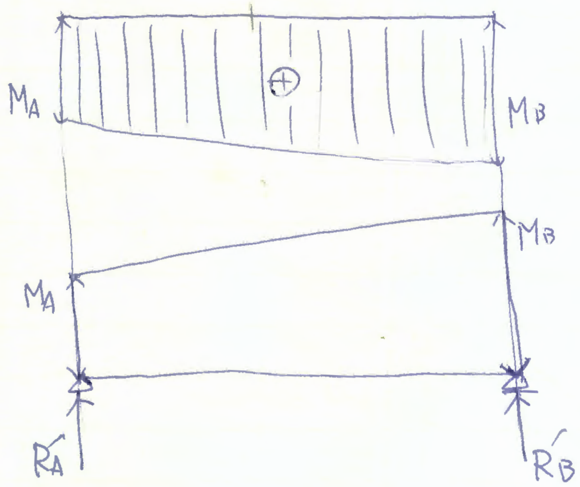


支点にモーメントをうける。



$$\Sigma M_B = M_A + R_A l - M_B = 0 \quad \therefore R_A = \cancel{-\frac{M_A}{l}}\ \frac{M_B - M_A}{l} . \qquad R_B = \cancel{\frac{M_A}{l}}\ \frac{M_A - M_B}{l}$$

$$\begin{cases} M_x = R_A x + M_A = \cancel{-\frac{M_A}{l}x + M_A}\ \frac{M_B - M_A}{l}x + M_A \\ S_x = \cancel{-\frac{M_A}{l}}\ \frac{M_B - M_A}{l} \end{cases}$$

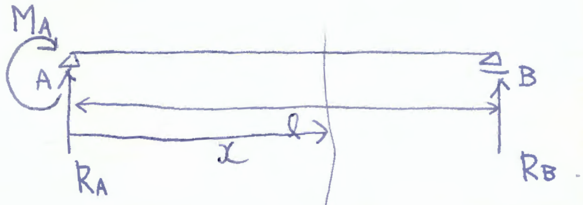


M 図 .

6/5 試験

6/12 演習（試験解答）

6/15 講ギ

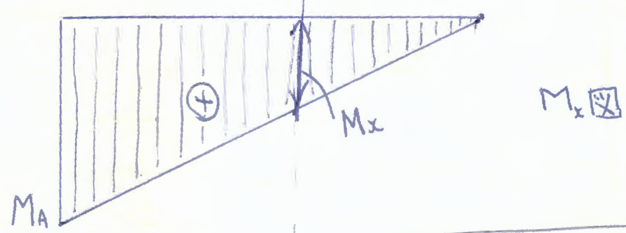


$$R_A = -\frac{M_A}{l}$$

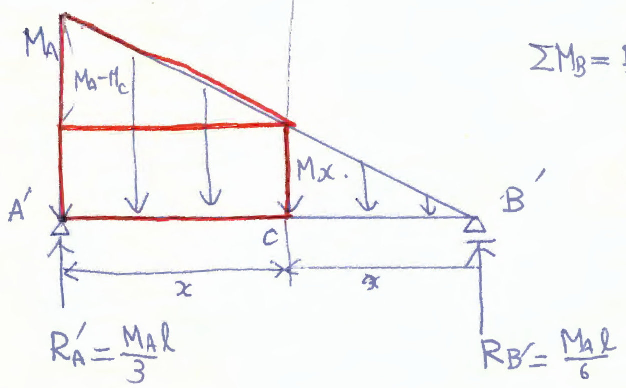
$$R_B = \frac{M_A}{l}$$

$$M_x = M_A + R_A x$$

$$= M_A - \frac{M_A}{l}x = M_A\left(1 - \frac{x}{l}\right)$$



M_x図



$$\Sigma M_B = R_A' l - \frac{M_A l}{2}\cdot\frac{2l}{3} = 0$$

$$\therefore R_A' = \frac{M_A l}{3} \qquad \therefore R_B' = \frac{M_A l}{6}$$

$$\theta_A = \frac{R_A'}{EI} = \frac{M_A l}{3EI}$$

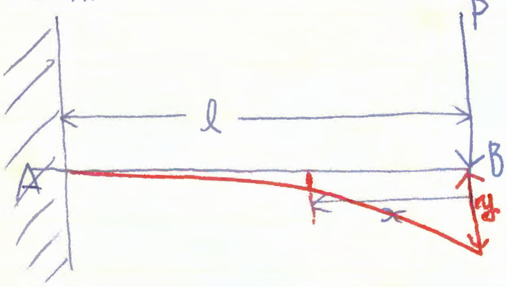
$$\theta_B = \frac{-R_B'}{EI} = -\frac{M_A l}{6EI}$$

$$M_c = R_A' x - \frac{M_x x^2}{2} - \frac{x^2(M_A - M_x)\times 1\times 2}{2\times 3}$$

$$= \frac{M_A l}{3}x - \frac{M_x x^2}{2} - \frac{(M_A - M_x)x^2}{3}$$

$$\therefore y_c = \frac{M_c}{EI} = \frac{x}{6EI}\left\{2M_A l - 3M_x x - 2(M_A - M_x)x\right\}$$

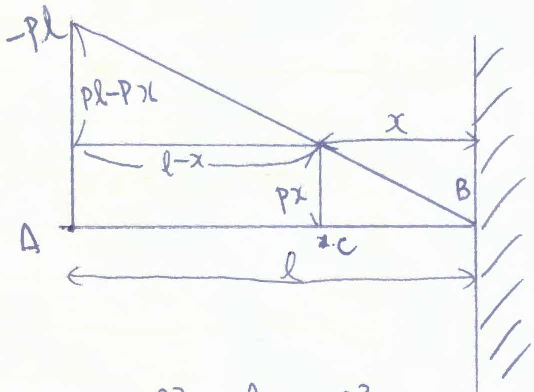
（片持ち梁）



$$M_x = -Px.$$

$$\begin{cases} \dfrac{d^2y}{dx^2} = \dfrac{-P}{EI}x. \\ \dfrac{dy}{dx} = -\dfrac{P}{EI}\left(\dfrac{1}{2}x^2 + c_1\right) \\ y = -\dfrac{P}{EI}\left(\dfrac{1}{6}x^3 + c_1x + c_2\right) \end{cases}$$

$$x = l \Rightarrow \begin{cases} y = 0, \\ \dfrac{dy}{dx} = 0. \end{cases}$$



共役梁 $\begin{cases}\text{自由端} \rightarrow \text{固定端} \\ \text{固定端} \rightarrow \text{自由端}\end{cases}$

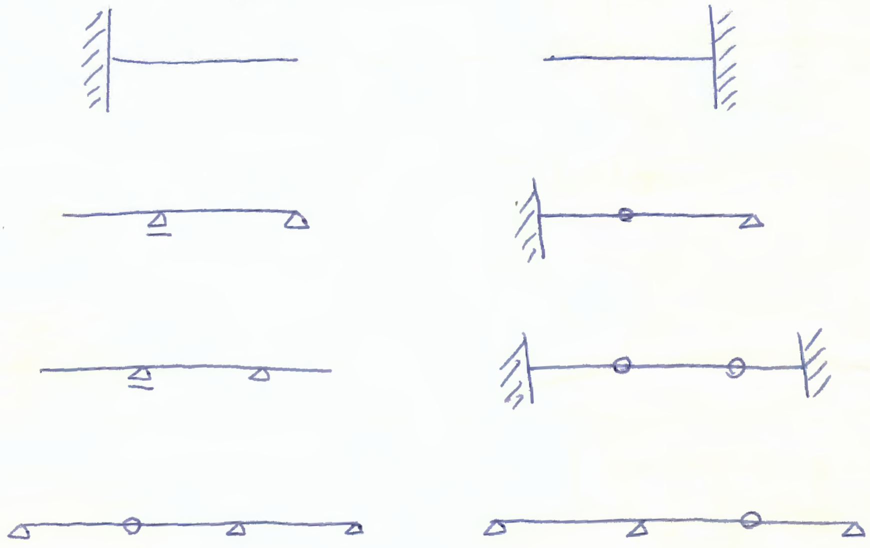
$$M_c = \frac{-Px(l-x)^2}{2} - \frac{P(l-x)^3}{3}$$

$$M_B = \frac{Pl^2}{2} \times \frac{2l}{3} = \frac{Pl^3}{3}$$

$$\therefore y_B = \frac{M_B}{EI} = \frac{Pl^3}{3EI}$$

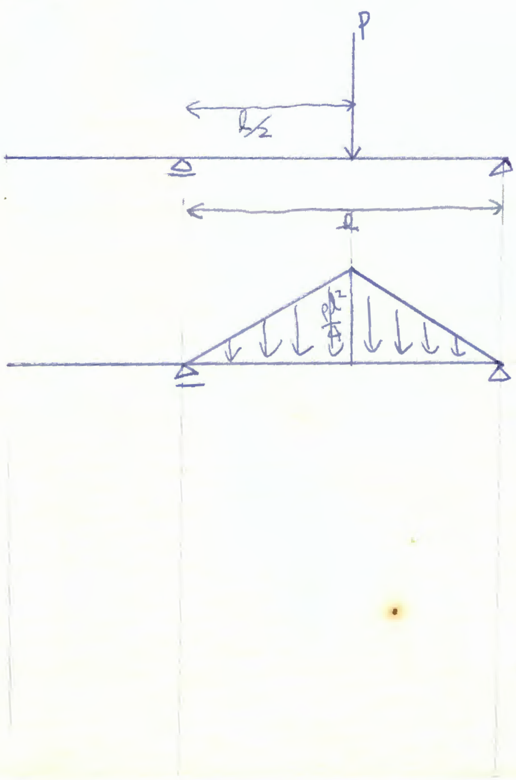
共役梁の色々

元の梁　　　　共役梁.

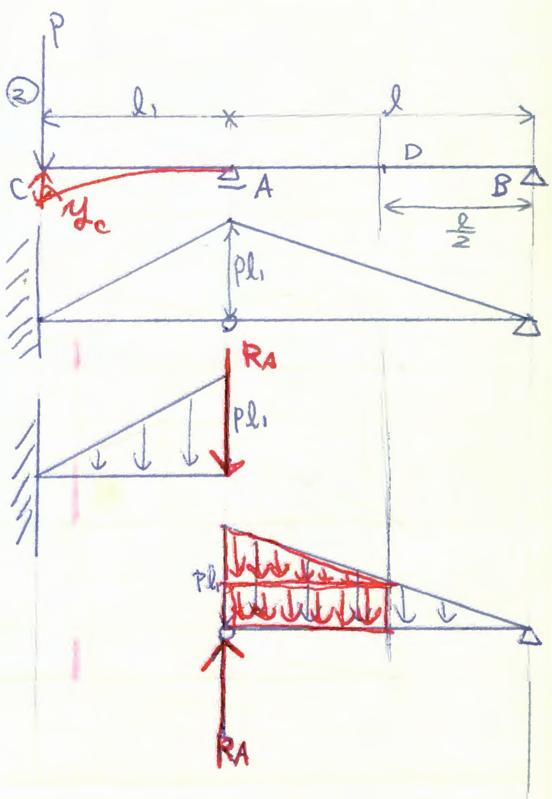


(張出梁)

①



②



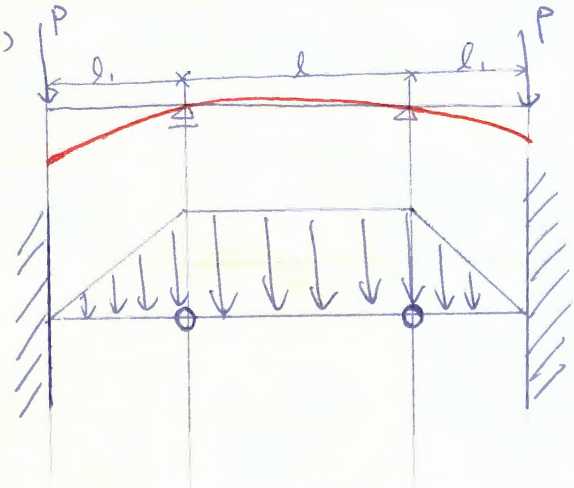
③の場合.

$$R_A l - \frac{P l_1 l}{2} \times \frac{2l}{3} = 0 \qquad R_A = \frac{P l_1 l}{3}$$

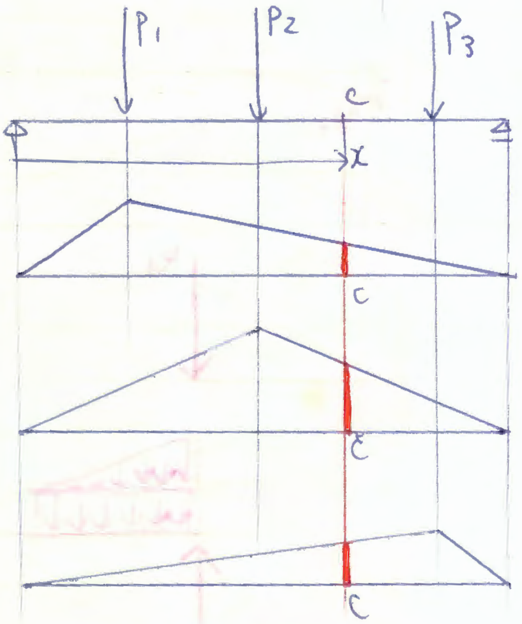
$$M_c = R_A \times l_1 + \frac{P l_1^2}{2} \times \frac{2 l_1}{3} = \frac{P l_1^2 l}{3} + \frac{P l_1^3}{3}$$

$$\therefore y_c = \frac{M_c}{EI} = \frac{P l_1^2}{3EI}(l + l_1)$$

(-ex)



(ex)



P_1による $M_{c_1} \rightarrow y_{c_1}$

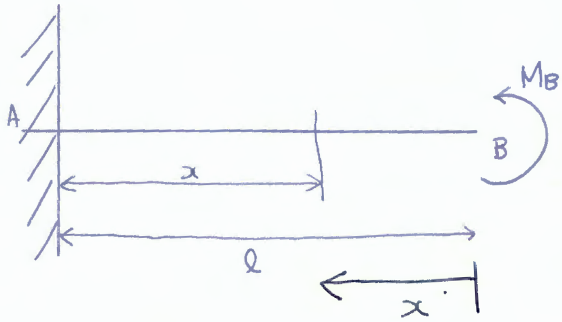
P_2による $M_{c_2} \rightarrow y_{c_2}$

P_3による $M_{c_3} \rightarrow y_{c_3}$

$$y_c = y_{c_1} + y_{c_2} + y_{c_3}$$

$$\theta_A = \theta_{A_1} + \theta_{A_2} + \theta_{A_2}$$

(ex1)



y_B を求めよ。

$$\frac{d^2y}{dx^2} = \frac{-M_B}{EI}$$

$$\frac{dy}{dx} = \frac{-M_B}{EI}x + c_1$$

$x=0 \longrightarrow dy/dx = 0 \quad \therefore c_1 = 0$

$$\frac{dy}{dx} = \frac{-M_B}{EI}x$$

$$y = \frac{-M_B}{2EI}x^2 + c_2$$

$x=0 \longrightarrow y=0 \quad \therefore c_2 = 0$

$$\therefore y = \frac{-M_B}{2EI}x^2$$

$$\theta_B = \left(\frac{dy}{dx}\right)_{x=l} = -\frac{M_B l}{EI}$$

$$y_B = (y)_{x=l} = -\frac{M_B l^2}{2EI}$$

$$\frac{d^2y}{dx^2} = -\frac{M_B}{EI}$$

$$\frac{dy}{dx} = -\frac{M_B}{EI}(x + c_1)$$

$$y = -\frac{M_B}{EI}\left(\frac{x^2}{2} + c_1x + c_2\right)$$

$$\begin{cases} x = l. & y = 0 \\ x = l & dy/dx = 0. \end{cases}$$

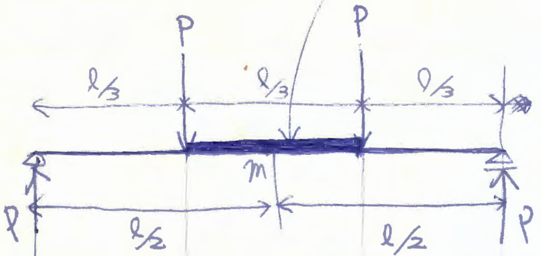
$$\therefore \frac{dy}{dx} = -\frac{M_B}{EI}(x - l)$$

$$y = -\frac{M_B}{EI}\left(\frac{x^2}{2} - lx + \frac{l^2}{2}\right)$$

$$-\frac{l^2}{2} \cancel{+ c_2 = 0}$$

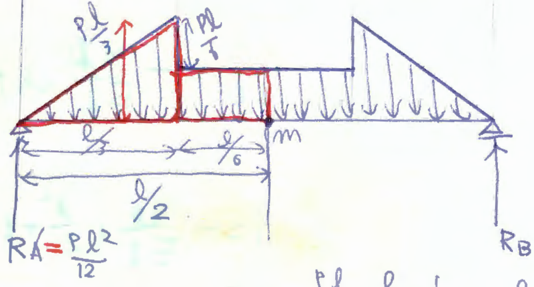
(ex2)

断面二次モーメントが2倍



y_mを求めよ。

~~たわみと分布荷重との関係~~



$\frac{M}{EI}$ 図

$$R_A' = \frac{Pl}{3} \times \frac{l}{3} \times \frac{1}{2} + \frac{l}{6} \times \frac{Pl}{6} = \frac{Pl^2}{18} + \frac{Pl^2}{36} = \frac{Pl^2}{12}$$

$$M_m = \frac{Pl^2}{12} \times \frac{l}{2} - \frac{Pl}{3} \times \frac{l}{3} \times \frac{1}{2} \times \left(\frac{l}{2} - \frac{2l}{9}\right) - \frac{Pl}{6} \times \frac{l}{6} \times \frac{l}{12}$$

$$= \frac{Pl^3}{24} - \frac{Pl^2}{18} \times \frac{5l}{18} - \frac{Pl^3}{36 \times 12}$$

$\frac{9-4}{18}$

$$= \left(\frac{1}{24} - \frac{5}{18 \times 18} - \frac{1}{36 \times 12}\right) Pl^3$$

12×2 18×18 $18 \times 12 \times 2$

$$= \frac{18 \times 18 - 5 \times 24 - 18}{12 \times 18 \times 18 \times 2} = \frac{186}{7776} Pl^3 = \frac{93Pl^3}{3888}$$

$$\therefore y_m = \frac{M_m}{EI} = \frac{93Pl^3}{3888EI}$$

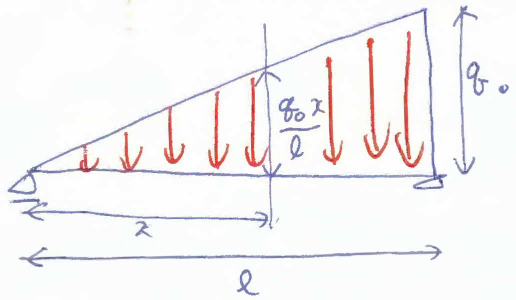
//

18
18
144
18
324
24
1296
648
7776

324
120
204
18
186

(ex3) たわみと分布荷重の関係.

$$\boxed{\begin{aligned} EI\frac{d^2y}{dx^2} &= -M \\ EI\frac{d^3y}{dx^3} &= -\frac{dM}{dx} = -S \\ EI\frac{d^4y}{dx^4} &= -\frac{d^2M}{dx^2} = -\frac{dS}{dx} = q. \end{aligned}}$$



$$EI\,d^4y/dx^4 = q_0 x/l$$

$$EI\,d^3y/dx^3 = q_0 x^2/2l + C_1$$

$$EI\,d^2y/dx^2 = q_0 x^3/6l + C_1 x + C_2$$

$$EI\,dy/dx = q_0 x^4/24l + C_1 x^2/2 + C_2 x + C_3$$

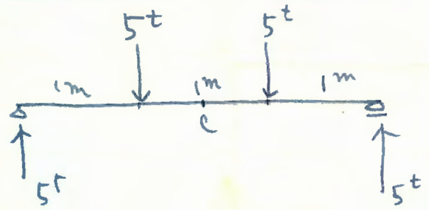
$$EI\,y = q_0 x^5/120l + C_1 x^3/6 + C_2 x^2/2 + C_3 x + C_4$$

$$\begin{cases} x=0 \rightarrow y=y''=0 \\ x=l \rightarrow y=y''=0 \end{cases} \qquad \begin{cases} C_2=0 & C_1=-q_0 l/6 \\ C_4=0 & C_3=7q_0 l^3/360 \end{cases}$$

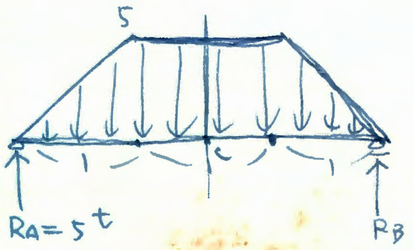
$$y=\frac{q_0}{EI}\left(\frac{x^5}{120l}-\frac{lx^3}{36}+\frac{7l^3x}{180}\right)=\frac{q_0 l^4}{360EI}\left\{3\left(\frac{x}{l}\right)^5-10\left(\frac{x}{l}\right)^3+7\left(\frac{x}{l}\right)\right\}$$

8章.

(2)



$$M_{max} = M_c = 5 \times 1.5 - 5 \times 0.5$$
$$= 5^{tm}.$$



$$R_A = 1 \times 5 \times \frac{1}{2} + \frac{1}{2} \times 5 = 5^{t}$$

$$\therefore \quad M_c = 5 \times 1.5 - \frac{5}{2} \times \left(\frac{1}{2} + \frac{1}{3}\right) - \frac{5}{2} \times \frac{1}{4}$$

$$= 7.5 - 2.1 - 0.6 = 4.8 \ {}^{tm^2}$$

故に $\quad y = \dfrac{M_c}{EI}$

より $\quad E = \dfrac{M_c}{yI} = \dfrac{4.8 \times 100 \times 12}{0.67 \times 15 \times 30^3} = \dfrac{5760}{271350}$

$$= 0.0212 \ {}^{t}/_{cm^2}$$
$$= 21.2 \ {}^{kg}/_{cm^2}$$

$$\frac{25}{12} \quad \frac{5}{6} \times \frac{5}{2} \quad 12)25 = 2.08 \quad 8)5 = 0.625$$

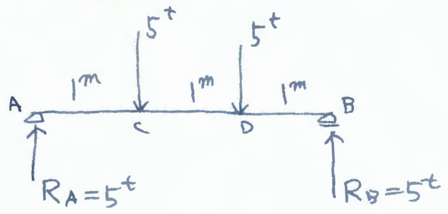
$$\begin{array}{r} 0.67 \\ \times\ 15 \\ \hline 335 \\ 67 \\ \hline 10.05 \end{array} \qquad \begin{array}{r} 270 \\ \times\ 10.05 \\ \hline 1350 \\ 27000 \\ \hline 271350 \end{array} \qquad \begin{array}{r} 4.8 \\ \times\ 12 \\ \hline 96 \\ 48 \\ \hline 57.60 \end{array}$$

$$\begin{array}{r} 0.0212 \\ 27135 \overline{)\,576000} \\ 54270 \\ \hline 33300 \\ 27135 \\ \hline 61650 \\ 54270 \\ \hline 73800 \end{array}$$

6/19（演習）

8−2.

微分方程式



AC 間 $(0 \leqq x_1 \leqq 1)$

$$M_{x_1} = R_A x_1 = 5x_1$$

$$\frac{d^2y_1}{dx_1^2} = -\frac{M_{x_1}}{EI} = -\frac{5x_1}{EI}$$

$$\therefore \frac{dy_1}{dx_1} = -\frac{5}{EI}\left(\frac{x_1^2}{2} + C_1\right)$$

$$y_1 = -\frac{5}{EI}\left(\frac{x_1^3}{6} + C_1 x_1 + C_2\right)$$

CD 間 $(1 \leqq x_2 \leqq 2)$

$$M_{x_2} = R_A x_2 - 5(x_2 - 1) = 5^{tm}$$

$$\frac{d^2y_2}{dx_2^2} = -\frac{5}{EI}$$

$$\frac{dy_2}{dx_2} = -\frac{5}{EI}(x_2 + C_1')$$

$$y_2 = -\frac{5}{EI}\left(\frac{x_2^2}{2} + C_1' x_2 + C_2'\right)$$

左右対称だから DB 間不要.

境界条件.

(i) $x_1 = 0 \longrightarrow y_1 = 0$

(ii) $x_2 = 1.5 \longrightarrow dy_2/dx_2 = 0$

(iii) $[y_1]_{x_1=1} = [y_2]_{x_2=1}$

(iv) $[dy_1/dx_1]_{x_1=1} = [dy_2/dx_2]_{x_2=1}$

(i) より $C_2 = 0$

(ii) より $C_1' = -1.5$

(iii) より $\frac{1}{6} + C_1 + C_2 = \frac{1}{2} + C_1' + C_2'$

$$\therefore C_1 - C_2' = \frac{1}{2} - \frac{1}{6} - 1.5 = -\frac{7}{6}$$

$$C_2' = C_1 + \frac{7}{6} = 1/6$$

(iv) より $\frac{1}{2} + C_1 = 1 + C_1'$ より

$$C_1 = C_1' + \frac{1}{2} = -1.0$$

$$\left\{\begin{aligned} \frac{dy_1}{dx_1} &= -\frac{5}{EI}\left(\frac{x_1^2}{2} - 1\right) \\ y_1 &= -\frac{5}{EI}\left(\frac{x_1^3}{6} - x_1\right) \\ \frac{dy_2}{dx_2} &= -\frac{5}{EI}(x_2 - 1.5) \\ y_2 &= -\frac{5}{EI}\left(\frac{x_2^2}{2} - 1.5x_2 + \frac{1}{6}\right) \end{aligned}\right.$$

$$x_2 = 1.5^m \longrightarrow y_2 = 0.67 \times 10^{-2}\ ^m$$

$$E = \frac{5 \times 0.958}{\frac{bh^3}{12} \times 0.67 \times 10^{-2}}$$

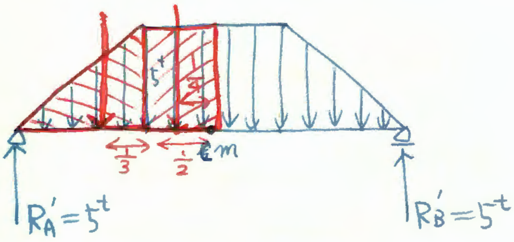
$$E = \frac{5 \times 0.958 \times 10^5}{0.67 \times \frac{135 \times 10^3}{4}} = 21.2\ \mathrm{kg/cm^2}$$

8−2 弾性荷重法.

$0 \leqq x \leqq 1 \quad M_x = 5x$

$1 \leqq x \leqq 2 \quad M_x = 5$

$2 \leqq x \leqq 3 \quad M_x = -5(x-3)$



最大たわみは梁の中央である。

その点mのモーメント

$$M_m = R_A' \times 1.5 - \frac{5}{2}\left(\frac{1}{3}+\frac{1}{2}\right) - \frac{5}{2} \times \frac{1}{4}$$

$$= \frac{115}{24}\,(t\,m) = \frac{115 \times 10^5}{24}\,(kg\,cm)$$

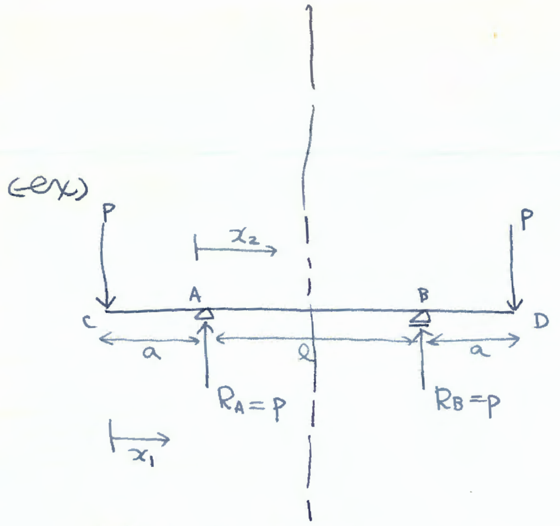
$$I = \frac{bh^3}{12} = \frac{15 \times 30^3}{12} = \frac{135 \times 10^3}{4}\,(cm^4)$$

$$y_{max} = \frac{M_m}{EI} \text{ より}$$

$$\therefore E = \frac{M_m}{y_{max}\, I} = \frac{\frac{115 \times 10^5}{24}}{0.67 \times \frac{135 \times 10^3}{4}}$$

$$= \frac{115 \times 4 \times 10^5}{24 \times 0.67 \times 135 \times 10^3} = 21.2\ kg/cm^2$$

(ex)



CA 間. $M=-Px_1$

AB 間. $M=-Pa$.

$$\frac{d^2y_1}{dx_1^2}=\frac{P}{EI}x_1$$

$$\frac{d^2y_2}{dx_2}=\frac{Pa}{EI}$$

境界条件.(B.C.)

(i) $x_1=a,\ x_2=0 \Rightarrow dy_1/dx_1 = dy_2/dx_2$

(ii) $x_1=a \Rightarrow y_1=0$

(iii) $x_2=0 \Rightarrow y_2=0$

(iv) $x_2=\ell/2 \Rightarrow dy_2/dx_2=0$

$$y_1=\frac{P}{6EI}\left[x_1^3-3a(\ell+a)x_1+a^2(3\ell+2a)\right]$$

$$\left.\begin{aligned}\frac{dy_1}{dx_1}&=\frac{P}{EI}\left(\frac{x_1^2}{2}+C_1\right)\\ y_1&=\frac{P}{EI}\left(\frac{x_1^3}{6}+C_1x+C_2\right)\end{aligned}\right\}\Rightarrow\left\{\begin{aligned}\frac{dy_1}{dx_1}&=\frac{P}{2EI}(x_1^2-a\ell-a^2)\\ &\cancel{y_1=\frac{Pa}{2EI}(2x-\ell)}\end{aligned}\right.$$

$$\left.\begin{aligned}\frac{dy_2}{dx_2}&=\frac{Pa}{EI}(x_2+C_1')\\ y_2&=\frac{Pa}{EI}\left(\frac{x_2^2}{2}+C_1'x_2+C_2'\right)\end{aligned}\right\}\Rightarrow\left\{\begin{aligned}\frac{dy_2}{dx_2}&=\frac{Pa}{2EI}(2x_2-\ell)\\ y_2&=\frac{Pa}{2EI}(x_2^2-\ell x_2)\end{aligned}\right.$$

(iv)より $C_1'=-\frac{\ell}{2}$

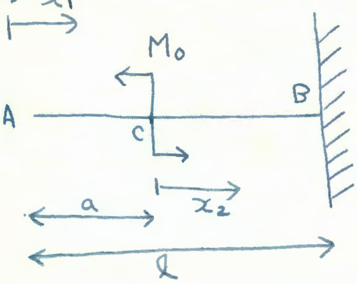
(iii)より $C_2'=0$

(ii)より $\frac{a^3}{6}+C_1a+C_2=0$

(i)より $\frac{a^2}{2}+C_1=aC_1'=-\frac{\ell a}{2}$ $\quad\therefore C_1=-\frac{1}{2}(a^2+\ell a)$

$$\therefore C_2=-\frac{a^3}{6}+\frac{a^2}{2}(a+\ell)=\frac{a^3}{3}+\frac{a^2\ell}{2}=\frac{a^2}{6}(3\ell+2a)$$

(ex) x_1



A点のたわみ角・たわみ.

AC 間 $M_{x_1} = 0 \quad (0 \leqq x_1 \leqq a)$

CB 間 $M_{x_2} = -M_0 \quad (0 \leqq x_2 \leqq (l-a))$

$$\frac{d^2 y_2}{dx_2^2} = \frac{M_0}{EI}$$

$$\frac{dy_2}{dx_2} = \frac{M_0}{EI}(x_2 + C_1)$$

$$y_2 = \frac{M_0}{EI}\left(\frac{x_2^2}{2} + C_1 x_2 + C_2\right)$$

$$\frac{d^2 y_1}{dx_1^2} = 0$$

$$\frac{dy_1}{dx_1} = \cancel{x_1} + C_1'$$

$$y_1 = \cancel{\frac{x_1^2}{2}} + C_1' x_1 + C_2'$$

B.C. i) $x_1 = a,\ x_2 = 0 \Rightarrow y_1 = y_2$

ii) $x_2 = l-a \Rightarrow dy_2/dx_2 = 0$

iii) $x_2 = l-a \Rightarrow y_2 = 0$

iv) $x_1 = a,\ x_2 = 0 \Rightarrow dy_1/dx_1 = dy_2/dx_2$

ii) より $C_1 = a - l$.

iii) より $\frac{(l-a)^2}{2} + -(l-a)^2 + C_2 = 0 \quad \therefore C_2 = \frac{(l-a)^2}{2}$

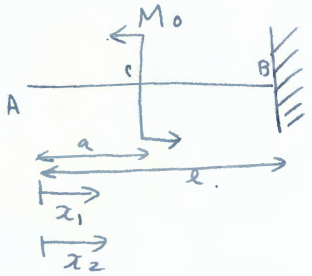
i) より $\frac{a^2}{2} + C_1' a + C_2' = \frac{M_0}{EI} \cdot \frac{(l-a)^2}{2}$

iv) より $a + C_1' = \frac{M_0}{EI}(a-l)$

$C_1' = -\frac{M_0}{EI}(l-a) - a$

$C_2' = \frac{a^2}{2} - \frac{M_0 a}{EI}(l-a) - a^2 - \frac{M_0}{EI}\frac{(l-a)^2}{2}$

$= -\frac{a^2}{2}$



とすれば．

$$\frac{dy_1}{dx_1} = C_1 = -\frac{M_0}{EI}(l-a)$$

$$\frac{dy_2}{dx_2} = \frac{M_0}{EI}(x_2 - l)$$

$$y_1 = C_1 x_1 + C_2$$

$x_2 = l \longrightarrow y_2 = 0$

$dy_2/dx_2 = 0$

$x_1 = x_2 = a \Rightarrow y_1 = y_2$

$$\frac{dy_1}{dx_1} = \frac{dy_2}{dx_2}$$

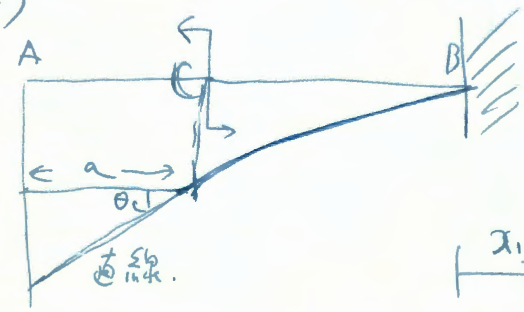
$C_1' = -l$, $C_2' = \frac{l^2}{2}$ $\qquad C_1 = -\frac{M_0}{EI}(l-a)$

$$C_2 = \frac{M_0}{2EI}(l^2 - a^2)$$

$$y_A = y_1 \big|_{x_1=0} = C_2 = \frac{M_0}{2EI}(l^2 - a^2)$$

$$\theta_A = \frac{dy_1}{dx_1}\Big|_{x_1=0} =$$

注)



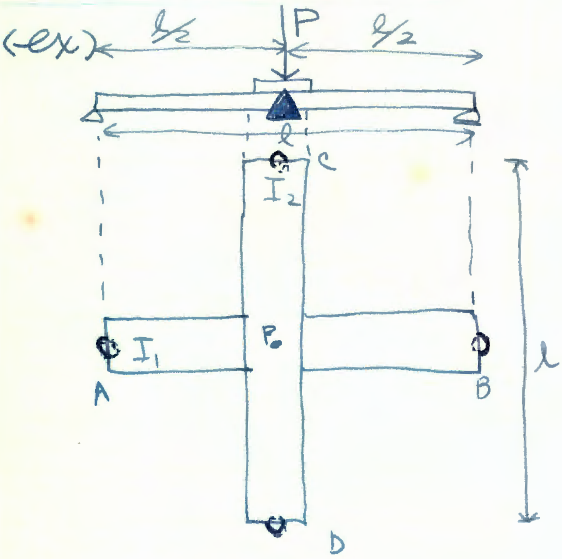
$\theta_A = \theta_C$.

$y_A = y_C + \theta_C \times a$

$$\frac{dy_1}{dx_1} = -\frac{dy_2}{dx_2}$$

6/22 講義 (8-11)

(ex)



同じ材料でつくったスパンの等しい二つの単純梁がその中央において交わり、交点に集中荷重Pをうけている。二つの梁の分担する荷重$P_1 P_2$を求めよ。
ただし断面二次モーメントをそれぞれI_1 I_2とする。

$$\begin{cases} \text{AB梁の分担荷重 } P_1 \\ \text{CD梁の分担荷重 } P_2 \end{cases}$$

AB梁の中央点のたわみ

$$y_1 = \frac{P_1 l^3}{48 E I_1}$$

CD梁の中央点のたわみ

$$y_2 = \frac{P_2 l^3}{48 E I_2}$$

$y_1 = y_2$ より

$$P_1 I_2 = P_2 I_1$$

また $P = P_1 + P_2$

$$\therefore \begin{cases} P_1 = \dfrac{I_1 P}{I_1 + I_2} \\ P_2 = \dfrac{I_2 P}{I_1 + I_2} \end{cases}$$

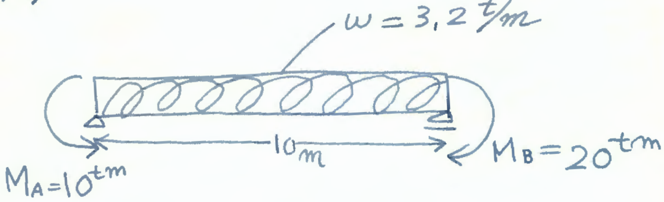
ヤング率 E_1 E_2
スパン l_1 l_2
とすると

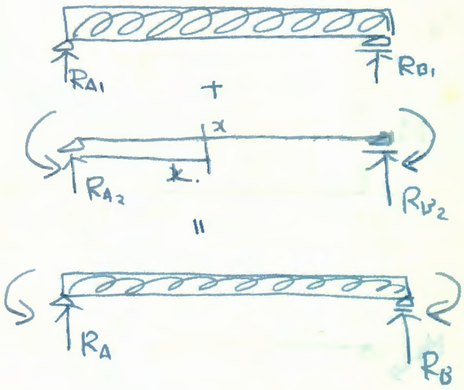
$$\frac{P_1 l_1^3}{48 E_1 I_1} = \frac{P_2 l_2^3}{48 E_2 I_2}$$

より

$$\begin{cases} P_1 = \dfrac{E_1 I_1 l_2^3}{E_1 I_1 l_2^3 + E_2 I_2 l_1^3} P \\ P_2 = \dfrac{E_2 I_2 l_1^3}{E_2 I_2 l_1^3 + E_1 I_1 l_2^3} P \end{cases}$$

(ex)





$$\left.\begin{array}{l} R_{A_1}+R_{B_1}=32^t \\ R_{A_1}=R_{B_1} \end{array}\right\} \rightarrow \begin{array}{l} R_{A_1}=16^t \\ R_{B_1}=16^t \end{array}$$

$$M_{max_1}=\frac{wl^2}{8}=\frac{3.2\times10^2}{8}=40^{tm}$$

$$R_{A_2}\times10-10+20=0$$

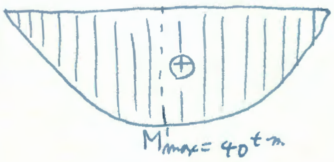
$$R_{A_2}=-1^t\ ,\quad R_{B_2}=1^t$$

$$M_{x_2}=R_{A_2}x-M_A$$
$$=-x-10$$

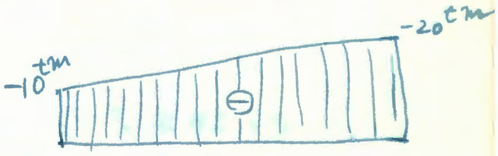
$$\begin{cases} R_A=R_{A_1}+R_{A_2} \\ R_B=R_{B_1}+R_{B_2} \end{cases}$$

$$M_x=M_{x_1}+M_{x_2}.$$

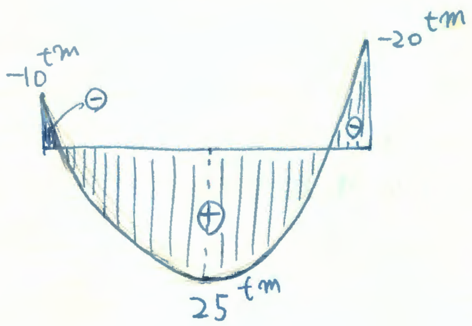
(M_{x_1} 図)



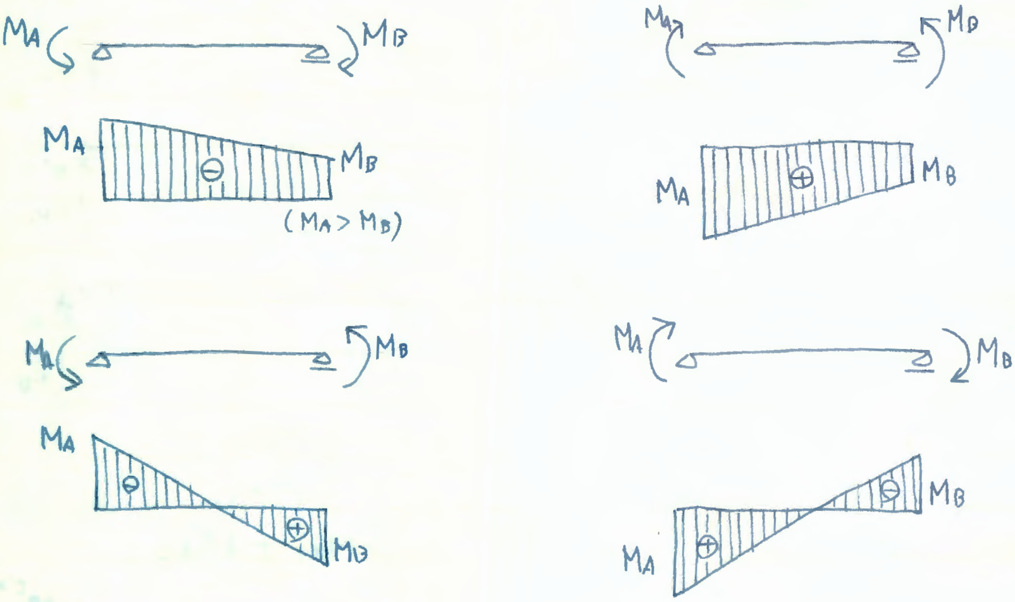
(M_{x_2} 図)



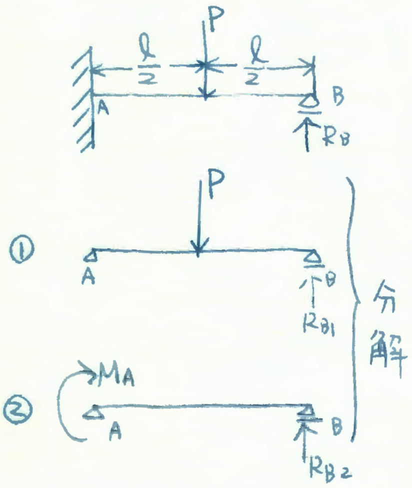
M_x 図.



○単モーメントによる単純梁の曲げモーメント



(一次不静定梁.)



梁①のy_Aと梁②のy_Aをたすと零.

$y_{A①} + y_{A②} = 0 \Rightarrow M_A =$ ______

①の曲げモーメント$M_{x①}$

② 〃 $M_{x②}$

⇓

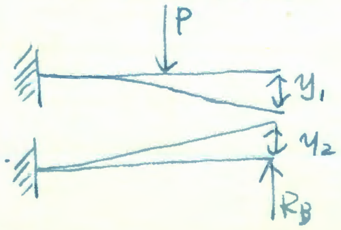
$M_x = M_{x①} + M_{x②}$

より曲げモーメント図が求まる。

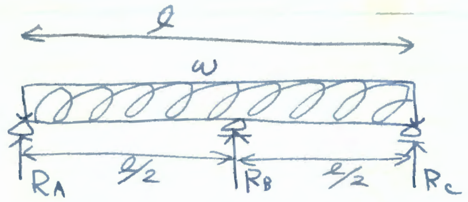
反力の出し方.

(1) $R_B = R_{B1} + R_{B2}$.

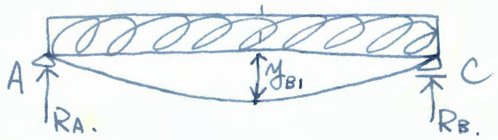
(2)

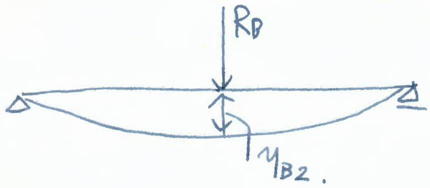


$y_1 + y_2 = 0$ より $R_B =$ ______



R_B をもとめる。





(方針) $y_{B1} + y_{B2} = 0 \Rightarrow R_B =$ ~~~

$$y_{B1} = \frac{5wl^4}{384EI}$$

$$y_{B2} = \frac{R_B l^3}{48EI}$$

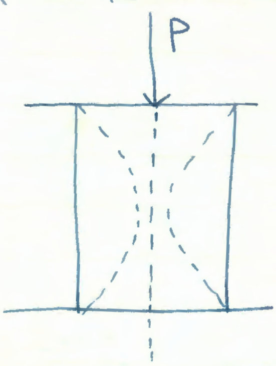
$$y_{B1} + y_{B2} = 0$$

より $R_B = -\dfrac{5wl \times 48\cancel{EI}}{384\cancel{EI}}$

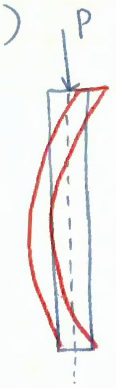
$$= -\frac{240wl}{384}$$

10章 柱 (column)

短柱 (Short column)
長柱 (Long column)

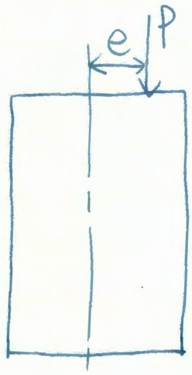


$(\sigma_c = P/A)$

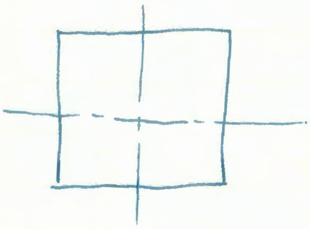


(座屈.)

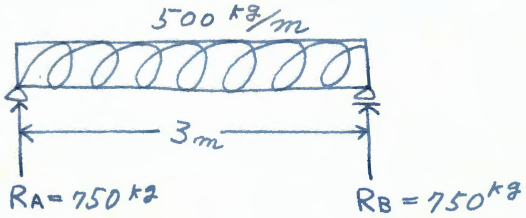
$\left(\dfrac{l}{d}\ \cancel{\gt 輪}\ の値 \right.$ によって区別.)



軸方向力
曲げモーメント



(8-1)



$$E = 10^5\ kg/cm^2 = 10^9\ kg/m^2$$

$$I = \frac{bh^3}{12} = \frac{0.1\times 0.2^3}{12} = 6.7\times 10^{-5}\ m^4$$

$$M_x = R_A x - \frac{q}{2}x^2 = 750x - 250x^2$$

$$\frac{d^2y}{dx^2} = -\frac{M_x}{IE} = \frac{250}{IE}(x^2 - 3x)$$

$$\frac{dy}{dx} = \frac{250}{IE}\left(\frac{x^3}{3} - \frac{3x^2}{2} + C_1\right) = \frac{250}{IE}\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2.25\right)$$

$$y = \frac{250}{IE}\left(\frac{x^4}{12} - \frac{x^3}{2} + C_1 x + C_2\right) = \frac{250}{IE}\left(\frac{x^4}{12} - \frac{x^3}{2} + 2.25x\right)$$

$$\left\{\begin{array}{llll} x = 1.5^m \rightarrow & dy/dx = 0 & \therefore C_1 = \frac{3}{2}(1.5)^2 - \frac{1}{3}(1.5)^3 = 2.25 \\ x = 0 \rightarrow & y = 0 & C_2 = 0 \end{array}\right\}$$

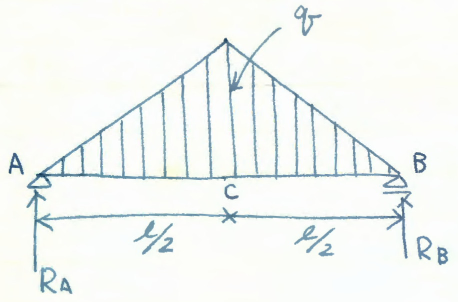
$$\therefore [y]_{x=1.5} = \frac{250}{6.7\times10^{-5}\times10^9}\left\{\frac{(1.5)^4}{12} - \frac{(1.5)^3}{2} + 2.25\times1.5\right\}$$

$$= 79\times10^{-4\ m} = 0.79\ cm$$

$$\left[\frac{dy}{dx}\right]_{x=0} = \frac{250}{6.7\times10^{-5}\times10^9}\times 2.25 = 0.0084$$

$$\therefore \alpha = \tan^{-1} 0.0084 = 29'$$

(8-3)



$$R_A = R_B = \frac{ql}{4}$$

AC間において.

$$M_x = \frac{ql}{4}x - x \times q \cdot \frac{x}{\frac{l}{2}} \times \frac{x}{3} \times \frac{1}{2}$$

$$= \frac{ql}{4}x - \frac{q}{3l}x^3$$

$$\frac{d^2y}{dx^2} = \frac{-1}{IE}\left(\frac{q}{3l}x^3 - \frac{ql}{4}x\right)$$

$$\frac{dy}{dx} = \frac{1}{IE}\left(\frac{q}{12l}x^4 - \frac{ql}{8}x^2 + C_1\right)$$

$$y = \frac{1}{IE}\left(\frac{q}{60l}x^5 - \frac{ql}{24}x^3 + C_1 x + C_2\right)$$

$x=0 \rightarrow y=0 \qquad \therefore C_2 = 0.$

$x = l/2 \rightarrow dy/dx = 0 \qquad \therefore C_1 = \frac{ql}{8}\left(\frac{l}{2}\right)^2 - \frac{q}{12l}\left(\frac{l}{2}\right)^4$

$$= \frac{ql^3}{32} - \frac{ql^3}{192} = \frac{5ql^3}{192}$$

$$\therefore y = \frac{1}{IE}\left(\frac{q}{60l}x^5 - \frac{ql}{24}x^3 + \frac{5ql^3}{192}x\right)$$

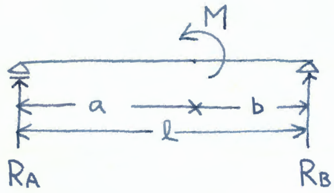
$$\therefore [y]_{x=l/2} = \frac{1}{IE}\left\{\frac{q}{60l}\left(\frac{l}{2}\right)^5 - \frac{ql}{24}\left(\frac{l}{2}\right)^3 + \frac{5ql^3}{192}\cdot\frac{l}{2}\right\}$$

$$= \cancel{\frac{1}{240}}\left(\frac{1}{1920} - \frac{1}{192} + \frac{1}{192} \times \frac{5}{2}\right)\frac{ql^4}{IE}$$

$$= \frac{ql^4}{120\,IE} \quad //$$

H.W

(8-5)



$R_A = \frac{M}{\ell}$ $\quad R_B = -\frac{M}{\ell}$

$0 \leqq x_1 \leqq a$

$$M_{x_1} = \frac{M}{\ell} x_1$$

$$\frac{d^2 y_1}{dx_1^2} = -\frac{M}{IE\ell} x_1$$

$$\frac{dy_1}{dx_1} = -\frac{M}{IE\ell}\left(\frac{x_1^2}{2} + C_1\right)$$

$$y_1 = -\frac{M}{IE\ell}\left(\frac{x_1^3}{6} + C_1 x_1 + C_2\right)$$

$a \leqq x_2 \leqq \ell$

$$M_{x_2} = -\frac{M}{\ell}(\ell - x_2)$$

$$\frac{d^2 y_2}{dx_2^2} = \frac{M}{IE\ell}(\ell - x_2)$$

$$\frac{dy_2}{dx_2} = \frac{M}{IE\ell}\left\{-\frac{(\ell - x_2)^2}{2} + C_1'\right\}$$

$$y_2 = \frac{M}{IE\ell}\left\{\frac{(\ell - x_2)^3}{6} + C_1' x_2 + C_2'\right\}$$

境界条件. i) $x_1 = 0$; $y_1 = 0$ $\qquad \therefore C_2 = 0$

ii) $x_2 = \ell$: $y_2 = 0$

$$C_1' \ell + C_2' = 0 \qquad \text{—— ①}$$

iii) $[dy_1/dx_1]_{x_1=a} = [dy_2/dx_2]_{x_2=a}$

$$\therefore \frac{a^2}{2} + C_1 = \frac{b^2}{2} - C_1' \qquad \therefore C_1 + C_1' = \frac{b^2 - a^2}{2} \text{ —— ②}$$

iv) $[y_1]_{x_1=a} = [y_2]_{x_2=a}$

$$\frac{a^3}{6} + C_1 a = -\frac{b^3}{6} - aC_1' - C_2'$$

$$\therefore aC_1 + aC_1' + C_2' = -\frac{a^3 + b^3}{6} \text{ —— ③}$$

①より $C_2' = -C_1' l$

②より $C_1 = \dfrac{b^2-a^2}{2} - C_1'$

$$\therefore a\left(\frac{b^2-a^2}{2} - \cancel{C_1'}\right) + \cancel{aC_1'} - C_1' l = -\frac{a^3+b^3}{6}$$

$$\therefore C_1' = \left\{\frac{a}{2}(b^2-a^2) + \frac{a^3+b^3}{6}\right\}\frac{1}{l}$$

$$= \left\{\frac{b^3}{6} + \frac{ab^2}{2} - \frac{a^3}{3}\right\}\frac{1}{l} = \left\{\frac{b^2(b+a+2a)}{6} - \frac{a^3}{3}\right\}\frac{1}{l}$$

$$\therefore C_2' = \frac{a^3}{3} - \frac{ab^2}{2} - \frac{b^3}{6}$$

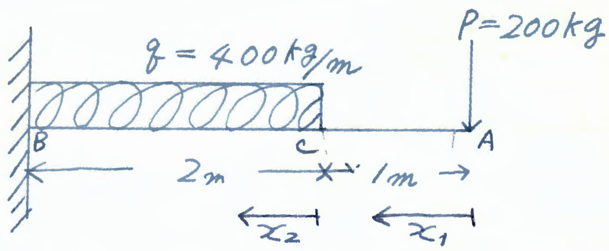
$$C_1 = \frac{b^2-a^2}{2} - \left\{\frac{b^3}{6} + \frac{ab^2}{2} - \frac{a^3}{3}\right\}\frac{1}{l}$$

$$= \left\{\frac{b^2(a+b)}{6} + \frac{a(a+b)(b-a)}{3}\right\}\frac{1}{l}$$

$$\left\{\begin{aligned} C_1' &= \frac{b^2}{6} + \frac{a(b-a)}{3} = \frac{b^2}{6} + \frac{ab}{3} - \frac{a^2}{3} = \frac{b^2+2ab-2a^2}{6} \\ \therefore C_1 &= \frac{b^2-a^2}{2} - \frac{b^2+2ab-2a^2}{6} = \frac{2b^2-2ab-a^2}{6} \\ C_2' &= \frac{2a^3-3ab^2-b^3}{6} \end{aligned}\right.$$

$$\therefore \text{Ans.}\left\{\begin{aligned} y_1 &= \frac{Mx}{6IEl}(a^2+2ab-2b^2-x^2) \\ y_2 &= \frac{M}{6IEl}\left\{(l-x)^3 + (b^2+2ab-2a^2)x + (2a^3-3ab^2-b^3)\right\} \\ &= \frac{M(l-x)}{6IEl}\left\{(l-x)^2 + 2a^2 - 2ab - b^2\right\} \end{aligned}\right.$$

(8−6)



$$-M_{x_2} = 200(x_2+1) + \frac{400}{2}\cdot x_2^2$$

AC間 $\dfrac{d^2y_1}{dx_1^2} = \dfrac{200}{IE}x_1$

$$\frac{dy_1}{dx_1} = \frac{200}{IE}\left(\frac{x_1^2}{2} + C_1\right)$$

$$y_1 = \frac{200}{IE}\left(\frac{x_1^3}{6} + C_1x_1 + C_2\right)$$

CB間 $\dfrac{d^2y_2}{dx_2^2} = \dfrac{200}{IE}(x_2^2 + x_2 + 1) = \cancel{x_2}$

$$\frac{dy_2}{dx_2} = \frac{200}{IE}\left(\frac{x_2^3}{3} + \frac{x_2^2}{2} + x_2 + C_1'\right)$$

$$y_2 = \frac{200}{IE}\left(\frac{x_2^4}{12} + \frac{x_2^3}{6} + \frac{x_2^2}{2} + C_1'x_2 + C_2'\right)$$

(境界条件) i) $x_2 = 2$; $dy_2/dx_2 = 0$

$$\therefore\ C_1' = -\frac{2^3}{3} - \frac{2^2}{2} - 2 = -\frac{20}{3}$$

ii) $x_2 = 2$; $y_2 = 0$

$$C_2' = -\frac{2^4}{12} - \frac{2^3}{6} - \frac{2^2}{2} + \frac{20}{3}\times 2 = \frac{26}{3}$$

iii) $x_1 = 1,\ x_2 = 0 \rightarrow dy_1/dx_1 = dy_2/dx_2$

$$\frac{1}{2} + C_1 = C_1' \qquad \therefore C_1 = -\frac{20}{3} - \frac{1}{2} = -\frac{43}{6}$$

iv) $x_1 = 1,\ x_2 = 0 \rightarrow y_1 = y_2$

$$\frac{1}{6} + C_1 + C_2 = C_2' \qquad \therefore C_2 = \frac{26}{3} - \frac{1}{6} + \frac{43}{6} = \frac{94}{6}$$

#

故に $\dfrac{dy_1}{dx_1} = \dfrac{200}{IE}\left(\dfrac{x_1^2}{2} - \dfrac{43}{6}\right)$

$$y_1 = \frac{200}{IE}\left(\frac{x_1^3}{6} - \frac{43}{6}x_1 + \frac{94}{6}\right)$$

$$\therefore \left[\frac{dy_1}{dx_1}\right]_{x_1=0} = \frac{200}{IE}\cdot\left(-\frac{43}{6}\right)$$

$$= \frac{-200 \times 43}{1674\times10^{-8}\times21\times10^{5}\times10^{4}\times6}$$

$$= -0.0041$$

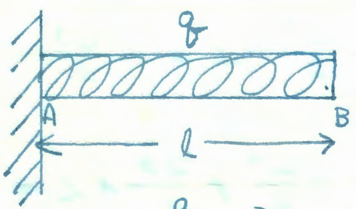
$$\therefore \alpha = \tan^{-1}|0.0041| = 14' \;//$$

$$[y_1]_{x_1=0} = \frac{200}{IE}\times\frac{94}{6} = \frac{200\times94}{1674\times10^{-8}\times21\times10^{5}\times10^{4}\times6}$$

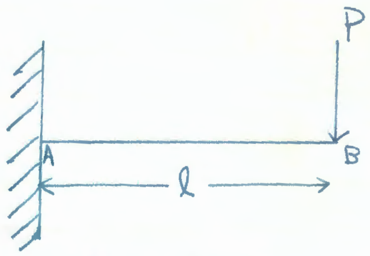
$$= 0.0089^{m} = 0.89^{cm} \;//$$

(8-7)

(1)



(2)



(1)

$$M_x = -\frac{q}{2}x^2$$

$$\frac{d^2y}{dx^2} = \frac{q}{2IE}x^2$$

$$\frac{dy}{dx} = \frac{q}{2IE}\left(\frac{x^3}{3} + C_1\right)$$

$$y = \frac{q}{2IE}\left(\frac{x^4}{12} + C_1x + C_2\right)$$

i) $x=l$: $dy/dx=0$ $\therefore C_1 = -l^3/3$

ii) $x=l$: $y=0$

$$C_2 = -\frac{l^4}{12} + \frac{l^4}{3} = \frac{l^4}{4}$$

(2)

$$M_x = -Px$$

$$\frac{d^2y}{dx^2} = \frac{P}{IE}x$$

$$\frac{dy}{dx} = \frac{P}{IE}\left(\frac{x^2}{2} + C_1'\right)$$

$$y = \frac{P}{IE}\left(\frac{x^3}{6} + C_1'x + C_2'\right)$$

i) $x=l$: $dy/dx=0$ $\therefore C_1' = -l^2/2$

ii) $x=l$: $y=0$

$$\therefore C_2' = -\frac{l^3}{6} + \frac{l^3}{2} = \frac{l^3}{3}$$

$$\therefore \quad y_1 = \frac{q}{2IE}\left(\frac{x^4}{12} - \frac{l^3}{3}x + \frac{l^4}{4}\right)$$

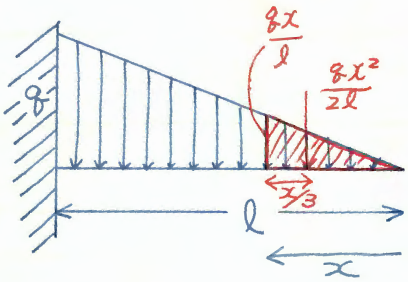
$$y_2 = \frac{P}{IE}\left(\frac{x^3}{6} - \frac{l^2}{2}x + \frac{l^3}{3}\right)$$

$x=0 \ : \ y_1 = y_2$

$$\therefore \frac{ql^4}{8IE} = \frac{Pl^3}{3IE}$$

$$\therefore \frac{ql}{8} = P/3 \qquad \therefore \frac{P}{q} = \frac{3l}{8} \ : \text{Ans.}$$

(8-8)



$$M_x = -\frac{x}{3} \times \frac{qx^2}{2l} = -\frac{q}{6l}x^3$$

$$\frac{d^2y}{dx^2} = +\frac{q}{6IEl}x^3$$

$$\frac{dy}{dx} = +\frac{q}{6IEl}\left(\frac{x^4}{4} + C_1\right)$$

$$y = +\frac{q}{6IEl}\left(\frac{x^5}{20} + C_1x + C_2\right)$$

$$\left.\begin{aligned} \left|\left[\frac{dy}{dx}\right]_{x=0}\right| &= \left|-\frac{ql^4}{24IEl}\right| = \frac{ql^3}{24IE} \\ \left[y\right]_{x=0} &= \frac{ql^4}{\cancel{6}30IE} \end{aligned}\right\} \text{Ans.}$$

$x = l : \ dy/dx = 0 \ , \ y = 0$

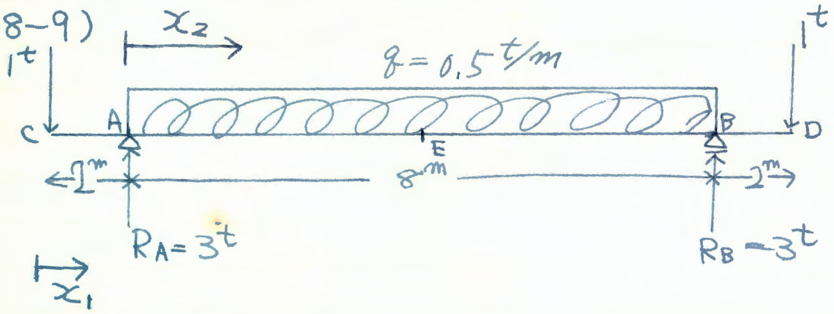
$$\therefore C_1 = -\frac{l^4}{4}$$

$$C_2 = -\frac{l^5}{20} + \frac{l^5}{4} = \frac{l^5}{5}$$

$$\therefore \frac{dy}{dx} = \frac{q}{6IEl}\left(\frac{x^4}{4} - \frac{l^4}{4}\right)$$

$$y = \frac{q}{6IEl}\left(\frac{x^5}{20} - \frac{l^4}{4}x + \frac{l^5}{5}\right)$$

(8-9)



$$M_{x_1} = -x_1 \quad , \quad M_{x_2} = \cancel{-x+3(x-2)} -2 - x_2 + 3x_2 - \frac{q}{2}x_2^2$$

$$= -\frac{x_2^2}{4} + 2x_2 - 2$$

$$\frac{d^2y_1}{dx_1^2} = \frac{1}{IE}x_1 \qquad \frac{d^2y_2}{dx_2^2} = \frac{1}{IE}\left(\frac{x_2^2}{4} - 2x_2 + 2\right)$$

$$\frac{dy_1}{dx_1} = \frac{1}{IE}\left(\frac{x_1^2}{2} + C_1\right) \qquad \frac{dy_2}{dx_2} = \frac{1}{IE}\left(\frac{x_2^3}{12} - x_2^2 + 2x_2 + C_1'\right)$$

$$y_1 = \frac{1}{IE}\left(\frac{x_1^3}{6} + C_1x_1 + C_2\right) \qquad y_2 = \frac{1}{IE}\left(\frac{x_2^4}{48} - \frac{x_2^3}{3} + x_2^2 + C_1'x + C_2'\right)$$

i) $x_2 = 4$; $dy_2/dx_2 = 0$

$$C_1' = -\frac{4^3}{12} + 4^2 - 2 \times 4 = 8 - \frac{16}{3} = \frac{8}{3}$$

ii) $x_2 = 0$; $y_2 = 0$

$$\therefore C_2' = 0$$

iii) $x_1 = 2$; $y_1 = 0$

$$C_2 = -\frac{2^3}{6} - 2C_1$$

iv) $x_1 = 2$, $x_2 = 0$ $\quad dy_1/dx_1 = dy_2/dx_2$

$$\frac{2^2}{2} + C_1 = C_1' = \frac{8}{3} \qquad \therefore C_1 = \frac{8}{3} - 2 = \frac{2}{3}$$

$$\therefore C_2 = -\frac{8}{6} - \frac{4}{3} = -\frac{8}{3}$$

$$\begin{cases} y_1 = \dfrac{1}{IE}\left(\dfrac{x_1^3}{6} + \dfrac{2}{3}x_1 - \dfrac{8}{3}\right) \\ y_2 = \dfrac{1}{IE}\left(\dfrac{x_2^4}{48} - \dfrac{x_2^3}{3} + x_2^2 + \dfrac{8}{3}x_2\right) \end{cases}$$

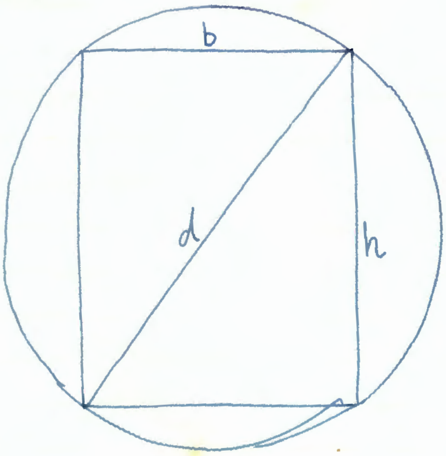
故に $[y_1]_{x_1=0} = \dfrac{1}{IE} \times \left(-\dfrac{8}{3}\right)$

$$= \frac{-8}{\frac{0.15\times0.3^3}{12} \times 10^9 \times 3\times10^{-3}} = 7901\times10^{-6}\,{}^{m} = 0.79\,{}^{cm}$$

$$[y_2]_{x_2=4} = \frac{1}{\frac{0.15\times0.3^3}{12}\times10^6}\left\{\frac{4^4}{48} - \frac{4^3}{3} + 4^2 + \frac{8}{3}\times4\right\}$$

$$= \frac{12}{0.15\times0.3^3\times10^6} \times \frac{32}{3} = 0.032\,{}^{m} = 3.2\,{}^{cm}$$

(8−10)



$$I = \frac{bh^3}{12} = \frac{b\cdot(d^2-b^2)^{\frac{3}{2}}}{12}$$

$$I' = \frac{1}{12}\left\{(d^2-b^2)^{3/2} - 3b^2(d^2-b^2)^{\frac{1}{2}}\right\}$$

$$= \frac{1}{12}(d^2-b^2)^{\frac{1}{2}}(d^2-4b^2)$$

$$= \frac{1}{12}\sqrt{(d+b)(d-b)}\,(d+2b)(d-2b)$$

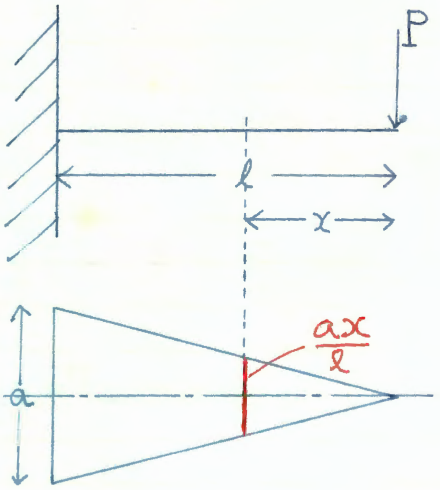
$d+b>0$, $d+2b>0$, $d-b>0$

$\therefore I'=0 \longrightarrow d-2b=0 \quad \therefore b=\dfrac{d}{2}$

このとき $h=\sqrt{d^2-\dfrac{d^2}{4}} = \dfrac{\sqrt{3}}{2}d$

H. W

(8-11)



$M_x = -Px.$

$I = \frac{bh^3}{12} = \frac{ah^3}{12l}x$

$\frac{d^2y}{dx^2} = \frac{12lP}{ah^3E}\cancel{x}$

$\frac{dy}{dx} = \frac{12lP}{ah^3E}\cancel{\left(\frac{x^2}{2} + C_1\right)}\ (x + C_1)$

$y = \frac{12lP}{ah^3E}\cancel{\left(\frac{x^3}{6} + C_1x + C_2\right)}\left(\frac{x^2}{2} + C_1x + C_2\right)$

B.C. $x = l$ $\quad dy/dx = 0 \quad C_1 = \cancel{-l^2/2}\ -l$

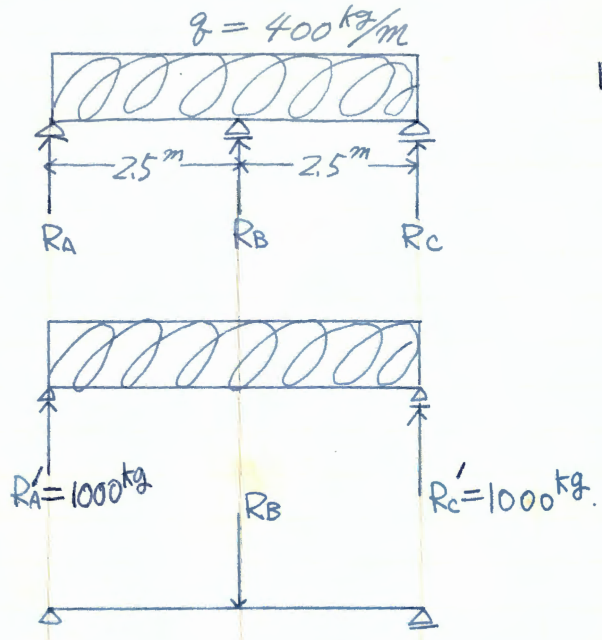
$;\quad y = 0 \quad C_2 = \cancel{-l^3/6 + l^3/2 = l^3/3}$

$-l^2/2 + l^2 = l^2/2.$

$\therefore y = \frac{12lP}{ah^3E}\left(\frac{x^2}{2} - lx + \frac{l^2}{2}\right)$

$= \frac{6lP}{ah^3E}(l-x)^2$ --- Ans

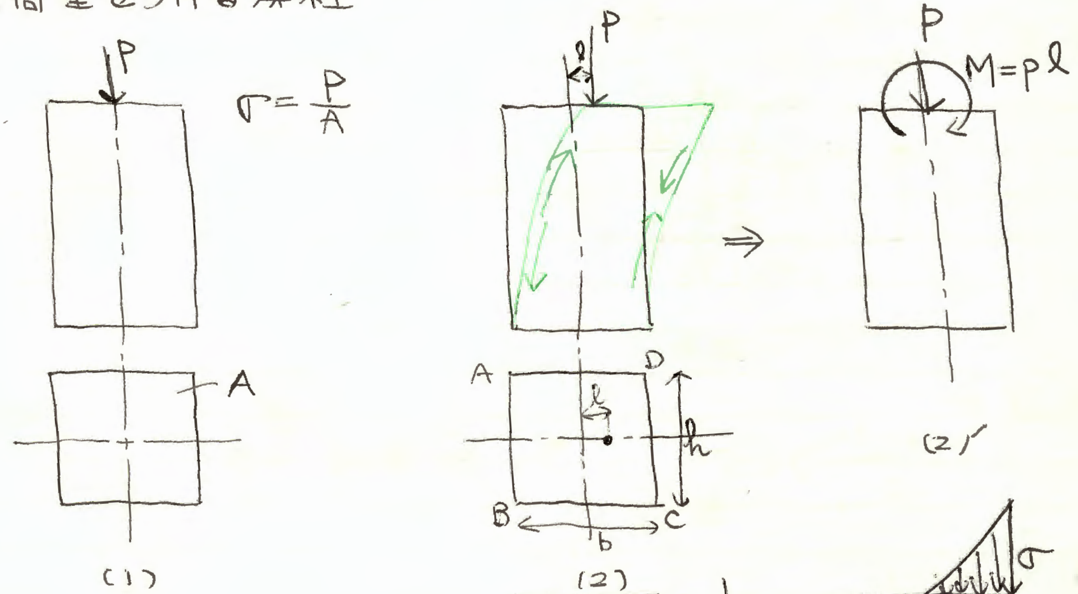
(8-13)

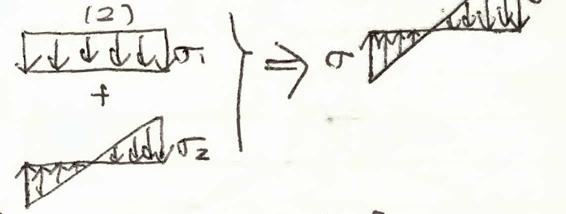


6/29 (講ギ)

柱.

偏心荷重をうける短柱





(2)'において

$$\sigma_1 = \frac{P}{A} = \frac{P}{bh}$$

$$\sigma_2 = \pm\frac{M}{I}y = \pm\frac{M}{W} = \pm\frac{Pl}{\frac{hb^2}{6}} \qquad \left(W = \frac{hb^2}{6}\right)$$

$$= \pm\frac{6P\cdot l}{hb^2}$$

$$\sigma = \sigma_1 + \sigma_2 = \frac{P}{bh} \pm \frac{6Pe}{hb^2}$$

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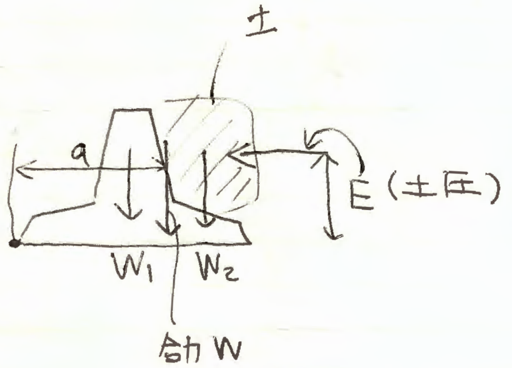
安定計算

① 転倒



$$M_D = W a$$

$$M = E y$$

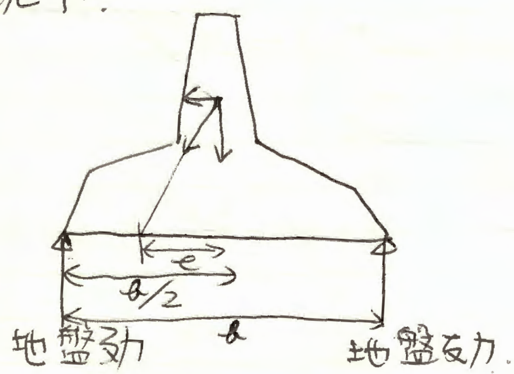
$$\frac{M_D}{M} > 2$$

② 滑動

$$\frac{W\mu}{E} > 1.5$$

μ：動摩擦係数

③ 沈下



$$\sigma < q_a$$

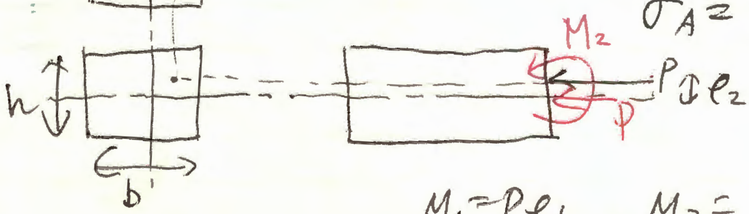
q_a；許容地耐力

$$\sigma = \frac{P}{bh} - \frac{6Pe}{hb^2} \Rightarrow \sigma = 0 \qquad e = \frac{b}{6} \text{ (Core Point)}$$

$$\sigma_P = \frac{P}{A} + \frac{M_1}{W_1} + \frac{M_2}{W_2}$$

$$\sigma_B = \frac{P}{A} - \frac{M_1}{W_1} - \frac{M_2}{W_2}$$

$$\sigma_A = \frac{P}{A} - \frac{M_1}{W_1} + \frac{M_2}{W_2}$$

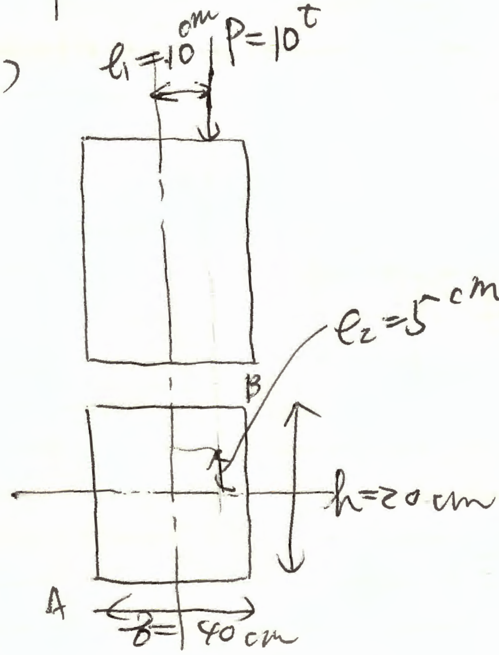


$$\sigma_C = \frac{P}{A} + \frac{M_1}{W_1} - \frac{M_2}{W_2}$$

$$M_1 = Pe_1, \quad M_2 = Pe_2, \quad W_1 = \frac{hb^2}{6}, \quad W_2 = \frac{bh^2}{6}$$

$$\sigma_A = \frac{P}{bh} - \frac{6Pe_1}{hb^2} + \frac{6Pe_2}{bh^2}$$

(例)



$$\sigma_A = \frac{P}{bh} - \frac{6Pe_1}{hb^2} - \frac{6Pe_2}{bh^2}$$
$$= -25 \text{ kg/cm}^2 \text{ (引)}$$

$$\sigma_B = \frac{P}{bh} + \frac{6Pe_1}{hb^2} + \frac{6Pe_2}{bh^2}$$
$$= 50 \text{ kg/cm}^2 \text{ (圧)}$$

6/29 演(補)

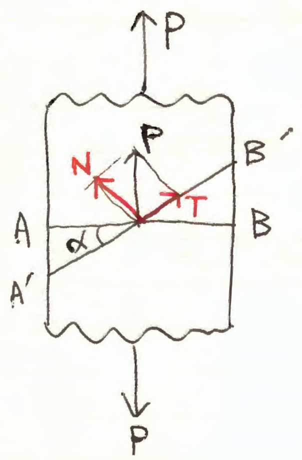
モールの応力円

組合わせ応力.

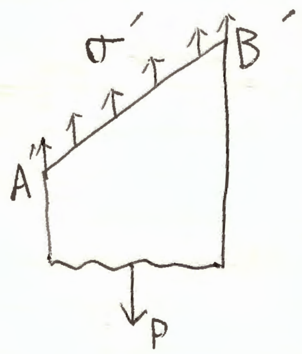
部材がうける単純応力 → $\underbrace{\text{(引張, 圧縮, 曲げ, ねじり)}}_{\text{垂直応力とせん断応力の組合わせ}}$

引張棒の斜断面上の応力.

(a)



(b)



(a)図

$$\begin{cases} N = P\cos\alpha \\ T = P\sin\alpha \end{cases}$$

(b)図 $\left.\begin{array}{l} A = \text{AB断面の面積} \\ A' = A'B' \quad 〃 \quad 〃 \end{array}\right\}$ とする.

$A' = \dfrac{A}{\cos\alpha}$, $\sigma = \dfrac{P}{A}$

(AB上の応力)

∴ A'B'断面上の応力σ'は

$$\sigma' = \frac{P}{A'} = \frac{P\cos\alpha}{A} = \sigma\cos\alpha$$

垂直分力をσn
平行 〃 をτ } とする

$$\begin{cases} \text{垂直応力} \rightarrow \sigma_n = \sigma'\cos\alpha = \sigma\cos^2\alpha & \text{—— (1)} \\ \text{せん断応力} \rightarrow \tau = \sigma'\sin\alpha = \dfrac{\sigma}{2}\sin 2\alpha & \text{—— (2)} \end{cases}$$

故に

垂直応力 σ_n は

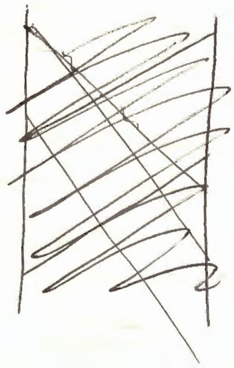
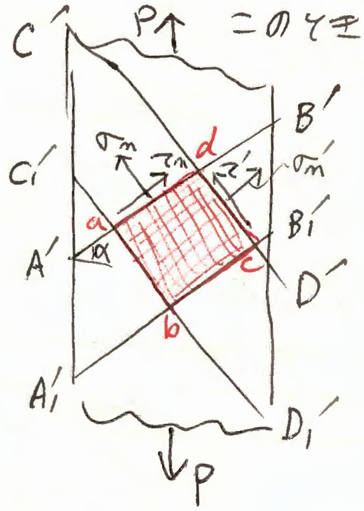
$\cos\alpha=1$ （$\alpha=0°$）の横断面で最大値 σ.

せん断応力 τ は

$\alpha=0°$ or $90°$ のとき $\tau=0$

$\alpha=45°$ や $135°$ → τ の絶対値最大

このとき $\tau_{max}=\frac{\sigma}{2}$

〈A'B' 断面に垂直な断面 C'D' 上の応力〉

$\sigma_n'=\sigma\cos^2(\alpha+90°)=\sigma\sin^2\alpha$ —— (3)

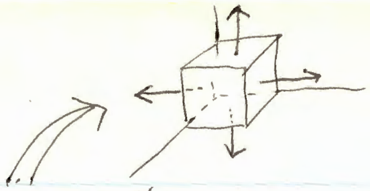
$\tau'=\frac{\sigma}{2}\sin 2(\alpha+90°)=\frac{\sigma}{2}\sin 2\alpha$ —— (4)

したがって abcd の各面には

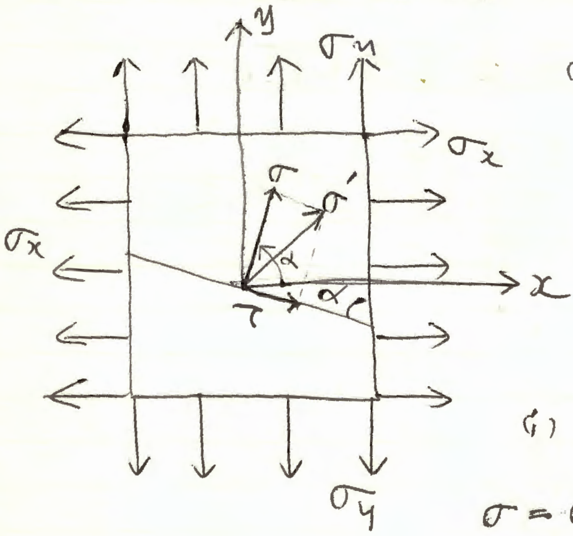
$$\begin{cases} \text{垂直応力} \quad \sigma_n=\sigma/2 \\ \text{せん断応力} \quad \tau=\sigma/2 \end{cases} \quad (\alpha=45°)$$

また

$$\begin{cases} \sigma_n+\sigma_n'=\sigma\cos^2\alpha+\sigma\sin^2\alpha=\sigma & \text{—— (5)} \\ \tau=\tau' & \text{—— (6)} \end{cases}$$



〈平面応力状態にある ある一点の応力〉



(i) σ_x による分応力 σ_n & τ_n

&(and)

$$\sigma_n = \sigma_x \sin^2\alpha \qquad \tau_n = \frac{\sigma_x}{2}\sin 2\alpha$$

(ii) σ_y による分応力

$$\sigma_n' = \sigma_y \cos^2\alpha \qquad \tau_n' = -\frac{\sigma_y}{2}\sin 2\alpha$$

(i) (ii) の合応力

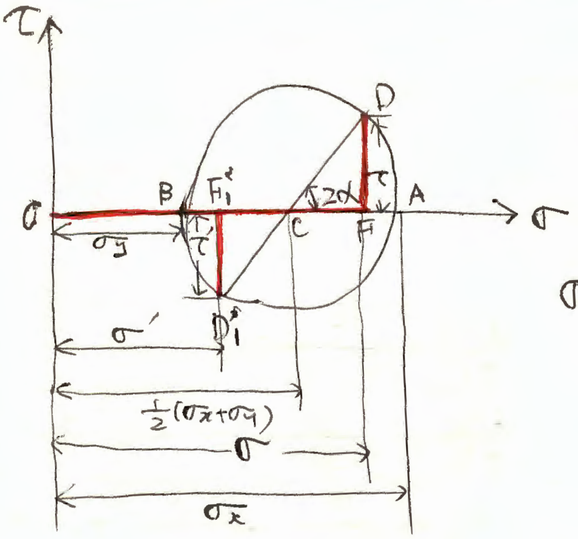
$$\begin{aligned}\sigma &= \sigma_n + \sigma_n' \\ &= \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha \\ &= \sigma_x \cdot \frac{1+\cos 2\alpha}{2} + \sigma_y \frac{1\mp\cos 2\alpha}{2} \\ &= \frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\alpha \quad \text{—(7)}\end{aligned}$$

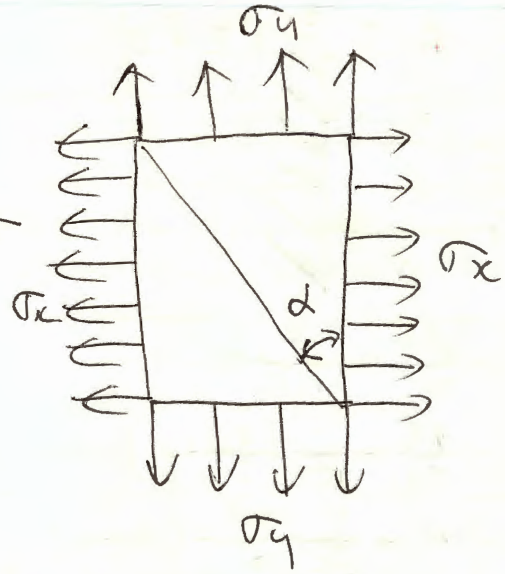
$$\tau = \tau_n + \tau_n' = \frac{1}{2}(\sigma_x-\sigma_y)\sin 2\alpha \quad \text{—(8)}$$

$\sigma_x > \sigma_y$ とすると

$\sigma_{max} = \sigma_x \ (\alpha=90°)$
$\sigma_{min} = \sigma_y \ (\alpha=0°)$
} せん断応力0

$\tau_{max} = \frac{1}{2}(\sigma_x-\sigma_y) \quad (\alpha=45°)$
$\tau_{min} = -\frac{1}{2}(\sigma_x-\sigma_y) \quad (\alpha=135°)$
} せん断応力は大きさ等しく方向が逆.





$\sigma_x > \sigma_y$, $\sigma_x > 0$, $\sigma_y > 0$.

σ_x, σ_y は主応力とする

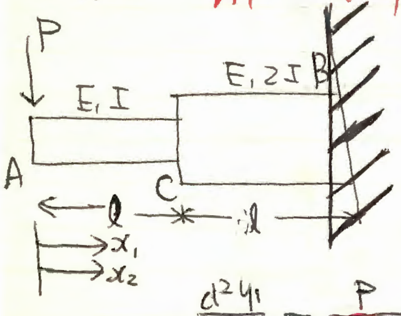
- OF：斜断面（垂直面とのなす角α）上の合垂直応力σを表わす。
- DF： 〃 〃 〃 合せん断応力τで 〃 。
- D_1F_1： 〃 （ 〃 α+90°） 〃 〃 τ' 〃 。

（証明）

$$OF = OC + CF = OC + (CD)\cos 2\alpha$$
$$= \frac{1}{2}(\sigma_x+\sigma_y) + \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\alpha$$
$$= \frac{1}{2}\sigma_x(1+\cos 2\alpha) + \frac{1}{2}\sigma_y(1-\cos 2\alpha)$$
$$= \sigma_x\cos^2\alpha + \sigma_y\sin^2\alpha = \sigma$$ ← (7)式より.

$$DF = (CD)\sin 2\alpha = \frac{1}{2}(\sigma_x-\sigma_y)\sin 2\alpha = \tau$$ ← (8)式より

(ex)　A, C 点のたわみ。



$M_x = -Px$

$$\frac{d^2y_1}{dx_1^2} = \frac{P}{IE}x_1 \qquad \frac{d^2y_2}{dx_2^2} = \frac{P}{2IE}x_2$$

$$\frac{dy_1}{dx_1} = \frac{P}{IE}\left(\frac{x_1^2}{2} + C_1\right) \qquad \frac{dy_2}{dx_2} = \frac{P}{2IE}\left(\frac{x_2^2}{2} + C_1'\right)$$

$$y_1 = \frac{P}{IE}\left(\frac{x_1^3}{6} + C_1x_1 + C_2\right) \qquad y_2 = \frac{P}{2IE}\left(\frac{x_2^3}{6} + C_1'x_2 + C_2'\right)$$

(B.C) i) $\boxed{x_2 = 2l}$ 固定端 $\begin{cases} dy_2/dx_2 = 0 \\ y_2 = 0 \end{cases}$

より $C_1' = -\frac{4l^2}{2} = -2l^2$

$$C_2' = -\frac{8l^3}{6} - (-2l^2) \times 2l$$

$$= -\frac{4}{3}l^3 + 4l^3 = \frac{8}{3}l^3$$

ii) $\boxed{\begin{cases} x_1 = l \\ x_2 = l \end{cases} \quad \begin{cases} \dfrac{dy_1}{dx_1} = \dfrac{dy_2}{dx_2} \\ y_1 = y_2 \end{cases}}$ C点で連続

$$\frac{l^2}{2} + C_1 = \frac{1}{2}\left(\frac{l^2}{2} - 2l^2\right)$$

$$\therefore C_1 = -\frac{3}{4}l^2 - \frac{l^2}{2} = -\frac{5}{4}l^2$$

$$\frac{l^3}{6} - \frac{5}{4}l^3 + C_2 = \frac{1}{2}\left(\frac{l^3}{6} - 2l^3 + \frac{8}{3}l^3\right)$$

$$\therefore C_2 = \left(\frac{1}{2} \times \frac{5}{6} - \frac{1}{6} + \frac{5}{4}\right)l^3 = \frac{3}{2}l^3$$

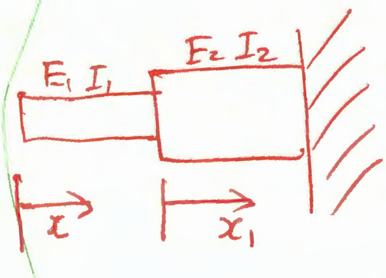
$\frac{5-2+15}{12} = \frac{18}{12}$　　$1-12+16$

故に
$$\begin{cases} \dfrac{dy_1}{dx_1} = \dfrac{P}{IE}\left(\dfrac{x_1^2}{2} - \dfrac{5}{4}l^2\right) \\ y_1 = \dfrac{P}{IE}\left(\dfrac{x_1^3}{6} - \dfrac{5}{4}l^2 x_1 + \dfrac{3}{2}l^3\right) \end{cases}$$

$$\begin{cases} \dfrac{dy_2}{dx_2} = \dfrac{P}{2IE}\left(\dfrac{x_2^2}{2} - 2l^2\right) \\ y_2 = \dfrac{P}{2IE}\left(\dfrac{x_2^3}{6} - 2l^2 x_2 + \dfrac{8}{3}l^3\right) \end{cases}$$

$x_1 = 0$ で
$$\begin{cases} \dfrac{dy_1}{dx_1} = -\dfrac{5Pl^2}{4IE} = \theta_A \\ y_1 = \dfrac{3Pl^3}{2IE} = \delta_A \end{cases} \quad (A\text{点})$$

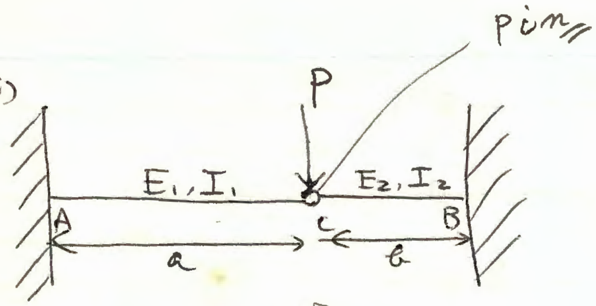
$x_1 = l$ で
$$\begin{cases} \dfrac{dy_1}{dx_1} = -\dfrac{3Pl^2}{4IE} = \theta_C \\ y_1 = \dfrac{5Pl^3}{12IE} = \delta_C \end{cases} \quad (C\text{点})$$



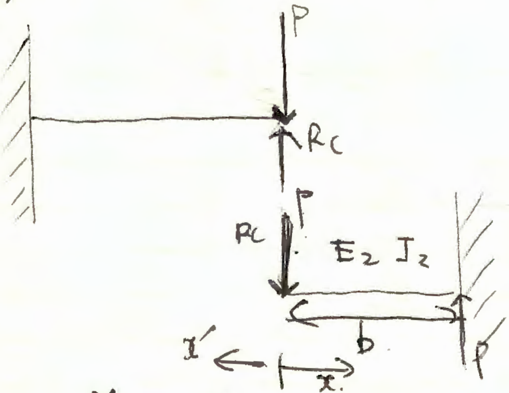
でもよい。

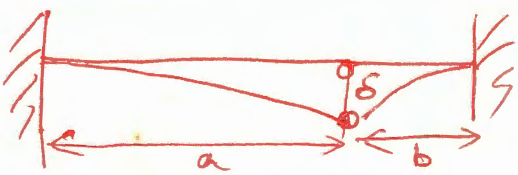
$x_2 = l$
$$\begin{cases} \dfrac{dy_2}{dx_2} = -\dfrac{3Pl^2}{4IE} \\ y_2 = \dfrac{1}{2}\left(\dfrac{5Pl^3}{6IE}\right) = \dfrac{5Pl^3}{12IE} \end{cases}$$

(ex)



$\boxed{\text{C点のたわみ?}}$





$$M_x = -Px$$

$$\frac{d^2y}{dx^2} = \frac{P}{I_2E_2}x$$

$$\frac{dy}{dx} = \frac{P}{I_2E_2}\left(\frac{x^2}{2} + C_1\right)$$

$$y = \frac{P}{I_2E_2}\left(\frac{x^3}{6} + C_1x + C_2\right)$$

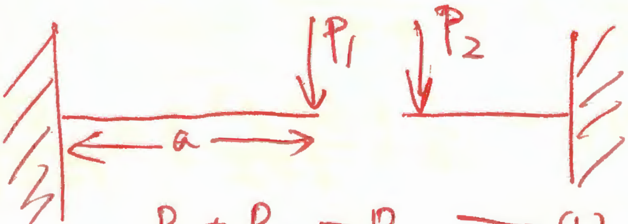
$$M_{x'} = 0$$

$$\frac{d^2y'}{dx'^2} = 0$$

$$\frac{dy'}{dx'} = C_1'$$

$$y' = C_1'x' + C_2'$$

$x = b$ $\left\{\begin{array}{ll} \dfrac{dy}{dx} = 0 & C_1 = -\dfrac{b^2}{2} \\ y = 0 & C_2 = -\dfrac{b^3}{6} + \dfrac{b^3}{2} = \dfrac{b^3}{3} \end{array}\right.$

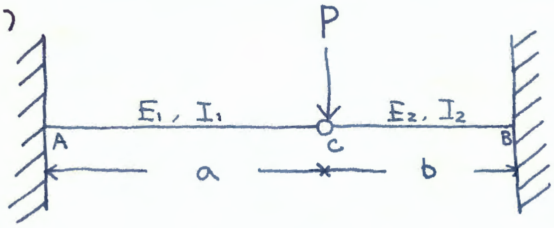


$P_1 + P_2 = P$ —(1)

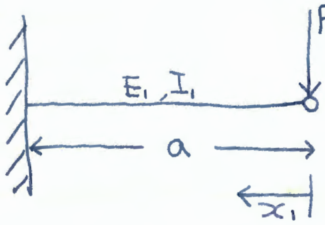
$\left.\begin{array}{l} y_1 = f(P_1) \\ y_2 = f(P_2) \end{array}\right\}$ $y_1 = y_2 - \delta$. (P_1とP_2の式) —(2)

7/3 (演)

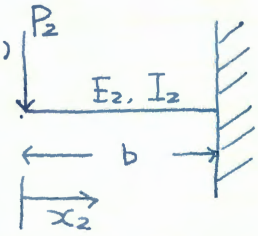
(ex1)



(1)



(2)



$M_1 = -P_1 x_1$

$\dfrac{d^2 y_1}{d x_1^2} = \dfrac{P_1}{E_1 I_1} x_1$

$\dfrac{d y_1}{d x_1} = \dfrac{P_1}{E_1 I_1}\left(\dfrac{x_1^2}{2} + C_1\right)$

$y_1 = \dfrac{P_1}{E_1 I_1}\left(\dfrac{x_1^3}{6} + C_1 x_1 + C_2\right)$

$x_1 = a$ で $\dfrac{d y_1}{d x_1} = 0$ $\therefore C_1 = -\dfrac{a^2}{2}$

$x_1 = a$ で $y_1 = 0$

$\therefore C_2 = -\dfrac{a^3}{6} + \dfrac{a^3}{2} = \dfrac{a^3}{3}$

$\therefore y_1 = \dfrac{P_1}{E_1 I_1}\left(\dfrac{x_1^3}{6} - \dfrac{a^2}{2} x_1 + \dfrac{a^3}{3}\right)$

$[y_1]_{x_1=0} = \dfrac{P_1 a^3}{3 E_1 I_1}$

$M_2 = -P_2 x_2$

(1)と同様に

$y_2 = \dfrac{P_2}{E_2 I_2}\left(\dfrac{x_2^3}{6} - \dfrac{b^2}{2} x_2 + \dfrac{b^3}{3}\right)$

$[y_2]_{x_2=0} = \dfrac{P_2 b^3}{3 E_2 I_2}$

ここで $[y_1]_{x_1=0} = [y_2]_{x_2=0}$ より

$\dfrac{P_1 a^3}{3 E_1 I_1} = \dfrac{P_2 b^3}{3 E_2 I_2}$ —— ①

また

$P_1 + P_2 = P$ —— ②

①, ② より

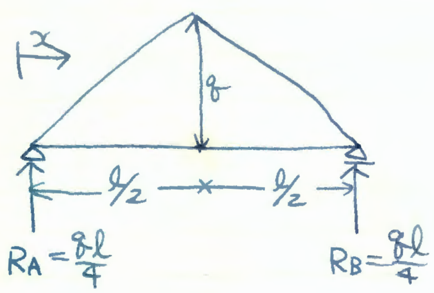
$P_1 = \dfrac{b^3 E_1 I_1}{b^3 E_1 I_1 + a^3 E_2 I_2} \cdot P$

故に C 点のたわみを δ_c とすると,

$\delta_c = \dfrac{P_1 a^3}{3 E_1 I_1} = \dfrac{a^3 b^3}{3(b^3 E_1 I_1 + a^3 E_2 I_2)} P$ --- Ans

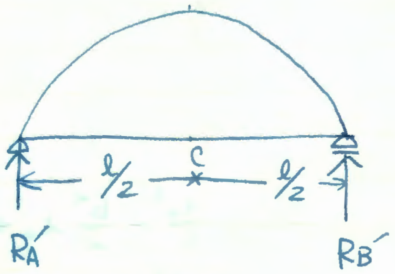
$\frac{1}{96} - \frac{1}{480} = \frac{5-1}{480}$

(ex2) 8-3 弾性荷重法.



$$R_A = R_B = \frac{ql}{4}$$

$$M_x = \frac{ql}{4}x - \frac{q}{3l}x^3$$



$$R_A' = \int_0^{l/2}\left(\frac{ql}{4}x - \frac{q}{3l}x^3\right)dx = R_B'$$

$$M_{x_c}' = R_A'\frac{l}{2} - \int_0^{l/2}\left(\frac{ql}{4}x - \frac{q}{3l}x^3\right)\left(\frac{l}{2} - x\right)dx$$

$$= \frac{l}{2}\left[\frac{ql}{8}x^2 - \frac{q}{12l}x^4\right]_0^{l/2} - \left[\frac{q}{15l}x^5 - \frac{ql}{24l}x^4 - \frac{ql}{12}x^3 + \frac{ql^2}{16}x^2\right]_0^{l/2}$$

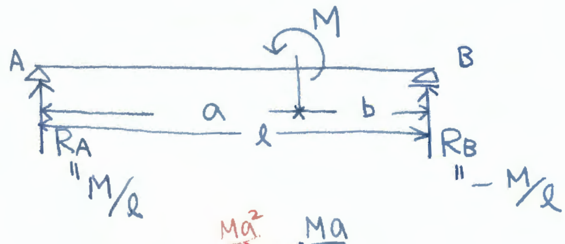
$$= \frac{l}{2}\left(\frac{ql^3}{32} - \frac{ql^3}{192}\right) - \left(\frac{ql^4}{480} - \frac{ql^4}{384} - \frac{ql^4}{96} + \frac{ql^4}{64}\right)$$

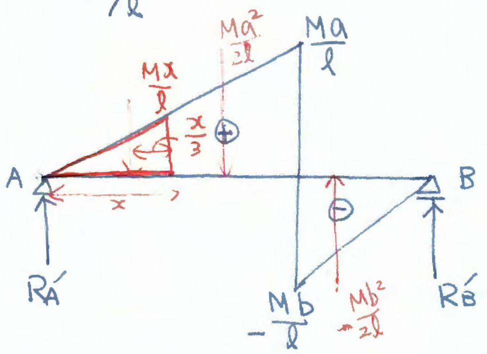
$$= \left(\frac{1}{64} - \frac{1}{384} - \frac{1}{480} + \frac{1}{384} + \frac{1}{96} - \frac{1}{64}\right)ql^4$$

$$= \frac{ql^4}{120}$$

$$\therefore\ y_c = \frac{ql^4}{120\,IE}$$

(ex 3) 8-5 弾性荷重法.





$$\Sigma M_B = R_A' l - \frac{Ma^2}{2l}\left(l - \frac{2}{3}a\right) + \frac{Mb^2}{2l}\cdot\frac{2b}{3} = 0$$

$$\therefore R_A' = \frac{M}{l^2}\left(\frac{a^2 l}{2} - \frac{a^3}{3} + \frac{b^3}{3}\right)$$

$$= \frac{M}{6l^2}\left(3a^2(a+b) - 2a^3 + 2b^3\right)$$

$$= \frac{M}{6l^2}\left(a^3 + 3a^2 b - 2b^3\right)$$

$$R_B' = \frac{Ma^2}{2l} - \frac{M}{6l^2}(a^3+3a^2b-2b^3) - \frac{Mb^2}{2l}$$

$$= \frac{M}{6l^2}\left\{3a^2(a+b) - (a^3+3a^2b-2b^3) - 3b^2(a+b)\right\}$$

$$= \frac{M}{6l^2}\left(2a^3 - 3ab^2 - b^3\right)$$

$0 \leqq x \leqq a$

$$M_x = R_A' x - \frac{Mx^2}{2l}\cdot\frac{x}{3}$$

$$= \frac{Mx}{6l^2}(a^3+3a^2b-2b^3) - \frac{Mx^3}{6l}$$

$$= \frac{Mx}{6l^2}\left\{a^2(a+b) + 2b(a^2-b^2)\right\} - \frac{Mx^3}{6l}$$

$$= \frac{Mx}{6l}\left(a^2 + 2ab - 2b^2 - x^2\right)$$

$$\therefore y = \frac{Mx}{6lEI}\left(a^2 + 2ab - 2b^2 - x^2\right)$$

$a \leqq x \leqq l$

$$M_x = R_B'(l-x) + \frac{M(l-x)^2}{2l}\cdot\frac{l-x}{3}$$

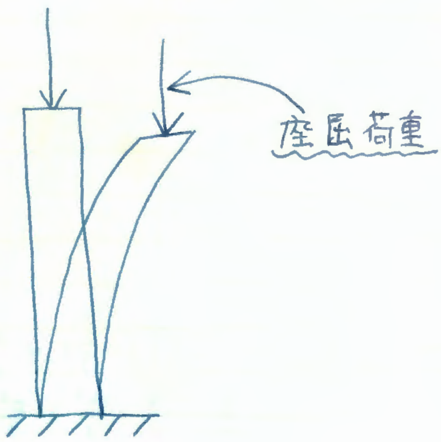
$$= \frac{M(l-x)}{6l}\left\{\frac{2a}{l}(a^2-b^2) - \frac{b^2}{l}(a+b) + (l-x)^2\right\}$$

$$= \frac{M(l-x)}{6l}\left\{2a^2 - 2ab - b^2 + (l-x)^2\right\}$$

$$\therefore y' = \frac{M(l-x)}{6lEI}\left\{2a^2 - 2ab - b^2 + (l-x)^2\right\}$$

7/6(講)

長柱の座屈



$\frac{l}{r} < 10$ 短柱

$\frac{l}{r} > 10$ 長柱

r：断面二次半径
l：部材の長さ

（$\frac{l}{r} = \lambda$：細長比）

Euler の長柱公式

$$\frac{l}{r} > 100$$

- 一端固定他端自由

$$\frac{P_b}{EI} = k^2$$

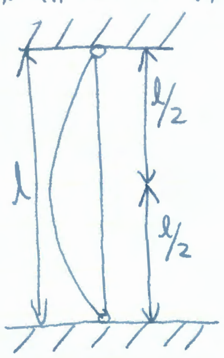
$$k = \frac{\pi}{2l}$$

$$P_b = \frac{\pi^2 EI}{4l^2}$$ --- 座屈荷重.

$$\sigma_b = \frac{P_b}{A} = \frac{\pi^2 E}{4\left(\frac{l}{r}\right)^2} = \frac{\pi^2 E}{4\lambda^2}$$

[(但し $r = \sqrt{\frac{I}{A}}$, $\lambda = \frac{l}{r}$)
$r^2 = \frac{I}{A}$: $A = \frac{I}{r^2}$]

◎ 両端回転端



$l \to l/2$

$$P_k = \frac{\pi^2 EI}{l^2} , \quad \sigma_k = \frac{\pi^2 E}{\lambda^2}$$

◎ 両端固定端 $l \to l/4$

$$P_k = \frac{4\pi^2 EI}{l^2} , \quad \sigma_k = \frac{4\pi^2 E}{\lambda^2}$$

◎ 一端固定他端回転端.

$$P_k = \frac{2\pi^2 EI}{l^2} .$$

n: 支点の条件による係数

$$P_k = \frac{n\pi^2 EI}{l^2}$$

◎ テトマイヤー（Tetmajer）の公式

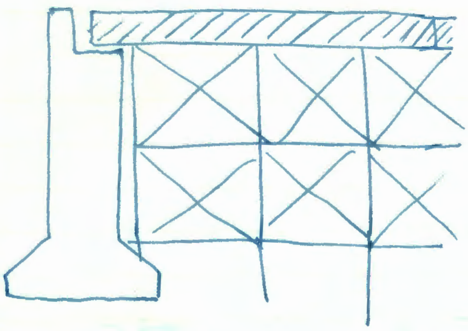
$\sigma_b = a - b\lambda$.

$\begin{cases} \text{軟鋼} & a = 3100, \ b = 11.4 \\ \text{木材} & a = 293, \ b = 1.94 \end{cases}$

◎ ジョンソン（Johnson）の公式

$\begin{cases} SM\text{-}50 & \sigma_b = 1800 - 0.001\lambda^2 \\ SS\text{-}50 & \sigma_b = 1600 - 0.0009\lambda^2 \\ SS\text{-}41 & \sigma_b = 1300 - 1.0006\lambda^2 \end{cases}$

● 支保工用鋼材の許容座屈応力度.



$\frac{l}{r} < 100 \quad \sigma_b = \sigma\left(1 - 0.0017\frac{l}{r}\right)$

$\frac{l}{r} > 100 \quad \sigma_b = \frac{0.3\sigma}{\left(\frac{l}{100r}\right)^2}$

〈σ：許容圧縮応力〉

1) 最大圧縮応力 → 左端.

$$\sigma = \frac{P}{A} - \frac{M}{I_z} y$$

$A = bh$

$I_z = bh^3/12$

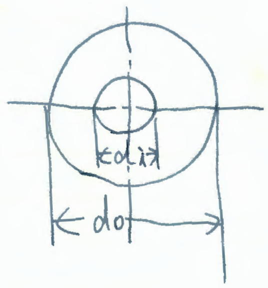
$M = P(\frac{h}{2} - a)$

$y = h/2$

$$\therefore \sigma = \frac{P}{bh} + \frac{P(\frac{h}{2}-a)\frac{h}{2}}{bh^3/12}$$

$$= \frac{P}{bh}\left(4 - \frac{6a}{h}\right) \Rightarrow \sigma = 50\ \mathrm{kg/cm^2}$$

2)



$$k = \frac{I_z}{A y_1}$$

$$I_z = \frac{\pi}{64}(d_o^4 - d_i^4)$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$y_1 = d_o/2$$

$$\therefore k = \frac{d_o^2 + d_i^2}{8 d_o}$$ 同心円

3)

断面2次半径 r

$$r = \sqrt{\frac{I}{A}} = \left[\frac{\pi}{64}(31.2^4 - 30^4) \Big/ \frac{\pi}{4}(31.2^2 - 30^2)\right]^{1/2}$$
$= 10.8$

細長比 $\lambda = \frac{l}{r} = \frac{1200}{18.8} \cong 111 > 100$ (長柱)

両端ピンの場合の座屈荷重 P_{cr} は

$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 21 \times 10^5 \times \frac{\pi}{64}(31.2^4 - 30^4)}{(1.2 \times 10^3)^2} = 96.4 \times 10^3\ \mathrm{kg}$$

安全率3 $\frac{P_{cr}}{3} = \frac{96.4 \times 10^3}{3} = 32.1 \times 10^3\ \mathrm{kg}$

$= 32.1\ \mathrm{ton}$

4)

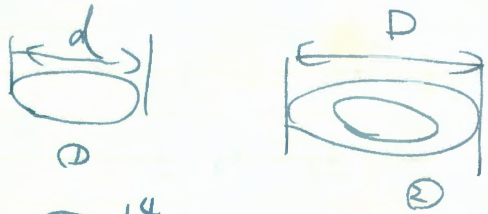
座屈荷重　$P_{cr}=\frac{\pi^2 EI}{l^2}$

安全率 3

$$\therefore 1000\times 3=\frac{\pi^2\times 84000\times\frac{\pi d^4}{64}}{900^2}$$

$\therefore d^4=5.98\times 10^4$　　$\therefore d=15.6\,cm$

5)



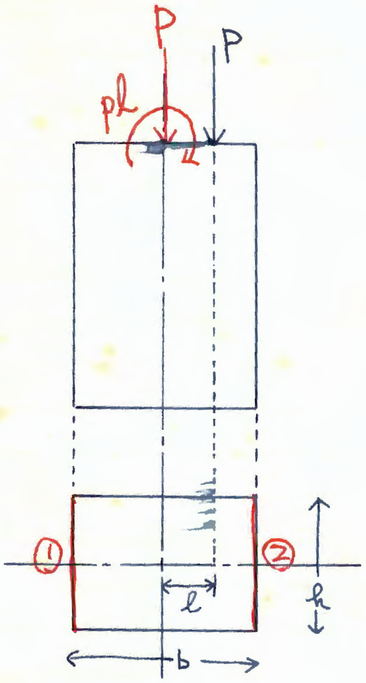
$I_1=\frac{\pi d^4}{64}$　　$I_2=\frac{\pi}{64}(D^4-d^4)$

$P_{cr}=\frac{\pi^2 EI}{l^2}$ ⟸ E, l は等しい

$I_1=I_2$ とすると

$$\frac{\pi d^4}{64}=\frac{\pi}{64}(D^4-d^4)\rightarrow\frac{d^4}{D^4}=\frac{1}{2}$$

$$\frac{d}{D}=0.84$$



$$\sigma_1 = \frac{P}{A} = \frac{P}{bh}$$

$$\sigma_2 = \pm \frac{M}{I} y = \pm \frac{M}{W} = \frac{Pl}{hb^2/6} = \frac{6Pl}{hb^2}$$

$$\therefore \sigma = \frac{P}{A} \pm \frac{M}{I} y$$

$$= \frac{P}{bh}\left(1 \pm \frac{6l}{b}\right)$$

[コア] $\sigma = \frac{P}{A} + \frac{M}{I} y_1 = P\left(\frac{1}{A} + \frac{l}{I} y_1\right) = 0 \longrightarrow$ 引張が零,

$$|l| = \frac{I}{A y_1} = |k_1| = \frac{r_z^2}{y_1}$$

$$r_z = \sqrt{\frac{I_z}{A}}.$$

長方形

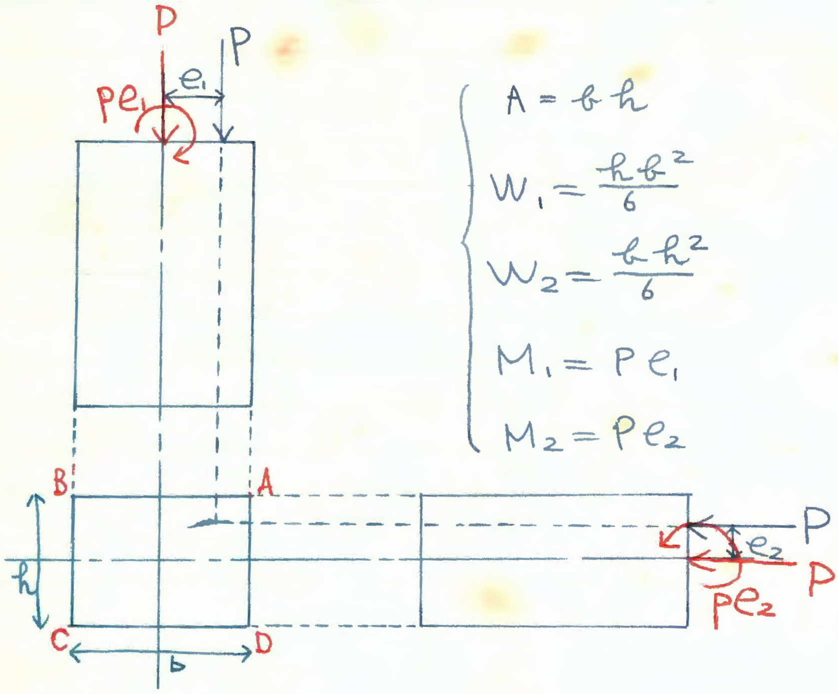
$$I_z = \frac{bh^3}{12}, \quad A = bh \qquad y_1 = y_2 = \frac{h}{2}$$

$$\therefore k_1 = k_2 = \frac{I_z}{A y_1} = \frac{h}{6} //$$

同様に $k_3 = k_4 = \frac{b}{6} //$

円 $I_z = \frac{\pi d^4}{64} \quad A = \frac{\pi d^2}{4} \qquad y_1 = y_2 = \frac{d}{2}$

$$k_1 = k_2 = \frac{d}{8} //$$



$$\begin{cases} A = bh \\ W_1 = \dfrac{hb^2}{6} \\ W_2 = \dfrac{bh^2}{6} \\ M_1 = Pe_1 \\ M_2 = Pe_2 \end{cases}$$

$$\sigma_A = \frac{P}{A} + \frac{M_1}{W_1} + \frac{M_2}{W_2} = \frac{P}{bh} + \frac{6Pe_1}{hb^2} + \frac{6Pe_2}{bh^2}$$

$$\sigma_B = \frac{P}{A} - \frac{M_1}{W_1} + \frac{M_2}{W_2} = \frac{P}{bh} - \frac{6Pe_1}{hb^2} + \frac{6Pe_2}{bh^2}$$

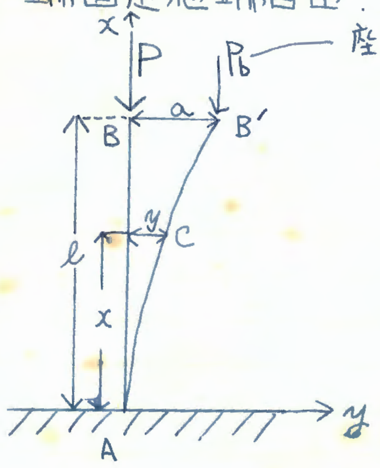
$$\sigma_C = \frac{P}{A} - \frac{M_1}{W_1} - \frac{M_2}{W_2} = \frac{P}{bh} - \frac{6Pe_1}{hb^2} - \frac{6Pe_2}{bh^2}$$

$$\sigma_D = \frac{P}{A} + \frac{M_1}{W_1} - \frac{M_2}{W_2} = \frac{P}{bh} + \frac{6Pe_1}{hb^2} - \frac{6Pe_2}{bh^2}$$

$\frac{1}{\omega} = \ldots$

Eulerの長柱公式 $\frac{l}{r} > 100$

(1) 一端固定他端自由



$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI}$$

$$M_x = -P_b(a-y)$$

$$\frac{d^2y}{dx^2} = \frac{P_b}{EI}(a-y)$$

$\frac{P_b}{EI} = k^2$ とおくと

$$\frac{d^2y}{dx^2} + k^2 y = k^2 a$$

$$y = A\sin kx + B\cos kx + a$$

$x=0 \rightarrow y=0 \quad \therefore B = -a$

$x=0 \rightarrow dy/dx = 0 \quad \therefore A = 0$

$$\therefore y = a(1-\cos kx)$$

$x = l \rightarrow y = a \quad \therefore a\cos kl = 0$

$a \neq 0 \quad \therefore kl = \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi, \cdots$

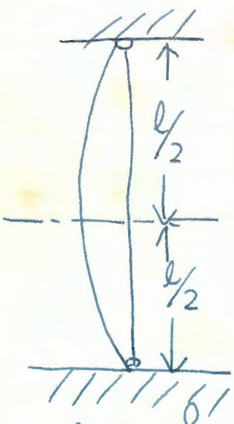
○ × ×

$$P_b = EIk^2 = \frac{\pi^2 EI}{4l^2}$$

$$\sigma_b = \frac{P_b}{A} = \frac{\pi^2 EI}{4l^2} \Big/ \frac{I}{r^2} = \frac{\pi^2 E}{4\left(\frac{l}{r}\right)^2} = \frac{\pi^2 E}{4\lambda^2}$$

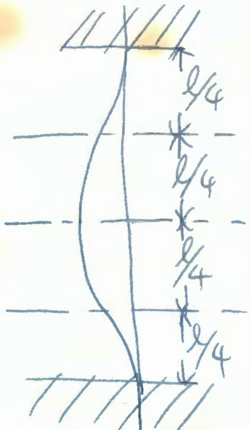
細長比

r = 断面二次半径 $= \sqrt{\frac{I}{A}}$



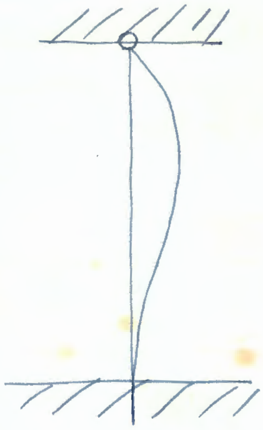
$l \to \frac{l}{2}$

$$\begin{cases} P_b = \dfrac{\pi^2 EI}{l^2} \\ \sigma_b = \dfrac{\pi^2 E}{\lambda^2} \end{cases}$$



$l \to \frac{l}{4}$

$$\begin{cases} P_b = \dfrac{4\pi^2 EI}{l^2} \\ \sigma_b = \dfrac{4\pi^2 E}{\lambda^2} \end{cases}$$



$l \to \frac{l}{\sqrt{2}}$

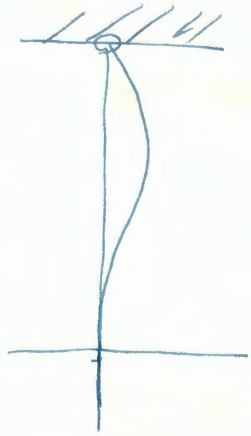
$$\begin{cases} P_b = \dfrac{2\pi^2 EI}{l^2} \\ \sigma_b = \dfrac{2\pi^2 E}{\lambda^2} \end{cases}$$

$$\begin{cases} P_b = \dfrac{n\pi^2 EI}{l^2} \\ \sigma_b = \dfrac{n\pi^2 E}{\lambda^2} \end{cases}$$

$n = \frac{1}{4}$

$n = 1$

$n = 4$



$n = 2.$

Rankineの公式

$$\sigma_b = \frac{\sigma_c}{1 + a\lambda^2}$$

圧縮強さ

細長比

const.

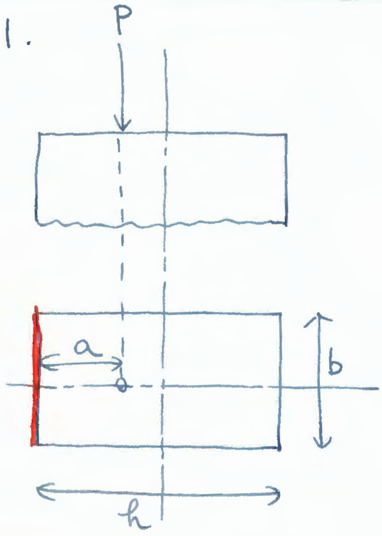
Tetmajerの公式

$$\sigma_b = a - b\lambda$$

Johnsonの公式

$$\sigma_b = \sigma_c - a\lambda^2$$

10−1.



左端辺で最大圧縮応力

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{M}{W}}$$

$$= \frac{P}{bh} + \frac{6P\left(\frac{h}{2}-a\right)}{bh^2}$$

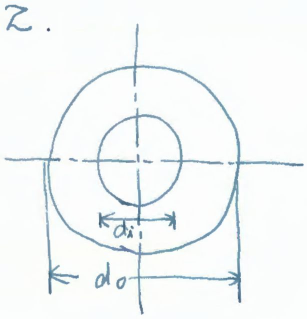
$$= \frac{P}{bh}\left\{1 + \frac{6}{h}\left(\frac{h}{2}-a\right)\right\}$$

$$= \frac{P}{bh}\left\{4 - \frac{6a}{h}\right\}$$

$b = 30\,cm$ $h = 60\,cm$ $a = 25\,cm$
$P = 60\,t$ より

$$\sigma_{max} = \frac{60\times10^3}{30\times60}\left\{4 - \frac{6\times25}{60}\right\} = 50\,(kg/cm^2)$$

10−2.

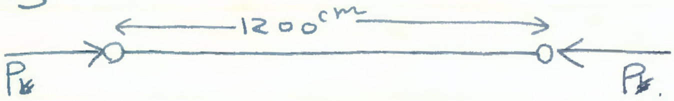


$$\boxed{k = \frac{I_z}{A y_1}}$$

$$\begin{cases} I_z = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) \\ A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) \\ y_1 = \frac{d_o}{2} \end{cases}$$

$$\therefore\ k = \frac{d_o^2 + d_i^2}{8 d_o}$$ //

10−3



$$\begin{cases} d_o = 31.2\ \text{cm} \\ d_i = 30.0\ \text{cm} \\ E = 21 \times 10^5\ \text{kg/cm}^2 \\ \text{安全率 } 3 \end{cases}$$

$$\boxed{r = \sqrt{\frac{I}{A}}} = \sqrt{\frac{\pi}{64}(31.2^4 - 30^4) \Big/ \frac{\pi}{4}(31.2^2 - 30^2)}$$

$$= \sqrt{\frac{31.2^2 + 30^2}{16}} = 10.8$$

細長比 $\boxed{\lambda = \frac{l}{r}} = \frac{1200}{10.8} \fallingdotseq 111 > 100$

オイラーの長柱公式

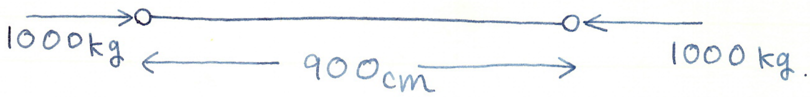
$$\boxed{P_b = \frac{\pi^2 E I}{l^2}} = \frac{\pi^2 \times 2.1 \times 10^6 \times \frac{\pi}{64}(31.2^4 - 30^4)}{(1.2 \times 10^3)^2}$$

$$= 97.06 \times 10^3$$

$$\frac{P_b}{3} = \frac{97.06}{3} \times 10^3 = 32.4 \times 10^3\ \text{kg}$$

$$= 32.4\ \text{ton}$$

10-4.



$E = 84{,}000\ \mathrm{kg/cm^2}$.　安全率3

$I = \dfrac{\pi d^4}{64}$

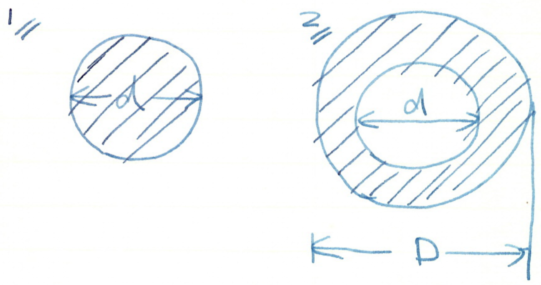
$$3P_b = \frac{\pi^2 E I}{l^2} = \frac{\pi^3 E d^4}{64 l^2}$$

$$\therefore d^4 = \frac{192\, l^2 P_b}{\pi^3 E} = \frac{192 \times 9^2 \times 10^4 \times 10^3}{\pi^3 \times 8.4 \times 10^4}$$

$$= 59.8 \times 10^3 = 59800\ \mathrm{cm^4}$$

$$\therefore d = \sqrt[4]{5.98} \times 10 = 15.6\ \mathrm{cm}.$$

10-5



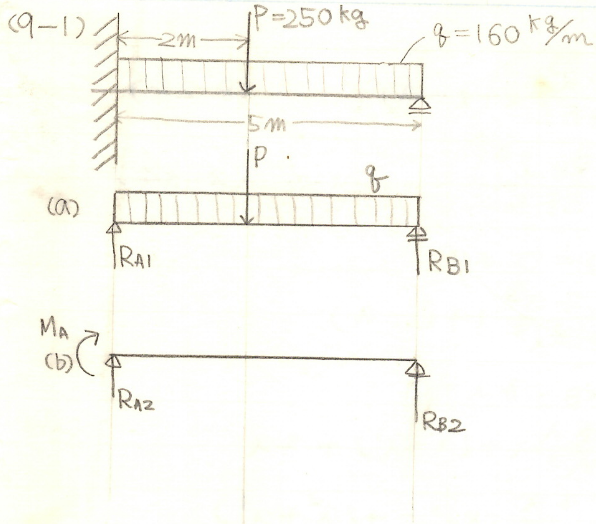
オイラーの長柱公式 ($\lambda > 100$)

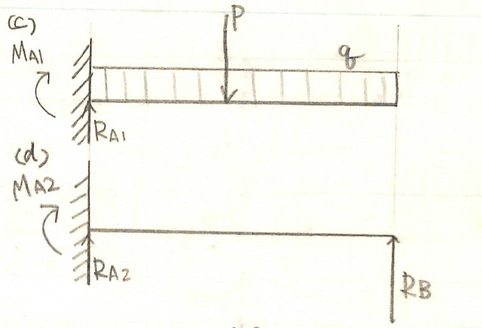
$$P_{cr_1} = \frac{\pi^2 E I_1}{l^2} \qquad P_{cr_2} = \frac{\pi^2 E I_2}{l^2}$$

$P_{cr_1} = P_{cr_2}$ より　$I_1 = I_2$.

$$\therefore \frac{\pi}{64} d^4 = \frac{\pi}{64}(D^4 - d^4)$$

$$\frac{\pi}{32} d^4 = \frac{\pi}{64} D^4 \quad \therefore \left(\frac{d}{D}\right)^4 = \frac{1}{2} \quad \therefore \frac{d}{D} = 0.84$$





(a) $R_{A1}+R_{B1}=P+5q=1050^{kg}$

$\Sigma M_B=5R_{A1}-250\times3-800\times25=0$

$\therefore R_{A1}=550^{kg}$ $\quad \therefore R_{B1}=500^{kg}$

$$\theta_{A1}=\frac{160\times5^3}{24EI}+\frac{250\times3}{6EI\times5}(5^2-3^2)$$

$$=\frac{2500}{3EI}+400/EI=\frac{3700}{3EI}$$

(b) $R_{A2}=-M_A/5$ $\quad R_{B2}=M_A/5$

$$\theta_{A2}=\frac{M_A\times5}{3EI}$$

$$\theta_A=\theta_{A1}+\theta_{A2}=\frac{3700+5M_A}{3EI}=0$$

$\therefore M_A=-740^{kg\cdot m}$

$\therefore R_A=R_{A1}+R_{A2}=550+148=698^{kg}$

$R_B=R_{B1}+R_{B2}=500-148=352^{kg}$

$0\leqq x\leqq 2$ $\quad S_x=698-160x$

$\qquad M_x=-740+698x-80x^2$

$2\leqq x\leqq 5$ $\quad S_x=448-160x$

$\qquad M_x=-740+698x-80x^2-250(x-2)$

$\qquad =-240+448x-80x^2$

(C) $R_{A1}=1050^{kg}$

$\Sigma M_{A}=M_{A1}+250\times2-800\times2.5=0$

$\therefore M_{A1}=1500$

$$y_{B1}=\frac{ql^4}{8EI}+\frac{Pa^2}{6EI}(3l-a)$$

$$=\frac{160\times5^4}{8EI}+\frac{250\times2^2}{6EI}(3\times5-2)$$

$$=\frac{44000}{3EI}$$

(D) $R_{A2}=-R_B$ $\quad M_{A2}=-lR_B=-5R_B$

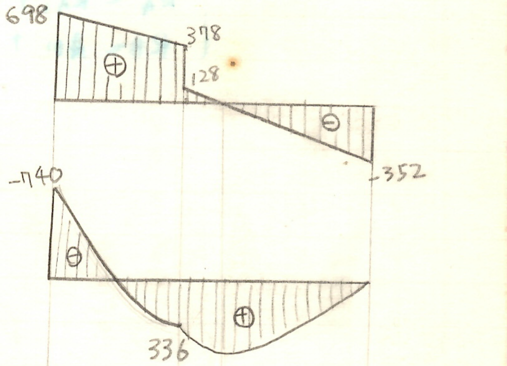
$$y_{B2}=-\frac{R_B5^3}{3EI}$$

$$y_B=y_{B1}+y_{B2}=\frac{44000-125R_B}{3EI}=0$$

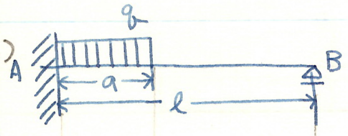
$\therefore R_B=352^{kg}$

$R_A=R_{A1}+R_{A2}=1050-352=698^{kg}$

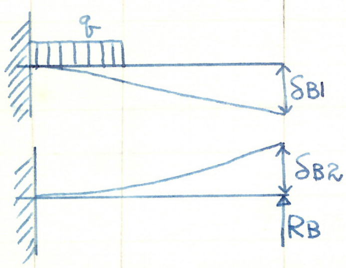
$M_A=M_{A1}+M_{A2}=1500-1760=740^{kg\cdot m}$



(9-2)



解1)



$$\begin{cases} \delta_{B1} = \dfrac{q a^3}{24EI}(4l - a) & \cdots (8\cdot 28) \\ \delta_{B2} = -\dfrac{R_B l^3}{3EI} & \cdots (8\cdot 34) \end{cases}$$

$\delta_{B1} + \delta_{B2} = 0$ より

$$R_B = \frac{q a^3}{8 l^3}(4l - a)$$

$$\Sigma V = R_A + R_B - qa = 0$$

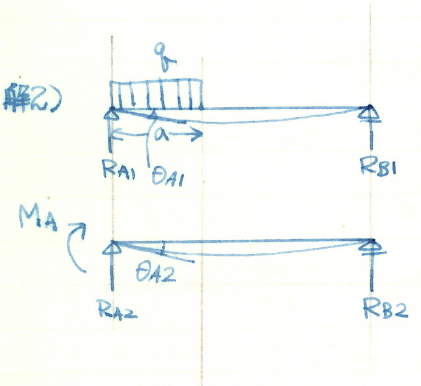
$$\therefore R_A = \frac{-qa}{8l^3}(4la^2 - a^3) + qa$$

$$\therefore R_A = \frac{qa}{8l^3}(8l^3 - 4la^2 + a^3)$$

$$\Sigma M_A = M_A + \frac{1}{2}qa^2 - \frac{qa^3}{8l^2}(4l - a) = 0$$

$$\therefore M_A = \frac{qa^2}{8l^2}(4l^2 - 4la + a^2)$$

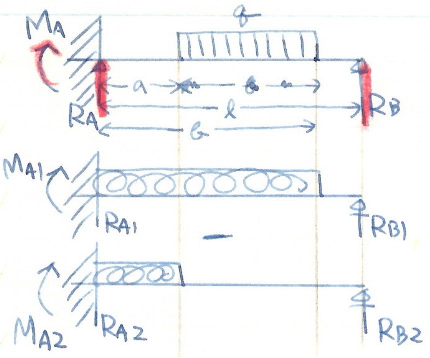
解2)



$$\begin{cases} R_{A1} = qa\left(1 - \dfrac{a}{2l}\right) \\ R_{B1} = \dfrac{qa^2}{2l} \\ \cancel{\delta}\,\theta_{A1} = \boxed{} \end{cases}$$

$$\begin{cases} R_{A2} = -M_A / l \\ R_{B2} = M_A / l \\ \theta_{A2} = l M_A / 3EI. & \cdots (8\text{-}23) \end{cases}$$

$$\begin{cases} \theta_{A1} + \theta_{A2} = 0 \longrightarrow M_A = \\ R_A = R_{A1} + R_{A2} = \\ R_B = A_{B1} + R_{B2} = \end{cases}$$



$$R_{A2} = \frac{qa}{8l^3}(8l^3 - 4la^2 + a^3)$$

$$R_{B2} = \frac{qa^3}{8l^3}(4l - a)$$

$$M_{A2} = \frac{qa^2}{8l^2}(4l^2 - 4la + a^2)$$

$$R_{A1} = \frac{qb}{8l^3}(8l^3 - 4lb^2 + b^3)$$

$$R_{B1} = \frac{qb^3}{8l^3}(4l - b)$$

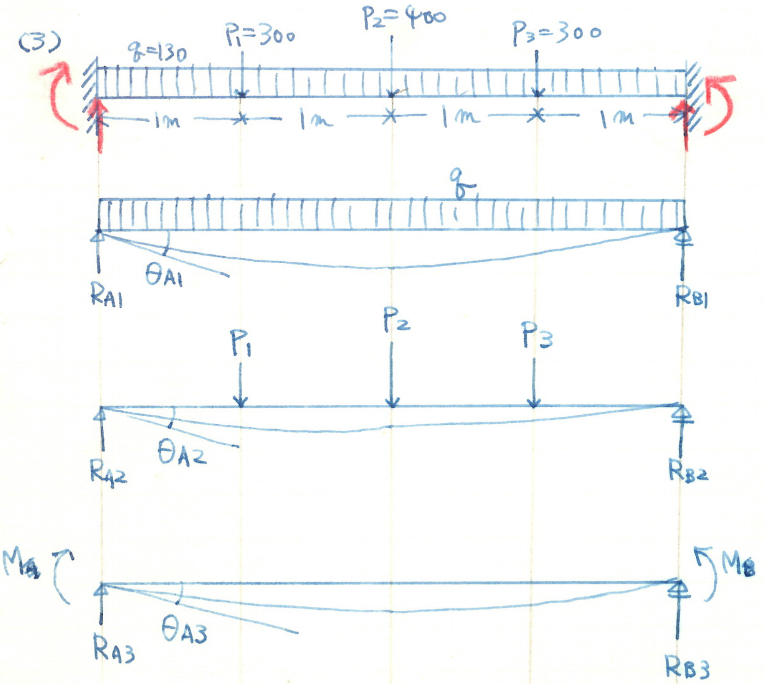
$$M_{A1} = \frac{qb^2}{8l^2}(4l^2 - 4lb + b^2)$$

$$R_A = R_{A1} - R_{A2} = \frac{q}{8l^3}\left\{8l^3(b-a) - 4l(b^3 - a^3) + (b^4 - a^4)\right\}$$

$$R_B = R_{B1} - R_{B2} = \frac{q}{8l^3}\left\{4l(b^3 - a^3) - (b^4 - a^4)\right\}$$

$$M_A = M_{A1} - M_{A2} = \frac{q}{8l^2}\left\{4l^2(b^2 - a^2) - 4l(b^3 - a^3) + b^4 - a^4\right\}$$

(3)



$$R_{A1} = R_{B1} = ql/2 = 260\ \text{kg.}$$

$$\theta_{A1} = \frac{ql^3}{24EI} = \frac{130\cdot 4^3}{24EI} = \frac{1040}{3EI}$$

$$R_{A2} = R_{B2} = 500\ \text{kg}$$

$$\theta_{A2} = \frac{1}{6EIl}\left\{3P_1(l^2-3^2) + 2P_2(l^2-2^2) + P_3(l^2-1^2)\right\} = \frac{1}{24EI}\cdot 20400 = \frac{850}{EI}$$

$$R_{A3} = R_{B3} = 0$$

$$\theta_{A3} = \frac{l}{6EI}(2M+M) = \frac{lM}{2EI} = \frac{2M}{EI}$$

$$\theta_A = \theta_{A1} + \theta_{A2} + \theta_{A3} = \frac{\overset{1040}{\cancel{1840}}}{3\cancel{EI}} + \frac{850}{\cancel{EI}} + \frac{\overset{4}{l}M}{2\cancel{EI}} = 0$$

$$\therefore M = -\frac{1}{2}\left(\frac{1040+2550}{3}\right) = -\frac{\overset{3590}{\cancel{4390}}}{6}\ \text{kg}\cdot\text{m} = -\frac{1795}{3}$$

最大曲げモーメントは梁の中央に働く。

$$\therefore M_{max} = \left(R_{A1}\times 2 - \frac{8\times 2^2}{2}\right) + \left\{R_{A2}\times 2 - P_1(2-1)\right\} + M$$

$$= 260\times 2 - 130\times 2 + 500\times 2 - 300\times 1 - \cancel{\frac{4390}{6}}\ \frac{1795}{3}$$

$$\cancel{= 685/3} = 1085/3\ \text{kg}\cdot\text{m} = 108500/3\ \text{kg}\cdot\text{cm}$$

$$\sigma = \frac{M_{max}}{I}y,\quad y = \underbrace{\frac{h}{2}}_{=b},\quad I = \frac{bh^3}{12},\quad (h = 2b) \rightarrow I = \frac{2b^4}{3}.$$

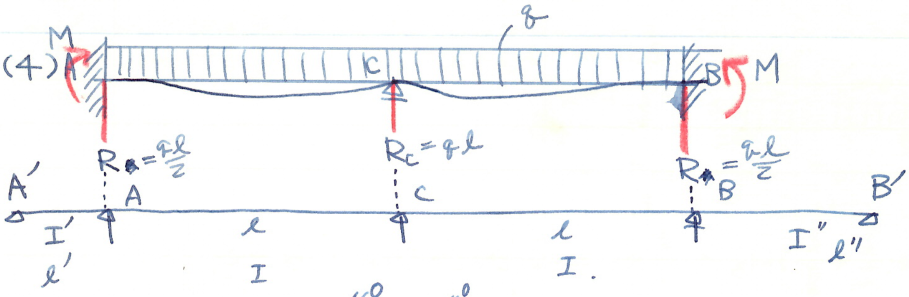
$$\sigma_a = \frac{M_{max}}{2b^4/3}\cdot b = \frac{3M_{max}}{2b^3}$$

$$\cancel{\therefore b^3 = \frac{3\times 685/3}{90\times 2\times 10^4}}$$

$$b^3 = \frac{3M_{max}}{2\sigma_a} = \frac{\cancel{3}\cdot(108500/\cancel{3})}{2\times 90} = \frac{5425}{9} = 627.78$$

$$b \fallingdotseq 8.5\ \text{cm} \qquad h \fallingdotseq 17.0\ \text{cm}\ /\!/$$

(4)



Ⓐ $\frac{\ell'}{I'} \overset{0}{\cancel{M_{A'}}} + 2\left(\overset{0}{\cancel{\frac{\ell'}{I'}}} + \frac{\ell}{I}\right) M_A + \frac{\ell}{I} M_C = 6E\left(0 - \frac{q\ell^3}{24EI}\right)$

$2M + M_C = -\frac{q\ell^2}{4}$ —— ①

Ⓒ $\frac{\ell}{I} M + 4\frac{\ell}{I} M_C + \frac{\ell}{I} M = 6E\left(-\frac{q\ell^3}{24EI} - \frac{q\ell^3}{24EI}\right)$

$\therefore\ \cancel{2}M + \cancel{4}2M_C = -\frac{q\ell^2}{\cancel{4}}$ —— ②

$2M + 4M_C = -\frac{q\ell^2}{2}$

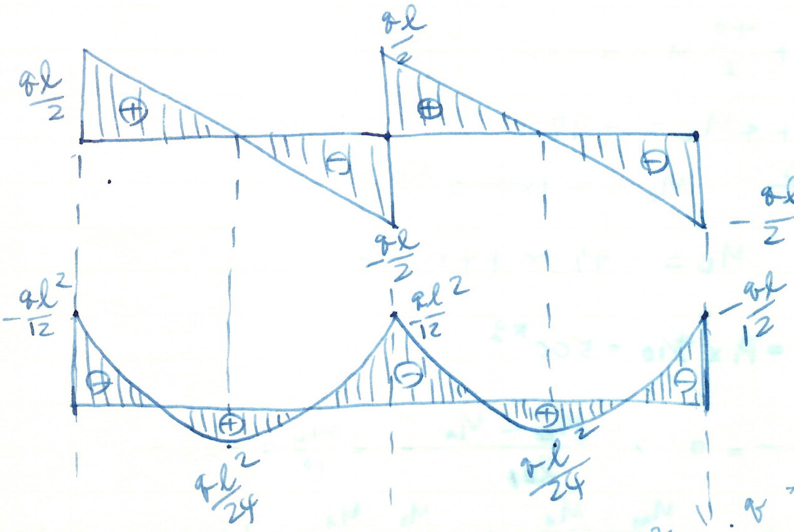
Ⓑ $\frac{\ell}{I} M_C + 2\left(\frac{\ell}{I} + \overset{0}{\cancel{\frac{\ell''}{I''}}}\right) M + \overset{0}{\cancel{\frac{\ell''}{I''} M_{B'}}} = 6E\left(-\frac{q\ell^3}{24EI} - 0\right)$

$\therefore\ M_C + 2M = -\frac{q\ell^2}{4}$ —— ③ (= ①)

①, ② より.

~~$M = M_C = -\frac{q\ell^2}{12}$~~

$3M_C = -\frac{q\ell^2}{4}$ $\therefore\ M_C = -\frac{q\ell^2}{12} = M.$



$\frac{\ell}{2} \cdot \frac{q\ell}{2} - \frac{\ell^2}{4} \cdot \frac{1}{2} \cdot q - \frac{q\ell^2}{12}$

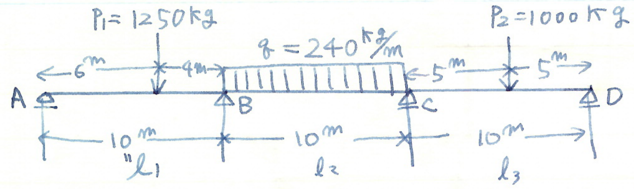
$\frac{1}{4} - \frac{1}{8} - \frac{1}{12}$

$\frac{6-3-2}{24}$

$\frac{1}{2} - \frac{1}{8} + \frac{1}{4}$

$\frac{1}{24} \times 2$

(5)



ABC $\quad \frac{l}{I}\underset{0}{M_A} + 2\left(\frac{l}{I}+\frac{l}{I}\right)M_B + \frac{l}{I}M_C = 6E\left\{\theta'_{Bl} - \theta'_{Br} + \cancel{\beta_B - \beta_C}\right\}$

$$\theta'_{Bl} = -\frac{P_1 a}{6EIl}(l^2 - a^2) = -\frac{1250\times 6}{6\times 10\times EI}(10^2 - 6^2)$$

$$= -\frac{8000}{EI}$$

$$\theta'_{Br} = \frac{q l^3}{24EI} = \frac{240\times 10^3}{24EI} = \frac{10000}{EI}$$

$$\therefore \frac{40}{I}M_B + \frac{10}{I}M_C = -6E\cdot\frac{18000}{EI}$$

$$\therefore 4M_B + M_C = -10800 \quad \text{—— ①}$$

BCD $\quad \frac{l}{I}M_B + 2\left(\frac{l}{I}+\frac{l}{I}\right)M_C + \frac{l}{I}\overset{0}{M_D} = 6E(\theta'_{Cl} - \theta'_{Cr})$

$$\theta'_{Cl} = -\frac{q l^3}{24EI} = -\frac{10000}{EI}$$

$$\theta'_{Cr} = \frac{P_2 b}{6EIl}(l^2 - b^2) = \frac{1000\cdot 5}{60EI}(10^2 - 5^2) = \frac{6250}{EI}$$

$$\therefore \frac{10}{I}M_B + \frac{40}{I}M_C = -6E\cdot\frac{16250}{EI}$$

$$M_B + 4M_C = -9750 \quad \text{—— ②}$$

$\frac{②\times 4 - ①}{15}$ $\quad M_C = -1880$ kgm

$$M_B = -9750 + 4\times 1880 = -2230 \text{ kgm}$$

• $R'_{Al} = 0$, $\quad R'_{Ar} = P_1 \times 4/10 = 500$ kg

$\frac{M_{A-1} - M_A}{l_{A0}} = 0$, $\quad \frac{M_B - M_A}{l_{B1}} = \frac{M_B}{10} = -223$ kg.

$\therefore R_A = R'_{Al} + R'_{Ar} + \frac{M_{A1} - M_A}{l_1} + \frac{M_B - M_A}{l_2} = 500 - 223 = 277$ kg

• $R'_{Bl} = 6/10\, P_1 = 750$ kg, $\quad R'_{Br} = 1200$ kg,

1950
223
35
2208

$$\frac{M_A - M_B}{l_1} = -\frac{M_B}{l_1} = 223^{kg} \qquad \frac{M_C - M_B}{l_2} = \frac{350}{10} = 35$$

$$R_B = 750 + 1200 + 223 + 35 = 2208^{kg}$$

∘ $R'_{Cl} = 1200^{kg}$ $\quad R'_{Cr} = 500^{kg}$

$$\frac{M_B - M_C}{l_2} = -35^{kg} \qquad \frac{M_D - M_C}{l_3} = -\frac{M_C}{l_3} = 188^{kg}$$

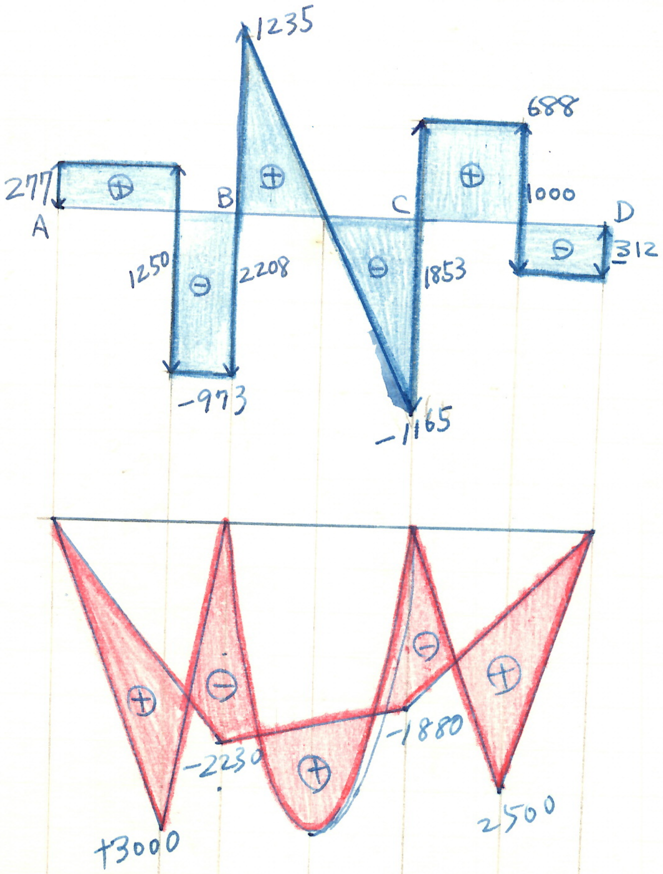
$$R_C = 1200 + 500 - 35 + 188 = 1853^{kg}$$

∘ $R'_{Dl} = 500^{kg}$, $\quad R'_{Dr} = 0$

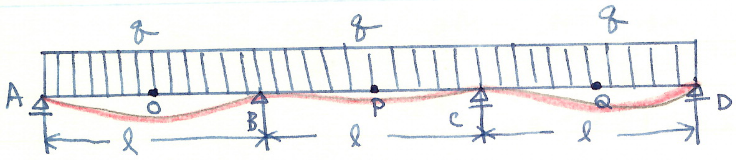
$$\frac{M_C - M_D}{l_3} = \frac{M_C}{l_3} = \cancel{35}\ -188\ kg \qquad \frac{M_{D'} - M_E}{l_4} = -\cancel{\frac{M_C}{l_4}} = 0$$

$$\therefore \cancel{R_D = 500 + 35 = 535^{kg}}$$

$$R_D = 500 - 188 = 312^{kg}$$



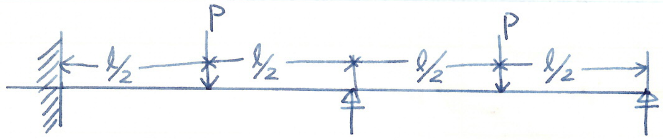
(6)

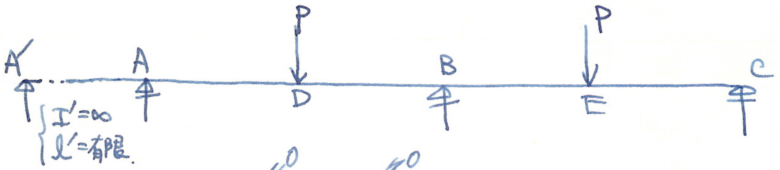


$EI = const.$

$\delta_O\ ,\ \delta_P\ ,\ \delta_Q = ?$

(7)





$\boxed{A'AB}$ $\quad \frac{l'}{I'}M_{A'} + 2\left(\frac{l'}{I'}+\frac{l}{I}\right)M_A + \frac{l}{I}M_B = 6E\left\{-\frac{P\cdot l/2}{6EIl}\left(l^2-\left(\frac{l}{2}\right)^2\right)\right\}$

$$= -\frac{3Pl^2}{8I}$$

$$2\frac{l}{I}M_A + \frac{l}{I}M_B = -\frac{3Pl^2}{8I}$$

$$\therefore 16M_A + 8M_B = -3Pl \quad \text{———} \ ①$$

$\boxed{ABC}$ $\quad \frac{l}{I}M_A + 2\left(\frac{l}{I}+\frac{l}{I}\right)M_B + \frac{l}{I}M_C = -2\cdot\frac{3Pl^2}{8I}$

$$\therefore \frac{l}{I}M_A + \frac{4l}{I}M_B = -\frac{3Pl^2}{4I}$$

$$4M_A + 16M_B = -3Pl \quad \text{———} \ ②$$

$$M_A = -\frac{3}{28}Pl \ , \quad M_B = -\frac{9}{56}Pl$$

∘ $R'_{Al} = 0, \quad R'_{Ar} = P/2.$

$$\frac{M_{A'} - M_A}{l'} = -\frac{M_A}{l} \qquad \frac{M_B - M_A}{l} = -\frac{3}{56}P$$

$$R_A = \frac{1}{2} - \frac{3}{56} = \frac{25}{56}P$$

∘ $R'_{Bl} = P/2, \quad R'_{Br} = P/2.$

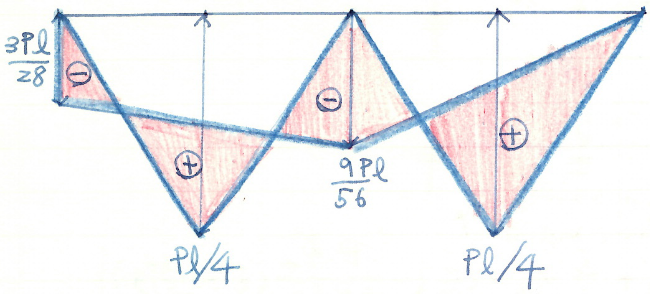
$$\frac{M_A - M_B}{l} = \frac{3}{56}P \qquad \frac{M_C - M_B}{l} = -\frac{M_B}{l} = \frac{9}{56}P$$

$$\therefore R_B = \left(\frac{1}{2}P\frac{1}{2} + \frac{3}{56} + \frac{9}{56}\right)P = \frac{68}{56}P = \frac{17}{14}P$$

∘ $R'_{Cl} = P/2, \quad R'_{Cr} = 0$

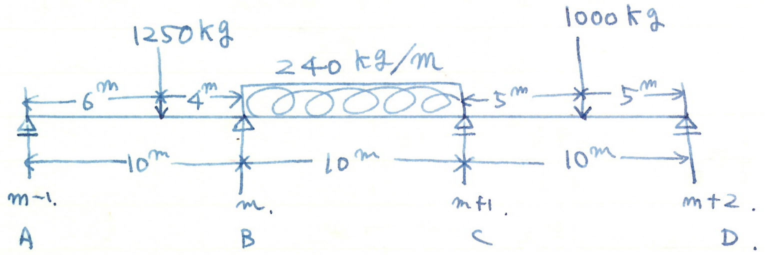
$$\frac{M_B - M_C}{l} = \frac{M_B}{l} = -\frac{9}{56}P, \qquad \frac{M_D - M_C}{l} = 0$$

$$\therefore R_C = \left(\frac{1}{2} - \frac{9}{56}\right)P = \frac{19}{56}P$$



(M図)

(8)



ABC 三連moment.

$$\frac{l}{I}M_A^{=0} + 2\left(\frac{2l}{I}\right)M_B + \frac{l}{I}M_C = 6E\left\{\theta'_{Bl} - \theta'_{Br} + \beta_m - \beta_{m+1}\right\}$$

$$\theta'_{Bl} = -\frac{Pa}{6EIl}(l^2 - a^2) = -\frac{1250\times 6}{6\times EI\cdot 10}\cdot 64 = -\frac{8000}{EI}$$

$$\theta'_{Br} = \frac{240\cdot 10^3}{24EI} = \frac{10000}{EI}$$

$$\beta_m - \beta_{m+1} = \frac{\delta_m - \delta_{m-1} - (\delta_{m+1} - \delta_m)}{l_m} = \frac{0.1+0.1}{l_m} = \frac{0.2}{10} = 0.02$$

$$\frac{4l}{I}M_B + \frac{l}{I}M_C = 6E\left\{-\frac{18000}{EI} + 0.02\right\}$$

$$4lM_B + lM_C = -6\cancel{E}\times 18000 + 0.12\times IE\times 0.1$$

$$4\cancel{0}M_B + \cancel{10}M_C = -10800\cancel{0}\cancel{E} + 0.12IE \times 0.1$$

BCD 三連moment.

$$\frac{l}{I}M_B + \frac{4l}{I}M_C + \frac{l}{I}M_D^{=0} = 6E\left\{\frac{10000}{EI} - \frac{6250}{EI} - \frac{0.1}{10}\right\} \quad 0.01$$

$$\cancel{10}M_B + 4\cancel{0}M_C = -9750\cancel{0} - 0.06EI\times 0.1$$

$$4M_B + M_C = -10800 + \overset{0.012}{\cancel{0.12}}IE = \cancel{30528}\ -6667.2$$

$$M_B + 4M_C = -9750 - \underset{0.006}{\cancel{0.06}}EI = \cancel{30414}\ -11816.4$$

より $\begin{cases} M_B = -990 \\ M_C = -\cancel{20}2707 \end{cases}$

$\circ\ R'_{Al} = 0 \qquad R'_{Ar} = P_1 \times 4/10 = 500^{kg}$

$$\frac{M_{A'} - M_A}{l'} = 0 \ , \quad \frac{M_B - M_A}{l_1} = \frac{M_B}{10} = -99$$

$$\therefore R_A = 500 - 99 = \boxed{401\ kg.}$$

$\circ\ R'_{Bl} = 6/10\, P_1 = 1750 \ , \quad R'_{Br} = 1200$

$$\frac{M_A - M_B}{l_1} = \frac{-M_B}{10} = +99 \quad , \quad \frac{M_C - M_B}{l_2} = -172.$$

$$\therefore R_B = 1750 + 1200 + 99 - 172 = \boxed{1877\ kg.}$$

$\circ\ R'_{Cl} = 1200 \ , \quad R'_{Cr} = 500$

$$\frac{M_B - M_C}{l_2} = 172 \qquad \frac{M_D - M_C}{l_3} = \frac{-M_C}{10} = 271$$

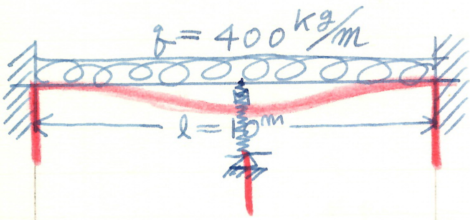
$$\therefore R_C = 1200 + 500 + 172 + 271 = \boxed{2143\ kg}$$

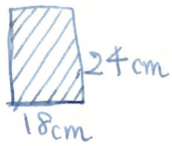
$\circ\ R'_{Dl} = 500 \ , \quad R'_{Dr} = 0$

$$\frac{M_C - M_D}{l_3} = \frac{M_C}{10} = -271 \quad , \quad \frac{M_{D'} - M_D}{l_4} = 0$$

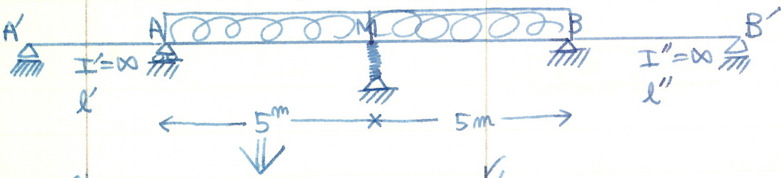
$$\therefore R_D = 500 - 271 = \boxed{229\ kg}$$

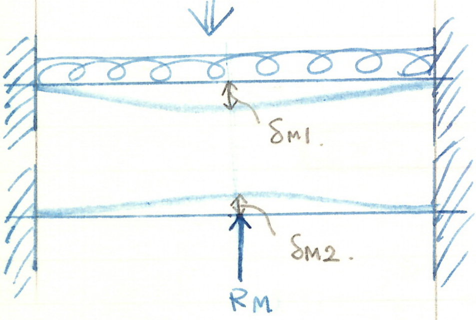
(9)





$E = 12 \times 10^4 \ kg/cm^2$



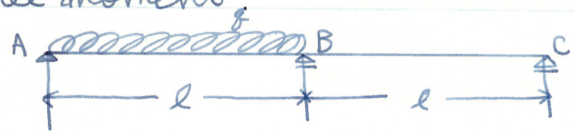


$$\delta_{M1} = \frac{q \cdot l^4}{384 EI}$$

$$\delta_{M2} =$$

three moment

(ex)

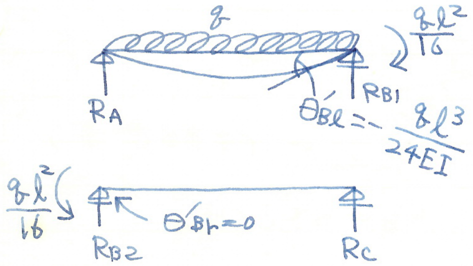


曲げモーメント，せん断力図を求めよ。

$$\frac{l}{I}\underset{0}{M_A} + 2\left(\frac{l}{I}+\frac{l}{I}\right)M_B + \frac{l}{I}\underset{0}{M_C} = 6E(\theta'_{Bl} - \theta'_{Br}) + 6E(\underbrace{\beta_m - \beta_{m+1}}_{0})$$

$$\frac{4l}{I}M_B = 6E\left(-\frac{ql^3}{24EI} - 0\right) = -\frac{ql^3}{\cancel{2}4I}$$

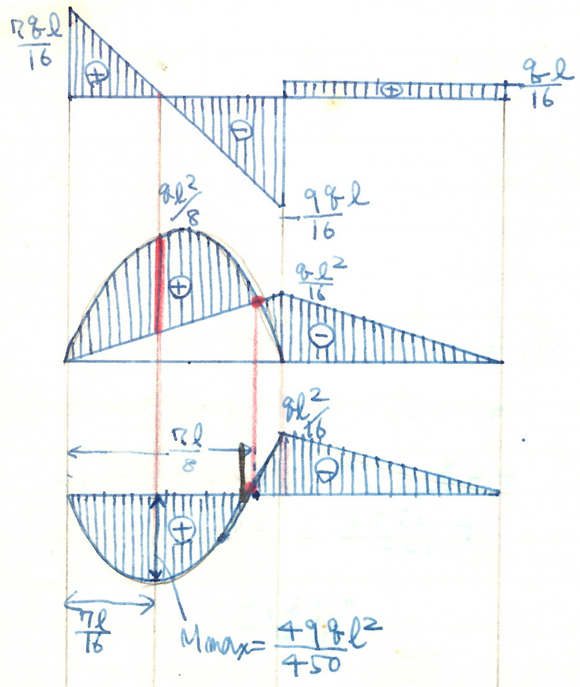
$$\therefore M_B = -\frac{ql^2}{16}$$



$$\begin{cases} \Sigma M_B = R_A l - \frac{ql^2}{2} + \frac{ql^2}{16} = 0 \qquad R_A = \frac{7ql}{16} \\ R_{B1} = ql - \frac{7ql}{16} = \frac{9}{16}ql \end{cases}$$

$$R_{B2} = ql/16 \qquad R_C = -ql/16$$

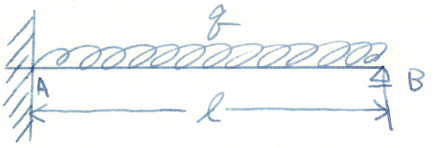
$$R_B = R_{B1} + R_{B2} = 5/8 \cdot ql$$



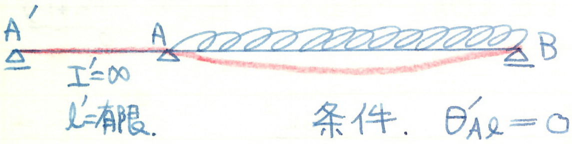
AB間. $M_x = \frac{7ql}{16}x - \frac{q}{2}x^2 = \frac{q}{2}x\left(\frac{7l}{8} - x\right)$

$$= -\frac{q}{2}\left\{x^2 - \frac{7l}{8}x + \left(\frac{7l}{16}\right)^2\right\} + \underbrace{\left(\frac{7l}{16}\right)^2 \times \frac{q}{2}}_{\frac{7^2 ql^2}{16^2 \times 2}}$$

(ex)



$EI = const.$



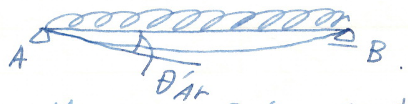
two span continuous beam.

条件. $\theta'_{Al} = 0$.

$$\frac{l'}{I'}M_{A'} + 2\left(\frac{l'}{I'} + \frac{l}{I}\right)M_A + \frac{l}{I}M_B = 6E(\theta'_{Al} - \theta'_{Ar})$$

$M_{A'} = 0$, $M_B = 0$, $\theta'_{Al} = 0$.

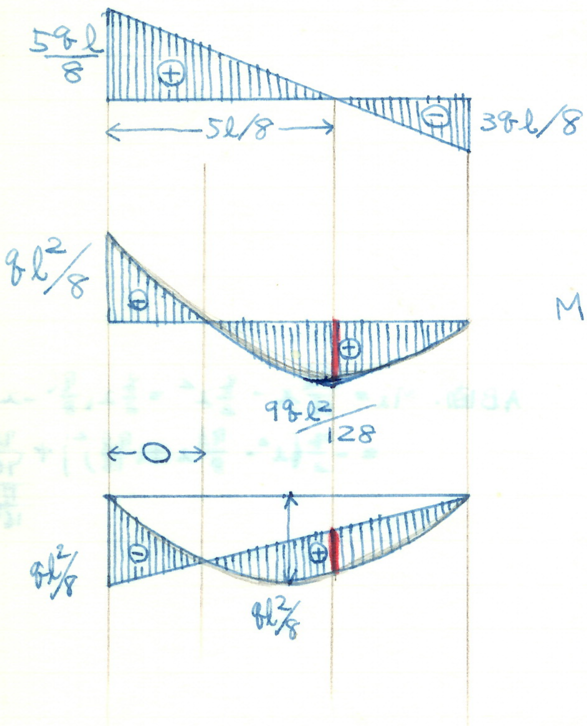
$$\theta'_{Ar} = \frac{ql^3}{24EI}$$



$$\therefore \frac{2l}{I}M_A = -\frac{ql^3}{4I} \qquad \therefore M_A = -ql^2/8 \text{ (不静定量)}$$

$$\Sigma M_A = -ql^2/8 + ql^2/2 - l \cdot R_B = 0 \quad \therefore R_B = 3ql/8.$$

$$R_A = ql - R_B = 5ql/8$$

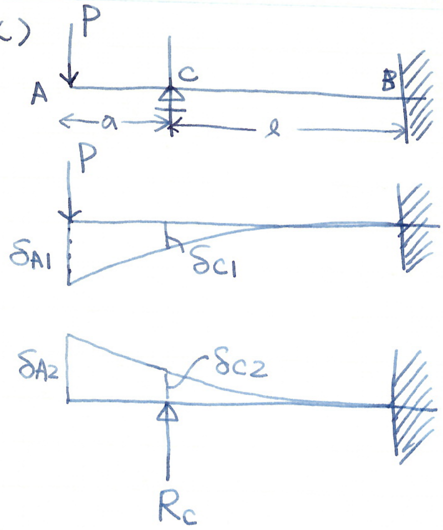


$$M_x = M_A + R_A x - \frac{qx^2}{2}$$

$$= -ql^2/8 + 5qlx/8 - qx^2/2$$

$$= -q/2\,(x^2 - 5lx/4 + 25l^2/64) + \underbrace{\frac{25ql^2}{128} - \frac{ql^2}{8}}_{\text{"}\frac{9ql^2}{128}}$$

(ex)



C点のたわみ＝零.

C点の反力＝？

A点のたわみ＝？

$$\frac{P}{6EI}\left\{l^2(3a+2l)\right\}$$

$$\delta_{c1} = \frac{P}{6EI}\left[a^3 - 3(a+l)^2 a + 2(a+l)^3\right]$$

$$\delta_{A1} = \frac{P(a+l)^3}{3EI} \quad \cdots (8.32)$$

$$\delta_{c2} = -\frac{R_c l^3}{3EI} \quad \cdots (8.32)$$

$$\delta_{A2} = -\frac{R_c l^2}{6EI}\left\{3(a+l) - l\right\}$$

$$\delta_{c1} + \delta_{c2} = 0 \quad \cdots\cdots (8.33)$$

$$R_c = \frac{P}{2l}(3a+2l)$$

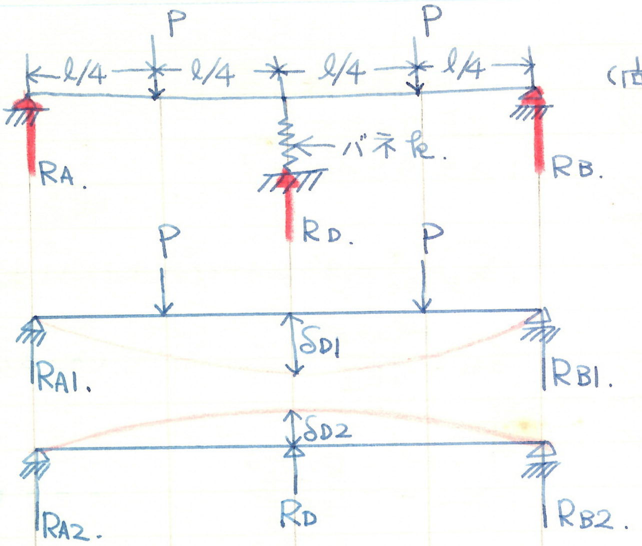
$$\delta_A = \delta_{A1} + \delta_{A2} = \frac{P(a+l)^3}{3EI} - \frac{P}{2\cancel{l}}(3a+2l)\frac{l^{\cancel{2}}}{6EI}(3a+2l)$$

$$= \frac{P(a+l)^3}{3EI} - \frac{P}{12EI}(3a+2l)^2$$

$$= \frac{P}{12EI}\left\{4(a+l)^3 - l(3a+2l)^2\right\}$$

$$\therefore \delta_A = \frac{P \cdot a^2}{12EI}(4a+3l)$$

(例)



(問)

$R_A, R_D, R_B = ?$

$$\delta_{D1} = 2 \times \frac{P b x}{6EIl}(l^2 - b^2 - x^2) \quad \text{--- (8·14)}$$

ただし $b = \frac{l}{4}$, $x = \frac{l}{2}$

$$\delta_{D1} = \frac{Pl^2}{3EIl \cdot 8} \cdot \frac{11}{16} l^2 = \frac{11Pl^3}{384EI}$$

$$\delta_{D2} = \frac{-R_D l^3}{48EI}$$

ここで $\delta_{D1} + \delta_{D2} = R_D / k$

$$= \frac{11Pl^3}{384EI} - \frac{R_D l^3}{48EI}$$

$$\therefore R_D = \frac{11Pl^3}{384EI} \Big/ \left(\frac{1}{k} + \frac{l^3}{48EI} \right)$$

$$= \frac{11 l^3 P}{\cancel{384EI}\ 8} \cdot \frac{\cancel{48kEI}}{48EI + kl^3}$$

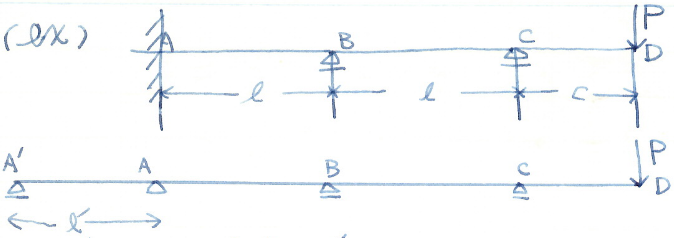
$$= \frac{11 kl^3}{8(48EI + kl^3)} \cdot P$$

$R_A = R_B$ $\quad 2R_A + R_D = 2P$.

$$R_A = R_B = \frac{1}{2}(2P - R_D) = \frac{16(48EI + kl^3) - 11kl^3}{2 \cdot 8(48EI + kl^3)} P$$

$$R_A = R_B = \frac{768EI + 5kl^3}{16(48EI + kl^3)} P$$

(ex)



$I' = \infty$ 条件 $\theta'_{Al} = 0$

A'−A−B. $\cancel{\frac{l'}{I'}} M'_A + 2\left(\cancel{\frac{l'}{I'}} + \frac{l}{I}\right) M_A + \frac{l}{I} M_B = 6E(\theta'_{Al} - \theta'_{Ar}) = 0$

$\therefore\ 2l M_A + l M_B = 0 \quad \therefore 2M_A + M_B = 0$ ——— ①

A−B−C. $\frac{l}{I} M_A + 2\left(\frac{l}{I} + \frac{l}{I}\right) M_B + \frac{l}{I} M_C = 6E(\theta'_{Bl} - \theta'_{Br}) = 0$

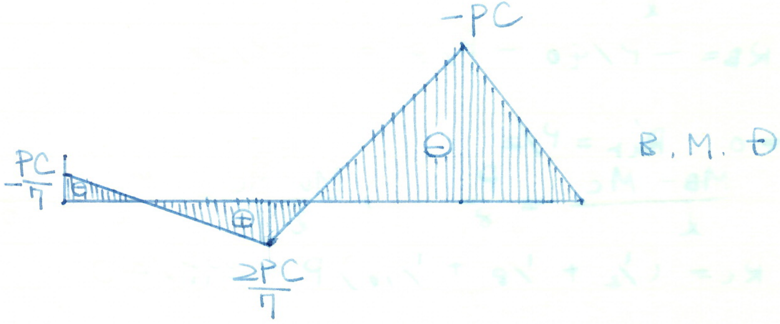
$\therefore\ M_A + 4M_B + M_C = 0$ ——— ②

$M_C = -PC$ ——— ③

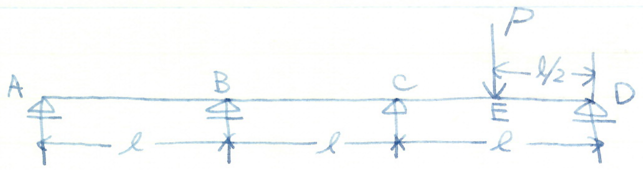
②, ③ → ×2. $2M_A + 8M_B = 2PC$. ——— ④

$$
\begin{array}{rl}
 & 2M_A + 8M_B = 2PC \\
-) & 2M_A + M_B = 0 \\
\hline
 & 7M_B = 2PC
\end{array}
\qquad \therefore M_B = 2PC/7
$$

$M_A = -4M_B - M_C = -8PC/7 + PC = -PC/7$



(ex)



B.M.D, S.F.D を求む。

three moment.

A — Ⓑ — C. $\cancel{M_A} + 4M_B + M_C = \frac{6EI}{l}(\theta'_{Bl} - \theta'_{Br}) = 0$, — ①

B — Ⓒ — D. $M_B + 4M_C + \cancel{M_D} = \frac{6EI}{l}(\theta'_{Cl} - \theta'_{Cr})$

$$= \frac{3\cdot\cancel{6EI}}{\cancel{l}}\left(-\frac{P\cdot l^{\cancel{2}}}{8\cdot\cancel{16EI}}\right) = -\frac{3Pl}{8} \quad \text{— ②}$$

①×4 − ② より.

$$M_B = \frac{3Pl}{8}\cdot\frac{1}{15} = \frac{Pl}{40}$$

$$M_C = -4M_B = -\frac{Pl}{10}$$

∘ $R'_{Al} = 0$, $R'_{Ar} = 0$.

$$\frac{0 - M_A}{l} = 0, \quad \frac{M_B - M_A}{l} = \frac{P}{40} \quad \therefore R_A = \frac{P}{40}$$

∘ $R'_{Bl} = 0$, $R'_{Br} = 0$.

$$\frac{M_A - M_B}{l} = -\frac{M_B}{l} = -\frac{P}{40} \quad \frac{M_C - M_B}{l} = -\frac{P}{8}$$

$\therefore R_B = -P/40 - P/8 = -3P/20$.

∘ $R'_{Cl} = 0$ $R'_{Cr} = P/2$

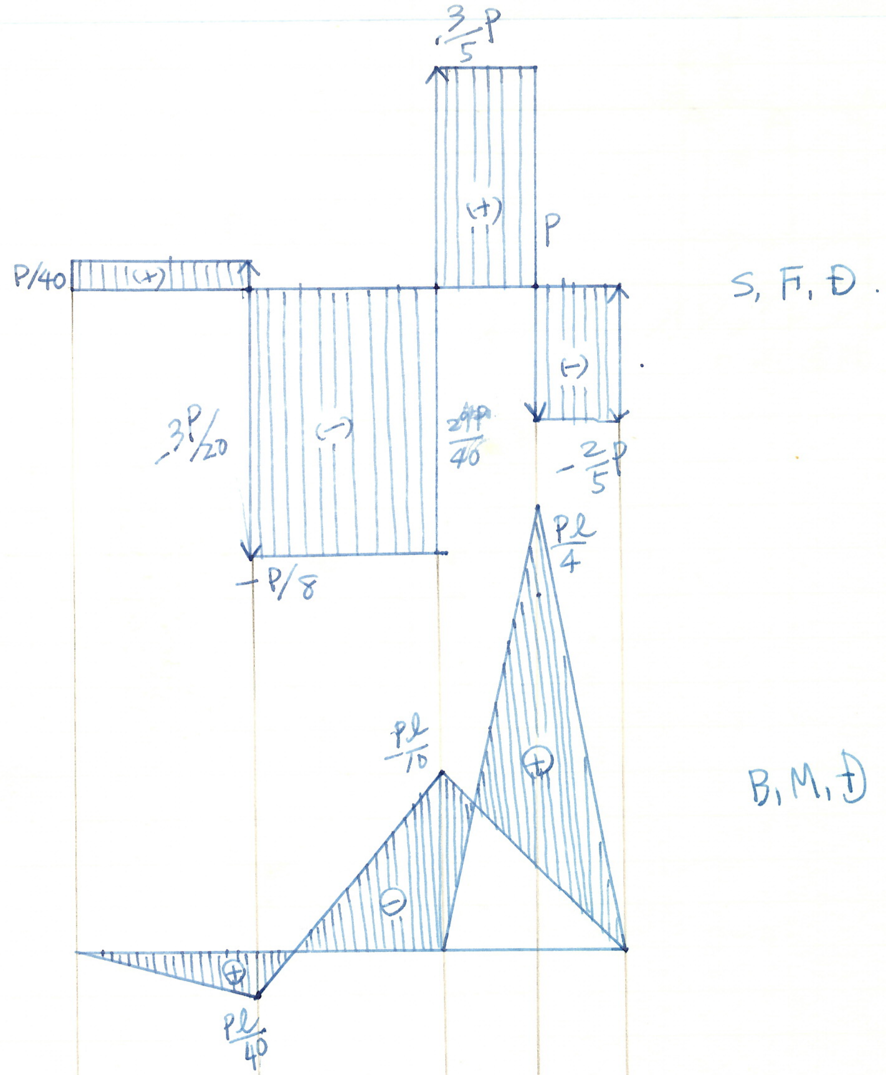
$$\frac{M_B - M_C}{l} = \frac{P}{8}, \quad \frac{M_D - M_C}{l} = -\frac{M_C}{l} = \frac{P}{10}$$

$\therefore R_C = (1/2 + 1/8 + 1/10)P = 29P/40$

∘ $R'_{Dl} = P/2$, $R'_{Dr} = 0$.

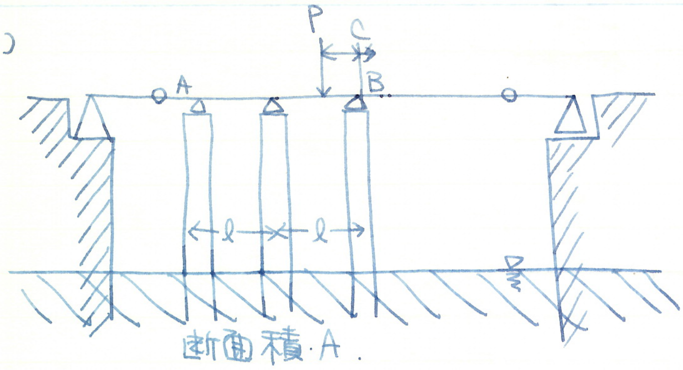
$$\frac{M_C - M_D}{l} = \frac{M_C}{l} = -\frac{P}{10}, \quad \frac{0 - M_D}{l} = 0$$

$\therefore R_D = P/2 - P/10 = 2P/5$

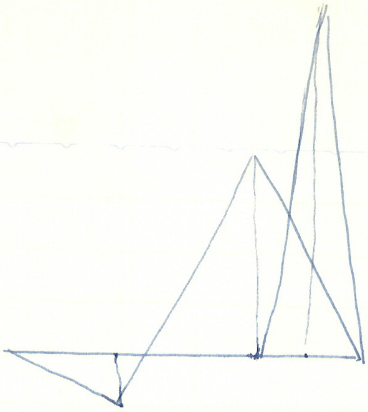


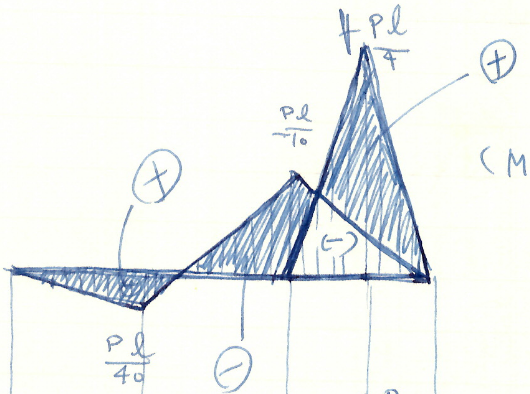
3/5 P
(+)
P
P/40
(+)
S, F, D.
(−)
(−)
3P/20
29P/40
−2/5 P
−P/8
Pl/4
Pl/10
B, M, D
Pl/40

(ex)

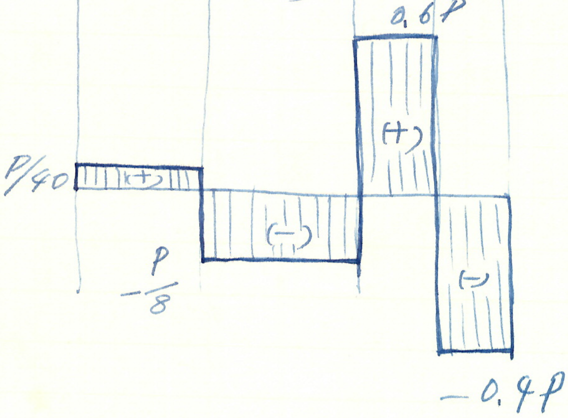


水の比重：γ



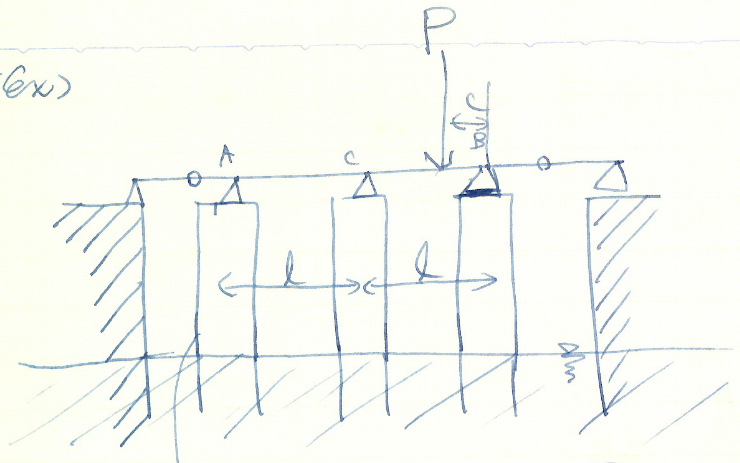


(M図)



(S図.)

(Ex)



浮力使う

水 γ 比重

ACBのB.M.D

$\Sigma M_B=0 \rightarrow A\gamma y_1 = \frac{Pc}{2l}$

$\Sigma M_A=0 \rightarrow A\gamma y_2 = \frac{(2l-c)}{2l}P$

$y_1\ y_2$は C支点をとりのぞいたときのA.B点の変位.

A.Bが $y_1\ y_2$ だけ沈んだ時C点の変位yは.

$$y = \frac{y_1+y_2}{2} = \frac{P}{2A\gamma}$$

荷重PによるC点のたわみを加えたものがC点の総変形量 δ.

$$\delta = \frac{Pc}{48EI}\left[3(2l)^2-4c^2\right]+\frac{P}{2A\gamma} \quad \cdots\cdots (1)$$

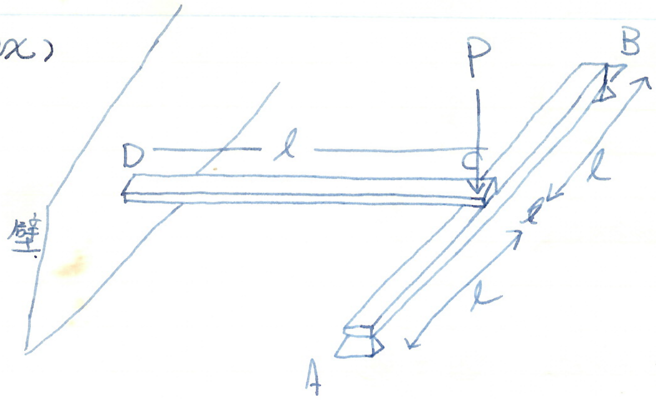
C点反力をXとすると.

$$\frac{X(2l)^3}{48EI}+\frac{X}{2A\gamma} \quad \cdots\cdots (2)$$

$$\frac{Pc}{48EI}\left[3(2l)^2-4c^2\right]+\frac{P}{2A\gamma}-\frac{X(2l^3)}{48EI}$$

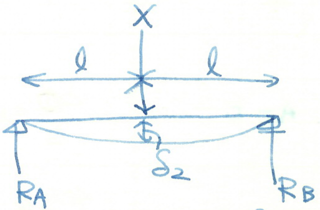
$$-\frac{X}{2A\gamma}=\frac{X}{A\gamma}$$

(ex)



EI : const.
Cの変位？

$$\delta_1 = \frac{(P-X)l^3}{3EI}$$



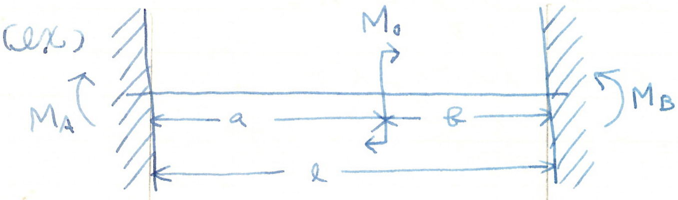
$$\delta_2 = \frac{X(2l)^3}{48EI} = \frac{Xl^3}{6EI}$$

$\delta_1 = \delta_2$ より $\frac{(P-X)\cancel{l^3}}{\cancel{3EI}} = \frac{X\cancel{l^3}}{\cancel{6EI}_{2}}$

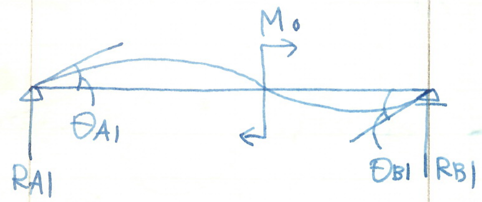
$$\therefore X = \frac{2}{3}P$$

$$\therefore \delta_c = (\delta_1 = \delta_2) = \frac{Pl^3}{9EI}$$

(ex)



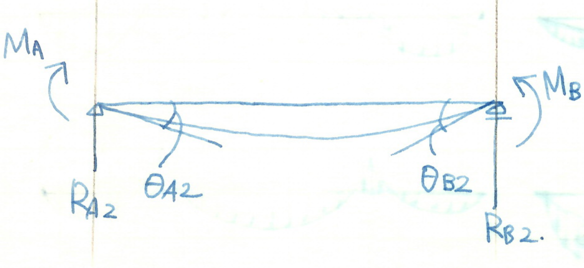
$$\begin{cases} \theta_{A1} = \dfrac{M_0}{6lEI}(l^2 - 3b^2) \\ \theta_{B1} = \dfrac{M_0}{6lEI}(l^2 - 3a^2) \end{cases}$$



$$\begin{cases} \theta_{A2} = \dfrac{l}{6EI}(2M_A + M_B) \\ \theta_{B2} = \dfrac{-l}{6EI}(M_A + 2M_B) \end{cases}$$

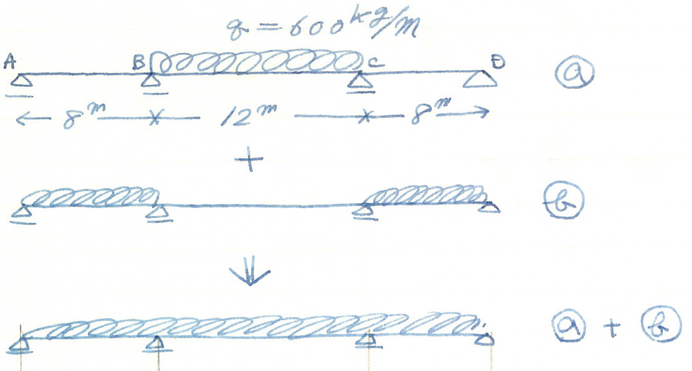
$\theta_{A1} + \theta_{A2} = 0$, $\theta_{B1} + \theta_{B2} = 0$

$$\therefore \left.\begin{aligned} M_A &= \frac{-M_0 b}{l^2}(2l - 3b) \\ M_B &= \frac{M_0 a}{l^2}(2l - 3a) \end{aligned}\right\}$$



7.901　17.9
5.29　5.2

(ex)



ⓐ. A – B – C. $\frac{\cancel{40}}{\cancel{I}}M_B+\frac{\cancel{8}}{\cancel{I}}M_C=\cancel{6E}\cdot\frac{-\overset{150}{\cancel{q}}\cdot l^{\overset{2}{\cancel{3}}}}{\underset{4}{\cancel{24EI}}}$

$40M_B+8M_C=-150\times12^3$

$5M_B+M_C=-75\times12^2\times3$ ——— ①

B – C – D. $\frac{12}{I}M_B+2\left(\frac{12}{I}+\frac{8}{I}\right)M_C=6E\cdot\left(-\frac{q\cdot12^3}{\underset{4}{\cancel{24EI}}}\right)$

$12M_B+40M_C=-150\times12^3$

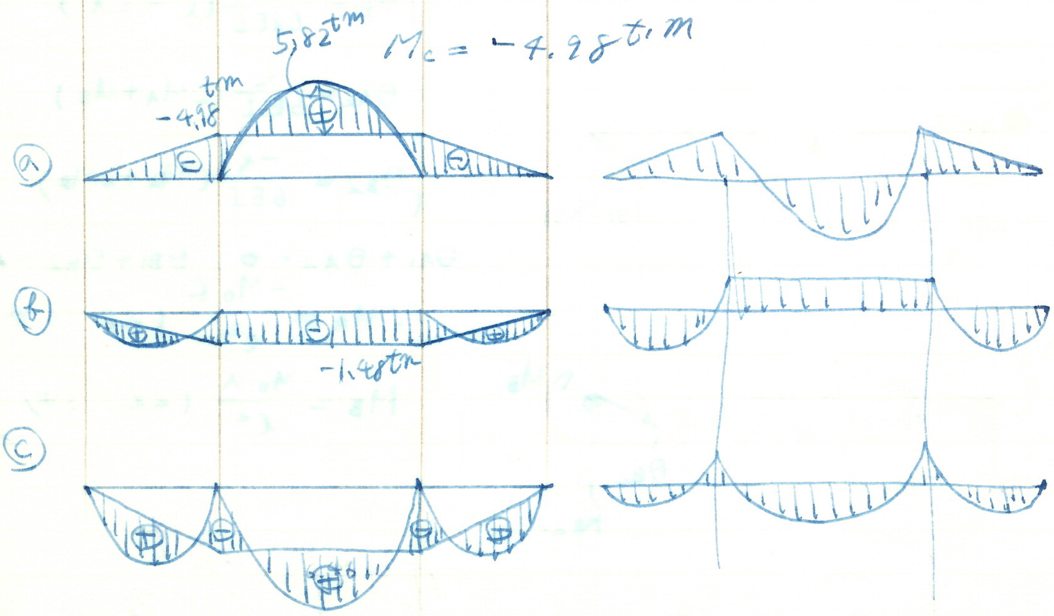
$3M_B+10M_C=-150\times12^2\times3$ ——— ②

①×10 − ②

$47M_B=-\cancel{75}\,150\times12^2\times3\times4$

$\therefore M_B=-\frac{150\times12^3}{47}=-4.98^{t\cdot m}$

$M_C=-4.98^{t\cdot m}$



⑨図 荷重なし.

支点 C が δ 沈下　　　　$\delta = 0.1m$

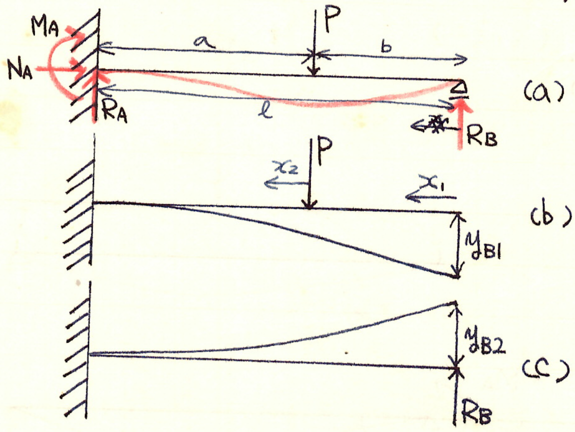
$$\begin{cases} M_B = -6.563\delta \\ M_C = 10.508\delta. \end{cases} \qquad \begin{cases} \Delta M_B = -0.66^{tm} \\ \Delta M_C = 1.05^{tm} \end{cases}$$

BC 間にて, C点の沈下 0.1m.

$$M_B = -4.98 - 0.66 = -5.63^{tm}$$

[地盤が悪いときは ゲルバー梁にする。]

一端固定.他端ローラー(集中荷重)



(a) より。

$$\begin{cases} N_A = 0 & \text{——①} \\ R_A + R_B = P & \text{——②} \\ M_A + Pa - R_B \ell = 0 & \text{——③} \end{cases}$$

(b) において。

$0 \leqq x_1 \leqq b$　$M=0$.

$\dfrac{d^2y_1}{dx_1^2} = 0$　　$\dfrac{dy_1}{dx_1} = c_1$

$\therefore y_1 = c_1 x_1 + c_2$

$0 \leqq x_2 \leqq a$.　$M = -Px_2$

$\dfrac{d^2y_2}{dx_2^2} = \dfrac{P}{EI} x_2$

$\dfrac{dy_2}{dx_2} = \dfrac{P}{EI}\left(\dfrac{x_2^2}{2} + c_1'\right)$

$y_2 = \dfrac{P}{EI}\left(\dfrac{x_2^3}{6} + c_1' x + c_2'\right)$

$x_2 = a \rightarrow \dfrac{dy_2}{dx_2} = \cancel{\dfrac{P}{EI}} = 0 \quad \therefore c_1' = -\dfrac{a^2}{2}$

$x_2 = a \rightarrow y_2 = \cancel{\dfrac{P}{EI}} = 0 \quad \therefore c_2' = \dfrac{a^3}{3}$

$x_1 = b \rightarrow \begin{cases} \dfrac{dy_1}{dx_1} = c_1 \\ y_1 = c_1 b + c_2 \end{cases}$

$x_2 = 0 \rightarrow \begin{cases} \dfrac{dy_2}{dx_2} = \dfrac{-P}{EI}\dfrac{a^2}{2} \\ y_2 = \dfrac{Pa^3}{3EI} \end{cases}$

$\therefore c_1 = -\dfrac{Pa^2}{2EI}$

$C_2 = \dfrac{Pa^3}{3EI} + \dfrac{Pa^2 b}{2EI}$

$\therefore [y_1]_{x_1=0} = y_{B1} = C_2 = \dfrac{Pa^2(3b+2a)}{6EI}$

(c) において。

$M = R_B x$.

$\dfrac{d^2y}{dx^2} = -\dfrac{R_B}{EI} x$

$\dfrac{dy}{dx} = -\dfrac{R_B}{EI}\left(\dfrac{x^2}{2} + c_1\right)$

$y = -\dfrac{R_B}{EI}\left(\dfrac{x^3}{6} + c_1 x + c_2\right)$

$\left[\dfrac{dy}{dx}\right]_{x=\ell} = 0 \quad \therefore c_1 = -\dfrac{\ell^2}{2}$

$[y]_{x=\ell} = 0 \quad \therefore C_2 = \dfrac{\ell^3}{3}$

$\therefore [y]_{x=0} = -\dfrac{R_B \ell^3}{3EI} = y_{B2}$.

$$y_{B1} + y_{B2} = \frac{Pa^2(3b+2a)}{6EI} - \frac{R_B \ell^3}{3EI} = 0$$

$$\therefore R_B = \frac{Pa^2(2a+3b)}{2\ell^3}$$

② より

$$R_A = \frac{P(2\ell^3 - 2a^3 - 3a^2 b)}{2\ell^3}$$

③ より。

$$M_A = \ell R_B - Pa$$

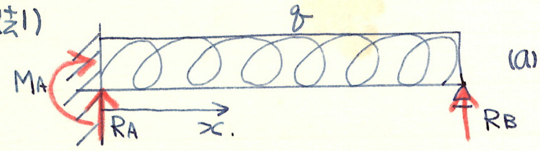
$$= \frac{P\{a^2(2a+3b) - 2\ell^2 a\}}{2\ell^2}$$

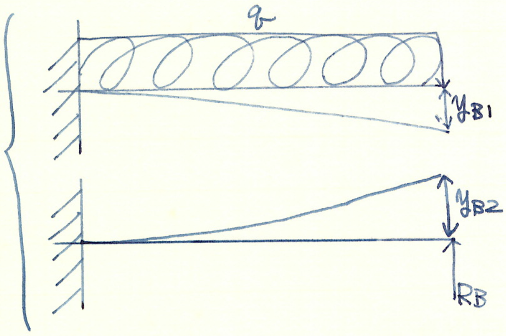
$$= \frac{Pa\{2a^2 + 3ab - 2(a+b)^2\}}{2\ell^2}$$

$$= \frac{Pab(3a - 4a - 2b)}{2\ell^2} = -\frac{Pab(a+2b)}{2\ell^2}$$

一端固定, 他端ローラー (等分布荷重)

(方法1)

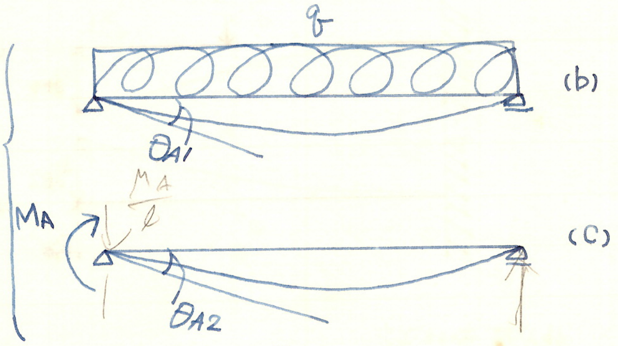




(方法2)

$\frac{ql}{2}x - \frac{qx^2}{2}$

$\frac{qx}{2}(l-x)$



(b)において

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left(\frac{qx^2}{2} - \frac{ql}{2}x\right)$$

$$\frac{dy}{dx} = \frac{1}{EI}\left(\frac{qx^3}{6} - \frac{ql}{4}x^2 + C\right)$$

$x = \frac{l}{2} \to dy/dx = 0$

$$\therefore C = \frac{ql^3}{\cancel{48}24}$$

$$\therefore \left[\frac{dy}{dx}\right]_{x=0} = \theta_{A1} = \frac{ql^3}{24EI}$$

(c)において

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left(\frac{M_A}{l}x - M_A\right)$$

$$\frac{dy}{dx} = \frac{M_A}{EI}\left(\frac{x^2}{2l} - x + C_1\right)$$

$$y = \frac{M_A}{EI}\left(\frac{x^3}{6l} - \frac{x^2}{2} + C_1x + C_2\right)$$

$x = 0,\ x = l \to y = 0$

$$C_2 = 0,\quad C_1 = \left(-\frac{l^2}{6} + \frac{l^2}{2}\right) \times \frac{1}{l} = \frac{l}{3}$$

$$\therefore \left[\frac{dy}{dx}\right]_{x=0} = \theta_{A1} = \frac{M_A l}{3EI}$$

$$\theta_{A1} + \theta_{A2} = \frac{ql^3}{24EI} + \frac{M_A l}{3EI} = 0$$

$$\therefore M_A = -\frac{ql^2}{8}$$

(a)において

$$\Sigma M_{B} = M_A - lR_B \cancel{=0} + \frac{ql^2}{2} = 0$$

$$\therefore R_B = \frac{ql}{2} + \left(-\frac{ql}{8}\right) = \frac{3ql}{8}$$

また

$$R_A = ql - R_B = \frac{5ql}{8}$$

$-\frac{ql}{8}x + \frac{ql^2}{8}$

$= -\frac{ql}{8}(x+l)$

$\frac{ql}{8}(l-x)$

$\frac{ql}{8}x - \frac{q}{8}l^2$

$\frac{ql}{8}(x-l)$

$\frac{1}{8} \times \frac{36}{64}$ 18 9 94

$-1 + \frac{25}{8} - \frac{100}{64}$

$-64 + 200 - 100$

せん断力．

$$S_x = R_A - qx = \frac{5ql}{8} - qx = \frac{q}{8}(5l - 8x)$$

曲げモーメント．

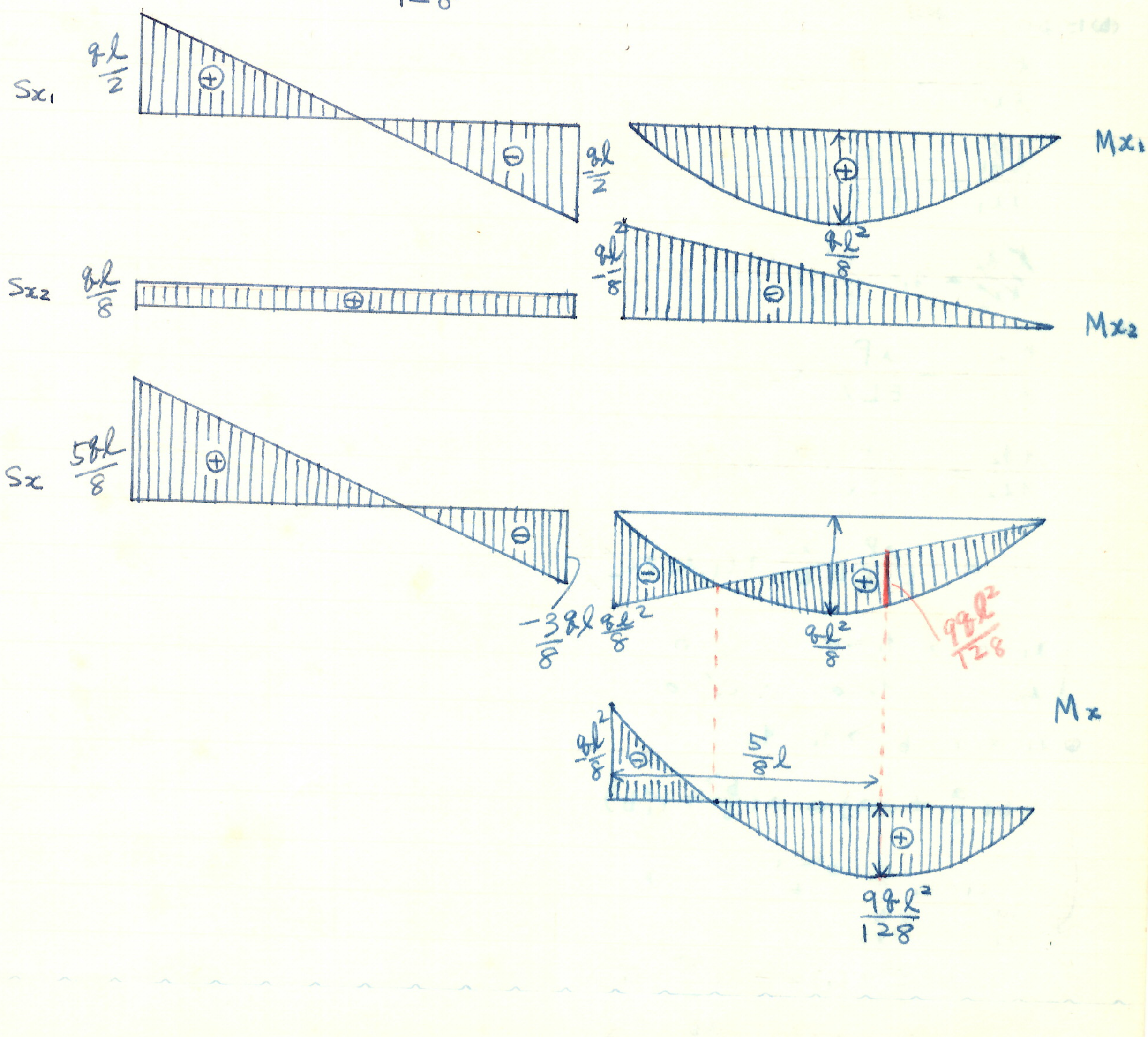
$$M_x = R_B(l-x) - \frac{q}{2}(l-x)^2$$

$$= (l-x)\left\{\frac{3ql}{8} - \frac{q}{2}(l-x)\right\} = \frac{q}{8}(l-x)(4x-l)$$

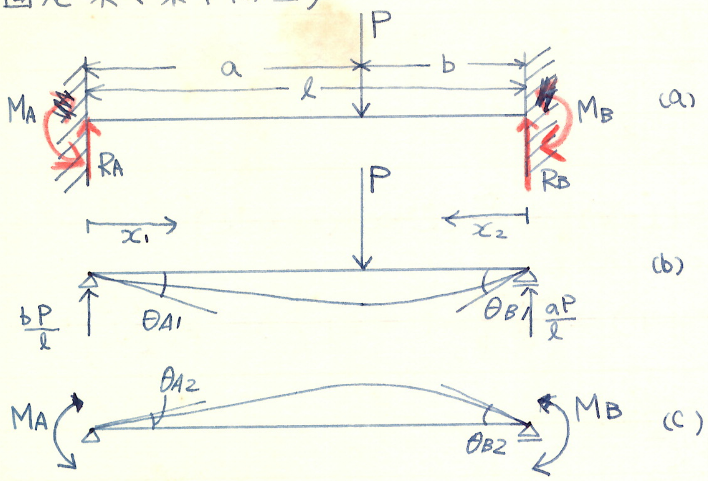
$$= \frac{ql^2}{8}\left(1-\frac{x}{l}\right)\left(4\frac{x}{l}-1\right) = \frac{ql^2}{8}\left\{-1 + 5\left(\frac{x}{l}\right) - 4\left(\frac{x}{l}\right)^2\right\}$$

$$S_x = 0 \rightarrow x = \frac{5}{8}l$$

$$M_{max} = \frac{9ql^2}{128}$$



固定梁 (集中荷重)



(b)において、~~dx~~

$$\frac{d^2y_1}{dx_1^2} = -\frac{bP}{EIl}x_1$$

$$\frac{dy_1}{dx_1} = -\frac{bP}{EIl}\left(\frac{x_1^2}{2} + C_1\right)$$

$$\cancel{\frac{d^2y_2}{dx_2^2}}\ y_1 = -\frac{bP}{EIl}\left(\frac{x_1^3}{6} + C_1x_1 + C_2\right)$$

$$\frac{d^2y_2}{dx_2^2} = -\frac{aP}{EIl}x_2$$

$$\frac{dy_2}{dx_2} = -\frac{aP}{EIl}\left(\frac{x_2^2}{2} + C_1'\right)$$

$$y_2 = -\frac{aP}{EIl}\left(\frac{x_2^3}{6} + C_1'x_2 + C_2'\right)$$

$$\begin{cases} x_1 = 0 \rightarrow y_1 = 0 & \therefore C_2 = 0 \\ x_2 = 0 \rightarrow y_2 = 0 & \therefore C_2' = 0. \end{cases}$$

$x_1 = a,\ x_2 = b. \rightarrow y_1 = y_2$

$$b\left(\frac{a^3}{6} + C_1a\right) = a\left(\frac{b^3}{6} + C_1'b\right)$$

$$C_1 - C_1' = \frac{b^2 - a^2}{6} \quad \text{—①}$$

$$\frac{dy_1}{dx_1} = -\frac{dy_2}{dx_2}$$

$$b\left(\frac{a^2}{2} + C_1\right) = -a\left(\frac{b^2}{2} + C_1'\right)$$

$$\therefore C_1 + C_1' = -\frac{ab}{2} \quad \text{—②}$$

$$\frac{①+②}{2} \quad C_1 = \frac{1}{2}\left\{\frac{b^2 - a^2}{6} - \frac{ab}{2}\right\} = \frac{b^2 - a^2 - 3ab}{12}$$

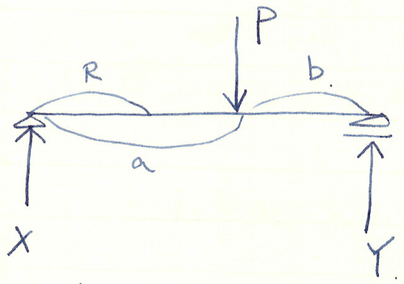
9/24 講ギ(星谷)

9章 不静定梁

c.f.
- 静定：釣合条件式のみで全ての反力を計算できる。
  荷重(given) 反力(?)
- 不静定：

anti-clockwise
(反時計回り)

$$\begin{cases} \Sigma H=0 \\ \Sigma V=0 \\ \Sigma M=0 \end{cases}$$



どの点においてのモーメントを考えても
結果は同じであることを示せ。

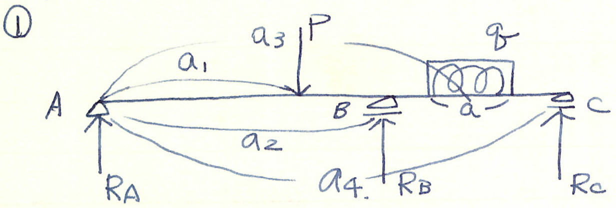
$$\Sigma M_R = RX + (a-R)P - (a+b-R)Y = 0$$

$$\therefore (X-P+Y) + aP - (a+b)Y = 0.$$

$$\therefore Y = \frac{a}{a+b}P.$$

$$(\because X+Y-P=\Sigma V=0)$$

①



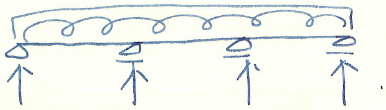
$R_A,\ R_B,\ R_C$: unknown reactions

$$\begin{cases} \Sigma V = R_A + R_B + R_C - P - q a = 0 \\ \Sigma M_A = P a_1 - R_B a_2 + q a a_3 - R_C a_4 = 0 \end{cases}$$

未知数：3ツ.
式　　：2ツ
} 1次不静定講造.

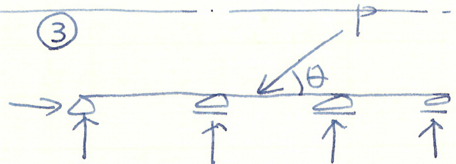
②

$\Sigma \cancel{H} V=0$
$\Sigma M=0$



(二次不静定)
4−2

③

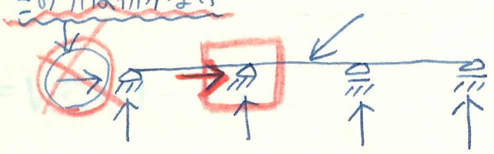


(二次不静定)
5−3

$\Sigma H=0$
$\Sigma V=0$
$\Sigma M=0$

④

この力は働かない



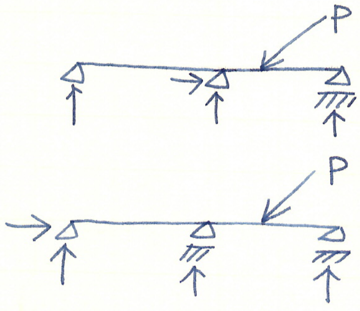
$\Sigma H = \Sigma V = \Sigma M = 0$. 式 3
未知数 ~~4~~5
(二次不静定)

Technical Term (技術用語)

エリザベス・テイラー
カーター
ジミー

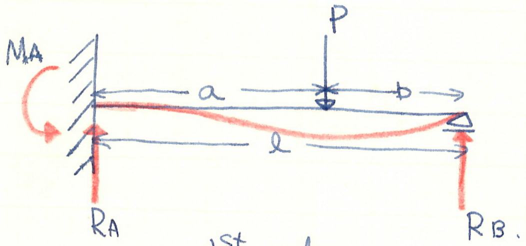
order　indeterminate　structure 構造物

(EX.)　何次不静定か。



unknown reaction 4.
式 3
故に 一次
First order.
1st

同様に 一次

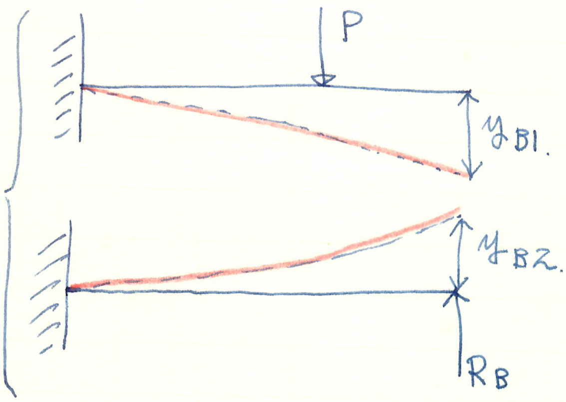


均合式.

$$\begin{cases} \Sigma V = R_A + R_B - P = 0 & \text{—— (1)} \\ \Sigma M = -M_A + Pa - R_B l = 0 & \text{—— (2)} \end{cases}$$

1st order

支点条件を1つ取り除けば静定となる。よって支点Bをとりのぞく。



静定基本型.

変形適合条件式

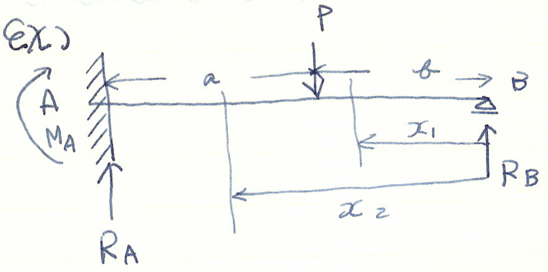
$$y_{B1} + y_{B2}(R_B) = 0. \quad \text{—— (3)}$$

$\therefore R_B =$

(1), (2), (3) より M_A, R_A, R_B が求まる。

「力学の講造物への応用」星谷著. ¥3,200

Ex.)



$0 \leqq x_1 \leqq b$.

$$M_{x_1} = R_B x_1$$

$$\frac{d^2 y_1}{d x_1^2} = -\frac{R_B}{EI} x_1$$

2回 積分.

$$\frac{dy_1}{dx_1} = -\frac{\cancel{R_B}\,1}{EI}\left(\frac{R_B x_1^2}{2} + C_1\right)$$

$$y_1 = -\frac{1}{EI}\left(\frac{R_B}{6} x_1^3 + C_1 x_1 + C_2\right)$$

$b \leqq x_2 \leqq a+b$.

$$M_{x_2} = -P(x_2 - b) + R_B x_2 = (R_B - P) x_2 + Pb$$

$$\frac{d^2 y_2}{d x_2^2} = -\left[(R_B - P) x_2 + Pb\right] / EI$$

$$\frac{dy_2}{dx_2} = \frac{1}{EI}\left[\frac{1}{2}(P - R_B) x_2^2 - Pb x_2 + C_1'\right]$$

$$y_2 = \frac{1}{EI}\left[\frac{1}{6}(P - R_B) x_2^3 - \frac{Pb}{2} x_2^3 + C_1' x_2 + C_2'\right]$$

① $x_1 = 0 \rightarrow y_1 = 0$

~~$x_2 = l \rightarrow y_2 = 0$~~

② $x_2 = l \rightarrow dy_2/dx_2 = 0$

③ $x_1 = x_2 = b$ で $dy_1/dx_1 = dy_2/dx_2$

④ $x_1 = x_2 = b$ で $y_1 = y_2$

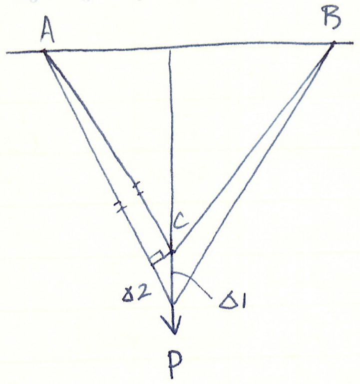
$$
\begin{cases}
C_1 = \dfrac{P}{2} b^2 - \dfrac{R_B}{2} l^2 + \dfrac{P}{2} l^2 - P b l. \\[2ex]
C_2 = 0 \\[2ex]
C_1' = \dfrac{R_B}{2} l^2 - \dfrac{P}{2} l^2 + P b l \\[2ex]
C_2' = -\dfrac{P b^3}{6}
\end{cases}
$$

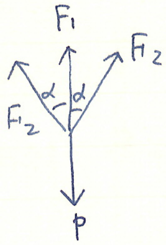
⑤ $x_2 = l \longrightarrow y_2 = 0$ より 反力R_Bが求まる。

$$
\begin{cases}
R_B = \dfrac{P a^2 (3l - a)}{2 l^3} \\[2ex]
R_A = \dfrac{P(2l^3 - 3 l a^2 + a^3)}{2 l^3}
\end{cases}
$$

[illegible].

解1)





$$F_1 + 2F_2\cos\alpha = P \quad \text{—— ①}$$

$$\delta_2 = \delta_1 \cos\alpha$$

$$\overline{AC} = L/\cos\alpha$$

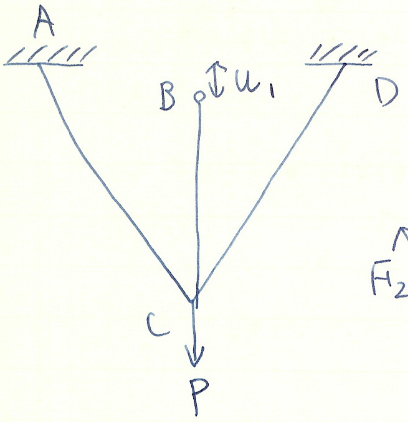
$$\therefore \underbrace{\frac{F_2 \cdot L/\cos\alpha}{AE}}_{\delta_2} = \underbrace{\frac{F_1 L}{AE}}_{\delta_1}\cos\alpha$$

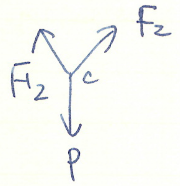
$$\therefore F_2 = F_1\cos^2\alpha \quad \text{—— ②}$$

$$\therefore F_1 = \frac{P}{1+2\cos^3\alpha}$$

$$F_2 = \frac{P}{1+2\cos^3\alpha}\cdot\cos^2\alpha$$

解2)

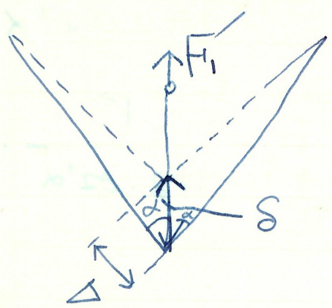


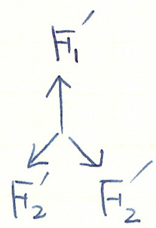


$2F_2\cos\alpha = P$

$$\therefore F_2 = \frac{P}{2\cos\alpha}$$

$$u_1 = \frac{\Delta_2}{\cos\alpha} = \frac{F_2(L/\cos\alpha)}{AE\cos\alpha} = \frac{PL}{AE\,2\cos^3\alpha}$$





$$F_2' = -\frac{F_1'}{2\cos\alpha}$$

$$\Delta = \frac{F_2'(L/\cos\alpha)}{AE} = -\frac{F_1'L}{AE\,2\cos^2\alpha}$$

$$\therefore \delta' = \frac{\Delta}{\cos\alpha} = \frac{-F_1'L}{AE\,2\cos^3\alpha}$$

C点の変位.

$u_1 + \delta = 0$

より

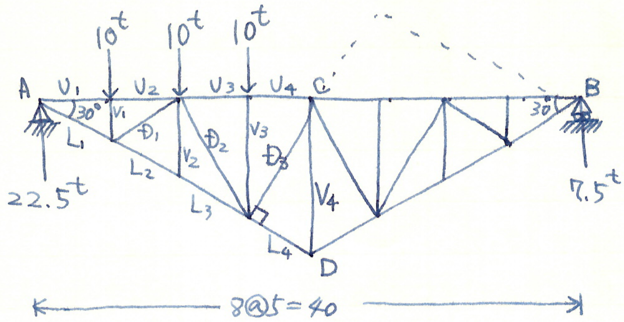
$$\delta = \delta' - \frac{F_1'L}{AE} = -\frac{F_1'L}{AE\,2\cos^3\alpha} - \frac{F_1'L}{AE} = -\frac{F_1'L}{AE}\left(\frac{1}{2\cos^3\alpha}+1\right)$$

B点の変位.

$$\frac{F_1'L}{AE}\left(\frac{1}{2\cos^3\alpha}+1\right) = \frac{PL}{AE\,2\cos^3\alpha}$$

$$\therefore F_1 = \frac{P}{1+2\cos^3\alpha}$$

(ex 1)

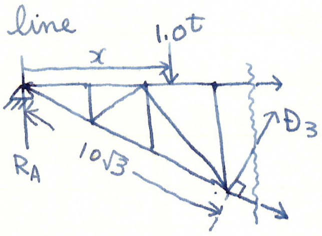


(イ) D_3, V_4 の Inf. line.

(ロ) D_3, V_4 の 部材力.

(ハ) 他の部材力.

(イ) Inf. line

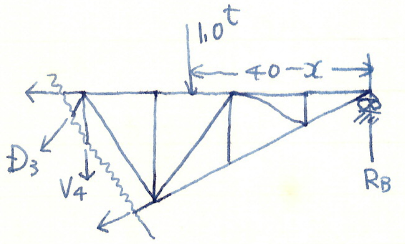


$0 \leqq x \leqq 15$

$$\Sigma M_A = 1.0 \cdot x - D_3 \cdot 10\sqrt{3} = 0$$

$$\therefore \ D_3 = \frac{\sqrt{3}\,x}{30}$$

$20 \leqq x$. $D_3 = 0$.



$0 \leqq x \leqq 15$

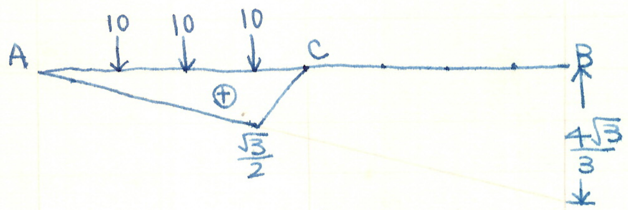
$$\Sigma M_B = V_4 \times 20 + \underbrace{D_3}_{\frac{\sqrt{3}x}{30}} \times 10\sqrt{3} = 0.$$

$$\therefore \ V_4 = -\frac{\sqrt{3}}{2} D_3 = -\frac{x}{20}$$

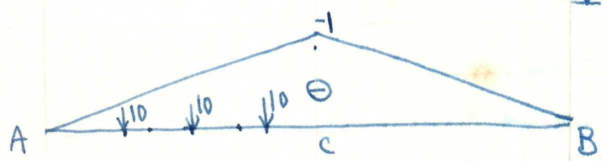
$20 \leqq x \leqq 40$.

$$\Sigma M_B = V_4 \times 20 + \underbrace{D_3}_{0} \times 10\sqrt{3} + 1.0 \times (40 - x) = 0$$

$$\therefore \ V_4 = \frac{x - 40}{20} = \frac{x}{20} - 2$$



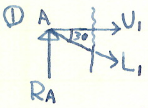
D_3 の Inf. Line



(ロ) $D_3 = 10\left(\sqrt{3}/6 + \sqrt{3}/3 + \sqrt{3}/2\right) = 10\sqrt{3}^{\,t}$ (ten)

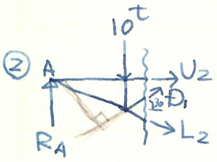
$V_4 = 10\left(-1/4 - 1/2 - 3/4\right) = -15^t$ (comp)

(ハ)各部材力.

① 

$\Sigma V = R_A - L_1 \sin 30^\circ = 0$. $\therefore L_1 = 22.5/\tfrac{1}{2} = 45^t$.

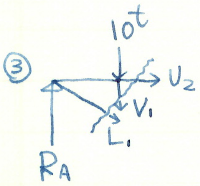
$\Sigma H = U_1 + L_1 \cos 30^\circ = 0$ $\therefore U_1 = -L_1 \cos 30^\circ = -45 \cdot \sqrt{3}/2 = -22.5\sqrt{3}^{\,t}$

② 

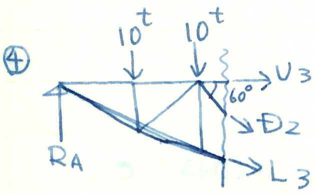
$\Sigma M_A = 10 \times 5 - D_1 \times 5 = 0$ $\therefore D_1 = 10^t$.

$\Sigma V = 22.5 - 10 + 10 \cdot \tfrac{1}{2} - L_2 \cdot \tfrac{1}{2} = 0$ $\therefore L_2 = 35^t$

$\Sigma H = U_2 + 10 \cdot \sqrt{3}/2 + 35 \cdot \sqrt{3}/2 = 0$ $\therefore U_2 = -22.5\sqrt{3}^{\,t}$

③ 

$\Sigma V = 22.5 - 10 - V_1 - 45 \cdot \tfrac{1}{2} = 0$ $\therefore V_1 = -10^t$

④ 

$\Sigma M_A = 10 \times 5 + 10 \times 10 + \tfrac{\sqrt{3}}{2} D_2 \times 10 = 0$

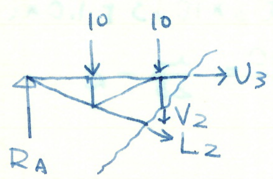
$\therefore D_2 = -10\sqrt{3}^{\,t}$

$\Sigma V = 22.5 - 20 - (-10\sqrt{3})\tfrac{\sqrt{3}}{2} - L_3/2 = 0$.

$\therefore L_3 = 35^t$

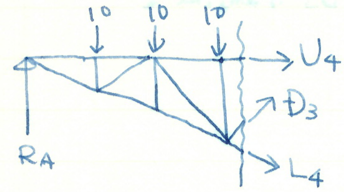
$\Sigma H = U_3 + D_2/2 + L_3 \cdot \sqrt{3}/2 = 0$

$\therefore U_3 = 5\sqrt{3} - 17.5\sqrt{3} = -12.5\sqrt{3}^{\,t}$

⑤ 

$\Sigma V = 22.5 - 20 - V_2 - L_2/2 = 0$.

$\therefore V_2 = -15^t$

⑥ 

$\Sigma M_A = 30 \times 10 - D_3 \cdot 10\sqrt{3} = 0$.

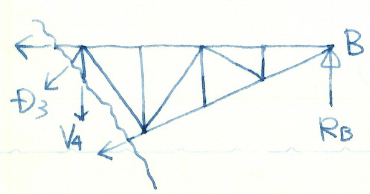
$\therefore D_3 = 10\sqrt{3}^{\,t}$

$\Sigma V = 22.5 - 30 + 10\sqrt{3} \cdot \sqrt{3}/2 - L_4/2 = 0$

$\therefore L_4 = 15^t$

$\Sigma H = U_4 + 5\sqrt{3} + 15 \cdot \sqrt{3}/2 = 0$

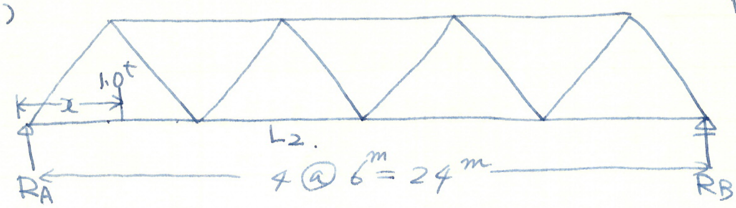
$\therefore U_4 = -12.5\sqrt{3}^{\,t}$

⑰ 

$\Sigma M_B = V_4 \times 20 + D_3 \times \sqrt{3}/2 \times 20 = 0$

$\therefore V_4 = -\tfrac{\sqrt{3}}{2} D_3 = -15^t$

(ex 2)

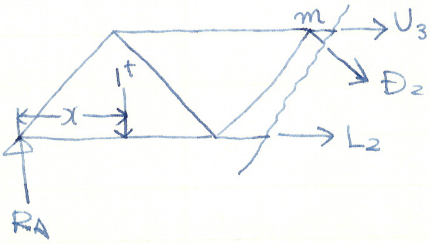


問.
等分布荷重 (1t/m), 集中荷重 (10t) がのるとき L_2 の最大部材応力を求めよ。

Inf. line を求める。

$R_A = \frac{24-x}{24} = 1 - \frac{x}{24}$.　　$R_B = \frac{x}{24}$.

$0 \leqq x \leqq 6$.



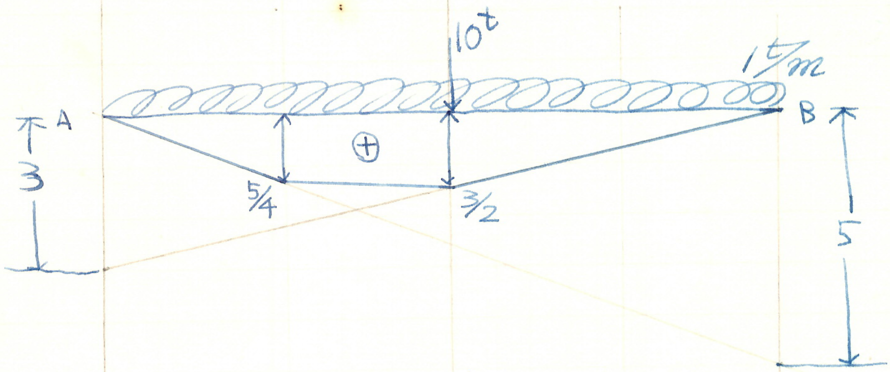
$\Sigma M_m = 3 \cdot L_2 + (9-x) \cdot 1 - 9 R_A = 0$.

$3L_2 + 9 - x - 9 + \frac{3x}{8} = 0$

$\therefore L_2 = \frac{5}{24} x$

$12 \leqq x \leqq 24$.　　$\Sigma M_m = 3L_2 - 9(1 - \frac{x}{24}) = 0$

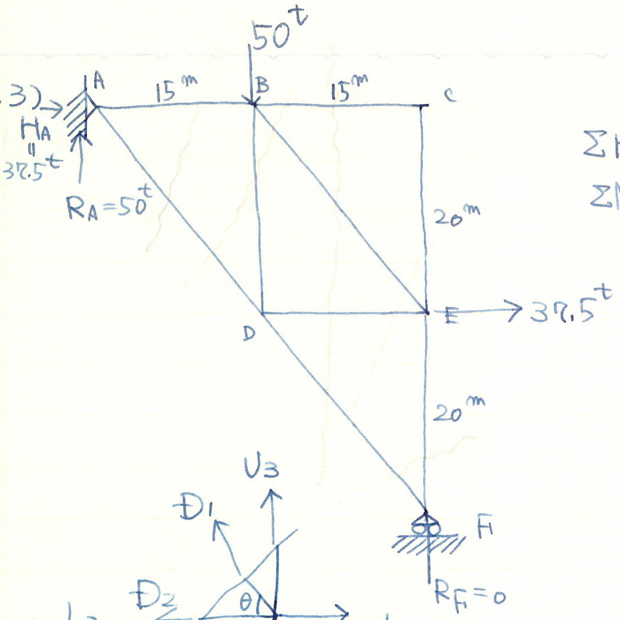
$\therefore L_2 = 3 - \frac{x}{8}$



$L_{2(max)} = \frac{3}{2} \times 10 + \left\{ 6 \times \frac{5}{4} \times \frac{1}{2} + 6 \times \left(\frac{5}{4} + \frac{3}{2} \right) \times \frac{1}{2} + 12 \times \frac{3}{2} \times \frac{1}{2} \right\} \times 1$

$= 15 + 21 = 36^t$ //

(ex 3)

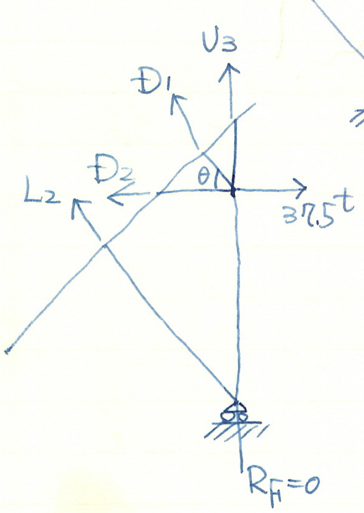


$$\Sigma H = H_A + 37.5 = 0 \quad \therefore H_A = -37.5^t$$

$$\Sigma M_A = 50 \times 15 - 37.5 \times 20 - R_F \times 30 = 0$$

$$\therefore R_F = 0.$$

$$\Sigma V = R_A - 50 + R_F = 0 \quad \therefore R_A = 50^t$$

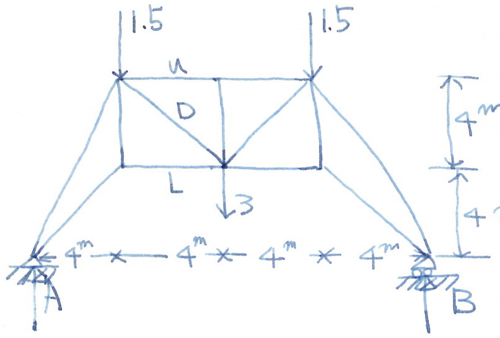


$$\Sigma H = 37.5 - D_2 - D_1 \cos\theta - L_2 \cos\theta = 0 \quad \text{—①}$$

$$\Sigma V = \underset{\overset{\|}{0}}{R_F} + U_3 + D_1 \sin\theta + L_2 \sin\theta = 0 \quad \text{—②}$$

10/30(演習)

(ex)



$R_A = R_B = 3$ //

$4R_L = 4R_A = 12.$

$R_L = 3$ //

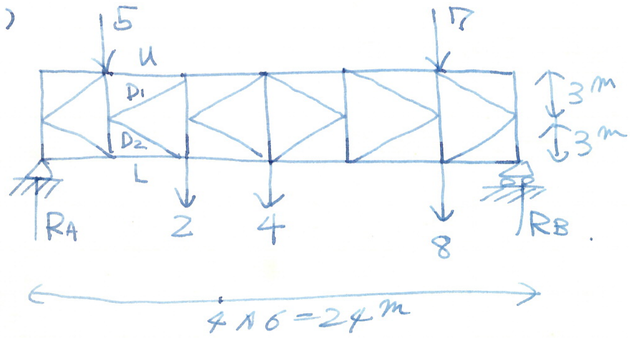
$4R_U + 1.5 \times 4 - R_A \times 8 = 0$

$R_U = -\frac{18}{4}$ //

$\Sigma H = 3 + (-\frac{18}{4}) + R_D \cos 45° = 0.$

$\frac{\sqrt{2}}{2} R_D = \frac{6}{4}$ // $\quad R_D = \frac{3\sqrt{2}}{2}$ //

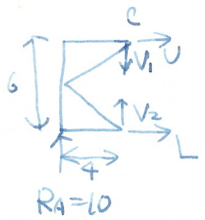
(ex)



$-24R_A + 5\times20 + 2\times16 + 4\times12 + 15\times4 = 0$

$\begin{cases} R_A = 10 \\ R_B = 26 - 10 = 16 \end{cases}$

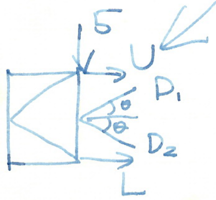
$\Sigma H = U + L = 0 \quad \therefore U = -L.$



$\Sigma M_c = L \times 6 - 10 \times 4 = 0 \quad \therefore L = 20/3$

$\Sigma H = U + L + D_1 \cos\theta + D_2 \cos\theta = 0$

$\therefore (D_1 + D_2)\cos\theta = 0$



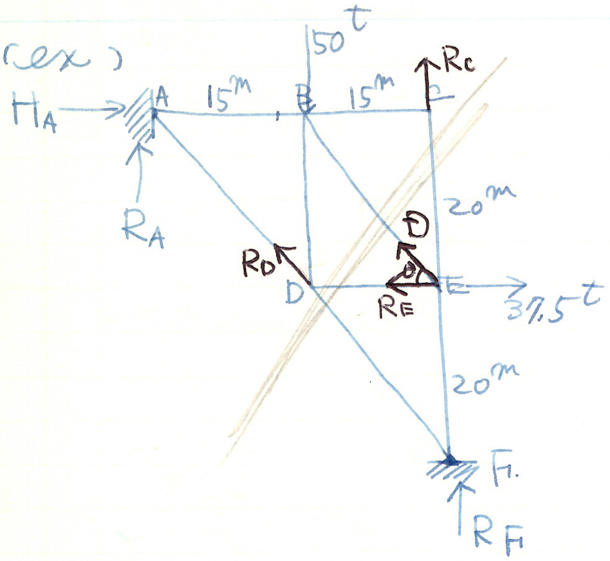
$D_1 + D_2 = 0 \quad \therefore D_1 = -D_2.$

$\Sigma V = 5 - 10 - D_1 \sin\theta + D_2 \sin\theta = 0$

$D_1 = -\frac{5}{2\sin\theta} = \frac{-5}{2\times 3/5}$

$= -25/6$

(ex)



$H_A = 37.5^t$ //

$\Sigma M_A = 50 \times 15 - 37.5 \times 20 - R_{F_1} \times 30 = 0$

$R_F = 0$ //

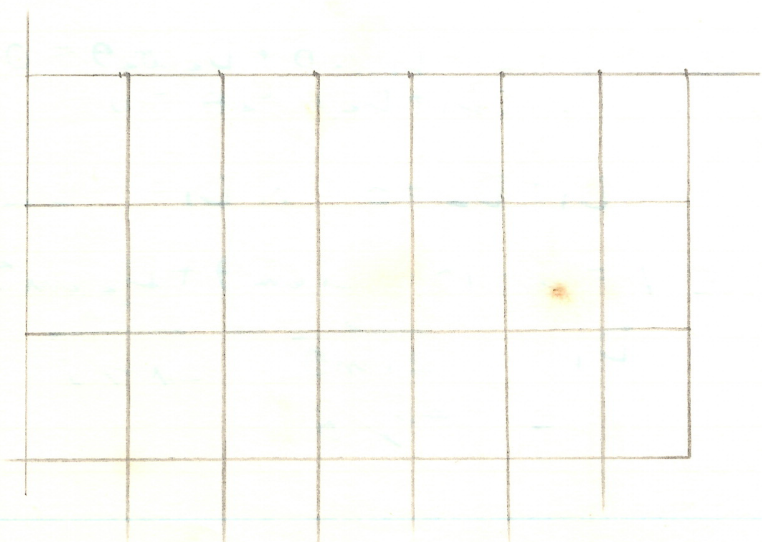
$R_A = 50^t$ //

$$\begin{cases} \Sigma H = 37.5 - R_E - \mathrm{D}\cos\theta - R_D \cos\theta = 0 & \text{①} \\ \Sigma V = R_F + R_C + \mathrm{D}\sin\theta + R_D \sin\theta = 0 & \text{②} \end{cases}$$

$R_{F_1} = 0$ より　$R_D = 0$　$R_C = 0$　.　$R_E = 3.75$.

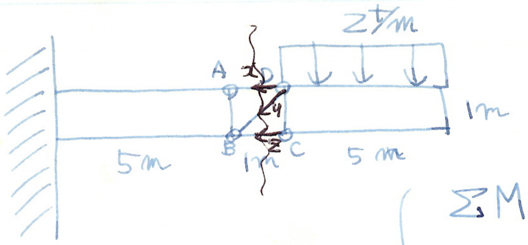
① より $\Sigma H = -\mathrm{D}\cos\theta = 0$　$\therefore \mathrm{D} = 0$.

② より　$\mathrm{D}\sin\theta = 0$　$\therefore \mathrm{D} = 0$.



(ex)

ⓐ



$$\Sigma M_D = z \times 1 + (2 \times 5) \times \frac{5}{2} = 0$$

$$z = -25$$

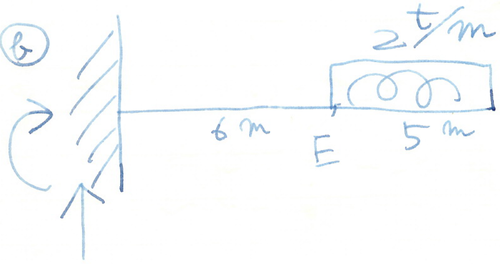
$$\Sigma M_B = -x \times 1 + (2 \times 5) \times (\frac{5}{2} + 1) = 0$$

$$x = 35.$$

$$\Sigma V = y \sin 45° + (2 \times 5) = 0$$

$$y = -\frac{10}{1/\sqrt{2}} = -10\sqrt{2}$$

ⓑ



$$M_E = -(2 \times 5) \times \frac{5}{2} = -2.5$$

$$S_E = 10$$

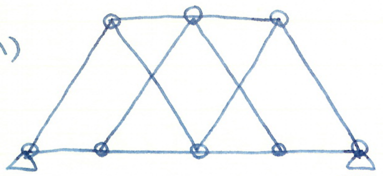
104 cm

104 cm

(11章の問題)

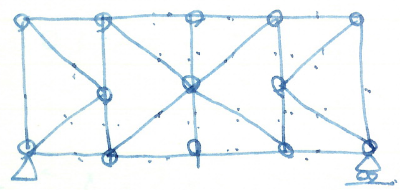
(1)

(a)



支点反力 $r=3$: 外的安定かつ静定。
部材の総数 $m=12$.
節点の 〃 $j=8$.
$2j-3=13 > m=12$ 内的不安定

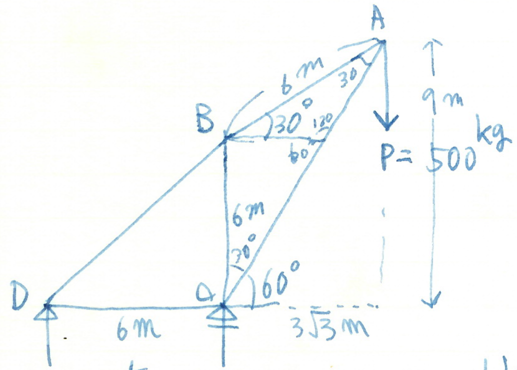
(b)



$r=3$ → 外的安定かつ静定
$m=\cancel{25}\ 24$
$j=13$
$m-(2j-3)$
$=\cancel{25}\ 24-(26-3)\ \cancel{\neq 0}$
$=1$

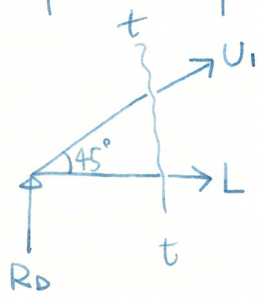
内的安定かつ一次不静定

(2)



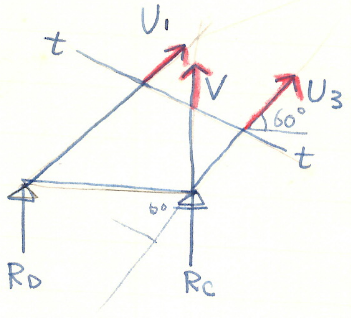
$\Sigma M_D = 6R_C - 3(2+\sqrt{3})500 = 0$

$R_C = (2+\sqrt{3})250^{kg}$
$R_D = 500-(3+\sqrt{3})250$
$= -([illegible]+\sqrt{3})250^{kg}$



$\frac{U_1}{\sqrt{2}} = -R_D \qquad \therefore U_1 = 250\sqrt{6}^{kg}$

$L = -\frac{U_1}{\sqrt{2}} = -250\sqrt{3}^{kg}$

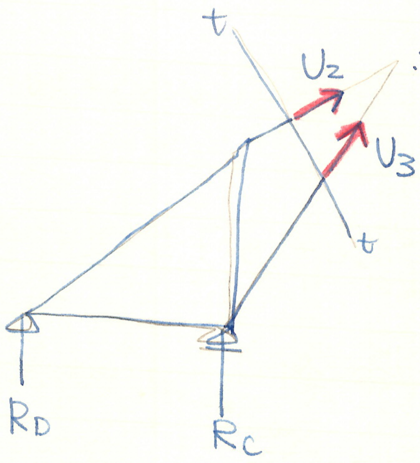


$$\cancel{\Sigma M_C = 6R_D + \frac{6}{\sqrt{2}}U_1 = 0}$$

$$\Sigma M_D = 6(V + R_C) + 3\sqrt{3}\,U_3 = 0.$$

$$2V + \sqrt{3}\,U_3 + 3\,(2+\sqrt{3})\overset{500}{\cancel{750}} = 0$$

$$\Sigma H = U_1\cos 45^\circ + U_3\cos 60^\circ = 0.$$



$$\therefore\ U_3 = -\frac{\cos 45^\circ}{\cos 60^\circ}U_1$$

$$= -\frac{1/\sqrt{2}}{1/2}\cdot 250\sqrt{6}$$

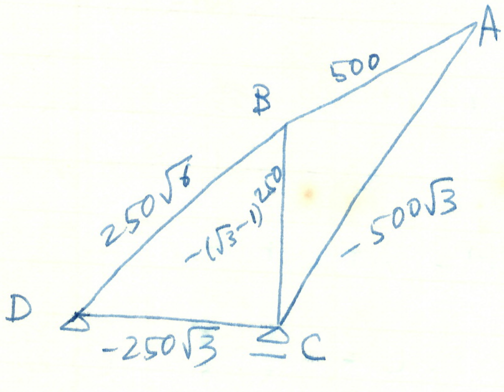
$$= -500\sqrt{3}\ \text{kg}\ //$$

$$V = -\frac{1}{2}\left\{(2+\sqrt{3})500\,\cancel{3} - 1500\right\}$$

$$= -(\sqrt{3}-1)250\ \text{kg}\ //$$

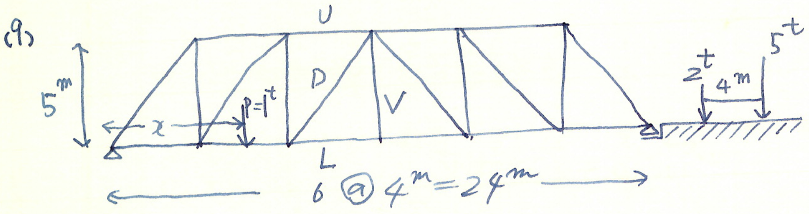
$$\frac{U_2}{\cancel{3}\sqrt{3}} + \frac{U_3}{2} = 0.$$

$$\therefore\ U_2 = -\frac{U_3}{\sqrt{3}} = \cancel{250\sqrt{6}}\,500\ \text{kg}\ //$$



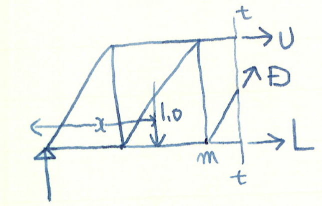
Ans //

(9)

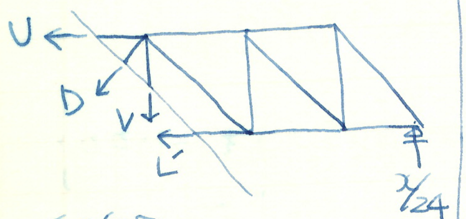


Inf. line を求める。

$R_A = (24-x)/24$. $R_B = x/24$



$1 - x/24$



$0 \leqq x \leqq 8$.

$$\Sigma V = V + D \cdot \frac{5}{\sqrt{41}} - \frac{x}{24} = 0$$

$$\therefore V = \frac{x}{24} - \frac{x}{24} = 0$$

$12 \leqq x \leqq 24$

$$\Sigma V = V + D \cdot \frac{5}{\sqrt{41}} - \frac{x}{24} + 1 = 0.$$

$$\therefore V = \frac{x}{24} - 1 - \left(\frac{x}{24} - 1\right)$$

$0 \leqq x \leqq 8$

$$\Sigma M_m = 8(1 - x/24) - 1(8 - x) + 5U = 0$$

$$\therefore U = -\frac{2}{15}x$$

$$\Sigma V = 1 - x/24 - 1.0 + D \times \frac{5}{\sqrt{41}} = 0$$

$$\therefore D = \frac{\sqrt{41}}{120}x .$$

$$\Sigma H = U + L + D \times \frac{4}{\sqrt{41}} = 0$$

$$\therefore L = \frac{2}{15}x - \frac{x}{30} = \frac{x}{10}$$

$12 \leqq x \leqq 24$.

$$\Sigma M_m = 8(1 - x/24) + 5U = 0$$

$$\therefore U = \frac{8}{5}\left(\frac{x}{24} - 1\right) = \frac{x}{15} - \frac{8}{5}$$

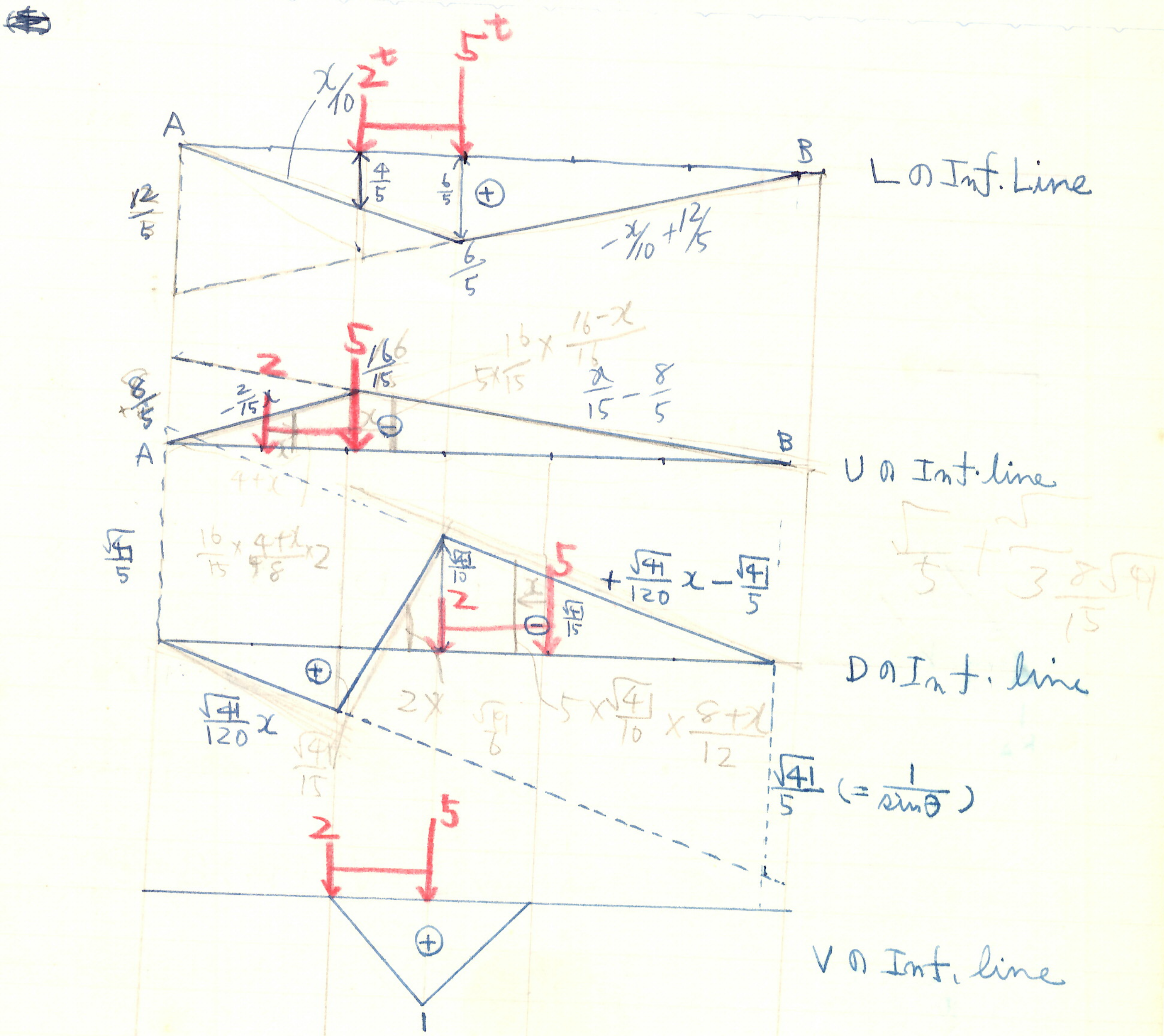
$$\Sigma V = 1 - x/24 + D \times \frac{5}{\sqrt{41}} = 0$$

$$\therefore D = \frac{\sqrt{41}}{5}\left(\frac{x}{24} - 1\right)$$

$$\Sigma H = U + L + D \times \frac{4}{\sqrt{41}} = 0$$

$$\therefore L = -\frac{x}{15} + \frac{8}{5} - \frac{4}{5}\left(\frac{x}{24} - 1\right)$$

$$= -\frac{x}{10} + \frac{12}{5}$$



$L_{max} = 2 \times 4/5 + 5 \times 6/5 = 7.6^t$

$V_{max} = 2 \times 0 + 5 \times 1 = 5.0^t$

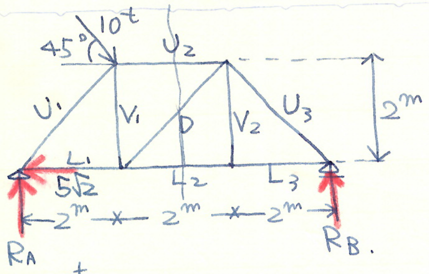
$$|U| = \frac{16}{15} \times \frac{4+x}{8} \times 2 + \frac{16}{15} \times \frac{16-x}{16} \times 5 = \frac{4}{15}(4+x) + \frac{1}{3}(16-x)$$

$$= -\frac{1}{15}x + \frac{16}{15} + \frac{16}{3} = -\frac{1}{15}x + \frac{96}{15} = -\frac{x}{15} + \frac{32}{5}$$

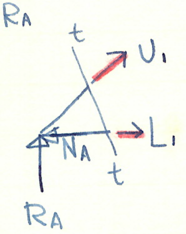
$U_{max} = U_{x=0} = \frac{32}{5} = 6.4$ $\quad U_{max} = -6.4^t$

$D_{max} = 5 \times \frac{\sqrt{41}}{15} + 2 \times \frac{\sqrt{41}}{10} = \frac{8\sqrt{41}}{15} = 3.4^t$ …答

(5)

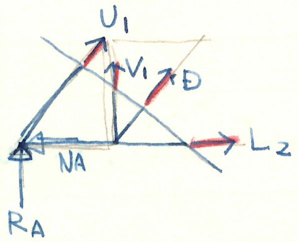


$$\Sigma M_A = 2\sqrt{2}\cdot 10 - 6R_B = 0$$
$$R_B = 10\sqrt{2}/3$$
$$R_A = 5\sqrt{2} - 10\sqrt{2}/3 = 5\sqrt{2}/3$$



$$\frac{U_1}{\sqrt{2}} = -R_A \qquad \therefore U_1 = -\sqrt{2}R_A = -\frac{10}{3}{}^{t}$$

$$L_1 + \frac{U_1}{\sqrt{2}} = N_A \qquad \therefore L_1 = 5\sqrt{2} + \frac{5\sqrt{2}}{3} = \frac{20}{3}\sqrt{2}$$



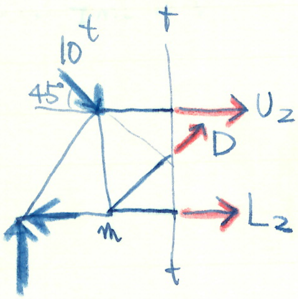
$$\Sigma M_A = 2V_1 + \sqrt{2}D = 0 \quad \text{—— } (*)$$

$$\cancel{\Sigma H = L_2 + D/\sqrt{2} + U_1/\sqrt{2} - N_A = 0}$$

$$\therefore \cancel{L_2 + D/\sqrt{2} - 5\sqrt{2}/3 - 5\sqrt{2} = 0} \text{ — } ②$$

$$\cancel{\Sigma V = R_A + U_1/\sqrt{2} + V_1 + D_1/\sqrt{2} = 0}$$

$$\cancel{-5\sqrt{2}/3 + 5\sqrt{2}/3 + V_1 + D/\sqrt{2} = 0}$$
— ③



$$\Sigma M_m = 2U_2 + 2R_A + 10\sqrt{2} = 0$$
$$\therefore U_2 = -5\sqrt{2} - 5\sqrt{2}/3 = -\frac{20\sqrt{2}}{3}{}^{t}$$

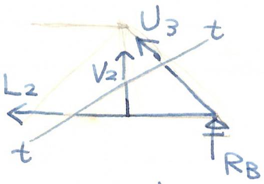
$$\Sigma H = L_2 + D/\sqrt{2} - \frac{20\sqrt{2}}{3} + \cancel{5\sqrt{2} - 5\sqrt{2}} = 0$$

$$\therefore L_2 + D/\sqrt{2} = 20\sqrt{2}/3 \text{ —— } ①$$

$$\Sigma V = 5\sqrt{2}/3 - 5\sqrt{2} + D/\sqrt{2} = 0$$
$$\therefore D = 20/3$$
$$\therefore L_2 = 10\sqrt{2}/3$$

$(*)$より

$$V_1 = -\frac{D}{\sqrt{2}} = -\frac{10\sqrt{2}}{3}{}^{t}$$

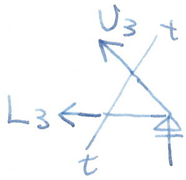


$L_2 = 10\sqrt{2}/3$

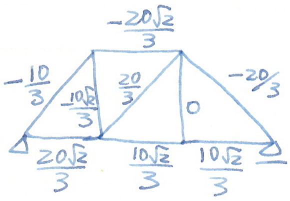
$\Sigma M_B = 2V_2 = 0 \quad \therefore V_2 = 0$

$\Sigma H = L_2 + U_3/\sqrt{2} = 0.$

$\therefore U_3 = -\sqrt{2}L_2 = -20/3$

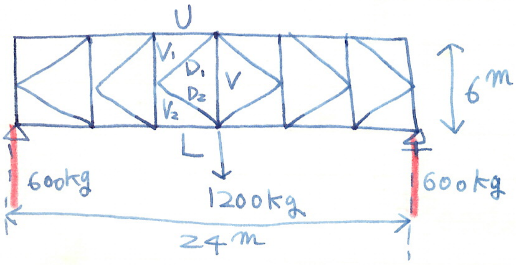


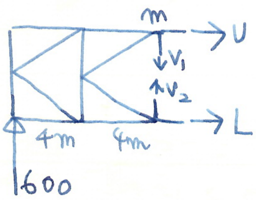
$L_3 + U_3/\sqrt{2} = 0 \quad \therefore L_3 = \frac{U_3}{\sqrt{2}} = +10\sqrt{2}/3$



Ans.

(8)



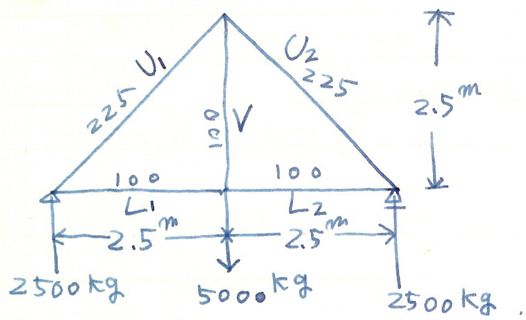


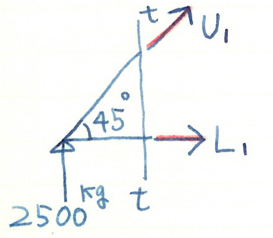
$\Sigma M_m = 8\times600 - 6\times L = 0 \quad \therefore L = 800^{kg}$

$\therefore U = -L = -800^{kg}.$

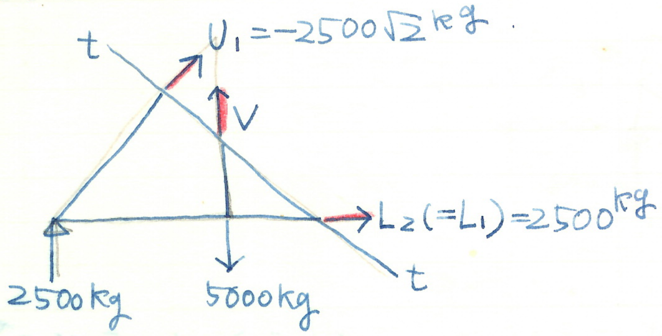
(10)

$E = 100000\ \mathrm{kg/cm^2}$





$$\begin{cases} U_1 = 2500\sqrt{2}\ \mathrm{kg} = U_2 \\ L_1 = 2500\ \mathrm{kg} = L_2 \end{cases}$$



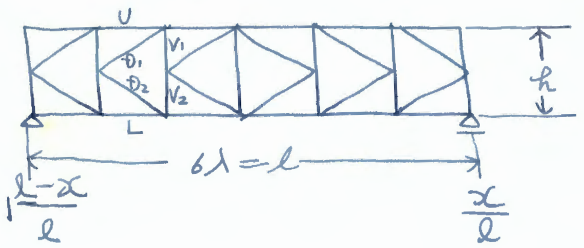
$V = 5000^{kg}$

Vによる伸び $\quad \Delta l_1 = \dfrac{Pl}{EA} = \dfrac{5000 \times 250}{100\,000 \times 100} = \dfrac{125}{1000} = 0.125\ \mathrm{cm}$

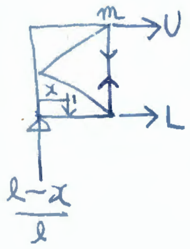
U_1の伸び (↗方向) $\quad \Delta l_2 = \dfrac{Pl}{EA} = \dfrac{-2500\sqrt{2} \times 100\sqrt{2}}{100000 \times 225} = -\dfrac{1}{45} = -0.022$

$\therefore \Delta l_2' = -0.022\sqrt{2}$

(11)



(i)



$0 \leqq x \leqq \lambda.$

$$U = -L,\ \Sigma M_m = \lambda R_A - (\lambda - x)\cdot 1 - hL = 0$$

$$\lambda \cdot \frac{l-x}{l} - \lambda + x - hL = 0 \quad \therefore L = (\cancel{})\frac{x}{h}\left(1 - \frac{\lambda}{l}\right)$$

$$U = -\frac{x}{h}\left(1 - \frac{\lambda}{l}\right)$$

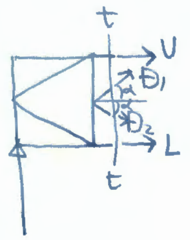
$\lambda \leqq x \leqq 6\lambda.$

$$U = -L,\ \Sigma M_m = \lambda R_A - hL = 0$$

$$\lambda\left(1 - \frac{x}{l}\right) - hL = 0 \qquad \therefore L = \frac{\lambda}{h}\left(1 - \frac{x}{l}\right)$$

$$U = -\frac{\lambda}{h}\left(1 - \frac{x}{l}\right)$$

(ii)



$$\Sigma H = U + L + (D_1 + D_2)\cos\alpha = 0.$$

$$\therefore D_1 + D_2 = 0 \quad \text{———} \ ①$$

$0 \leqq x \leqq \lambda.$

$$\Sigma V = 1 - x/l - 1 + (D_1 - D_2)\sin\alpha = 0.$$

$$\therefore D_1 - D_2 = \frac{x}{l \sin\alpha}$$

$$\therefore D_1 = \frac{x}{2l\sin\alpha}, \quad D_2 = -\frac{x}{2l\sin\alpha}$$

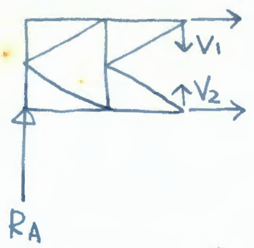
$2\lambda \leqq x \leqq 6\lambda.$

$$\Sigma V = 1 - x/l + (D_1 - D_2)\sin\alpha = 0$$

$$\therefore D_1 - D_2 = -\frac{1}{\sin\alpha}\left(1 - \frac{x}{l}\right)$$

$$\therefore D_1 = -\frac{1}{2\sin\alpha}\left(1 - \frac{x}{l}\right), \quad D_2 = \frac{1}{2\sin\alpha}\left(1 - \frac{x}{l}\right)$$

(iii)



$0 \leqq x \leqq 2\lambda$.

$V_1 - V_2 + 1 - R_A = 0$.

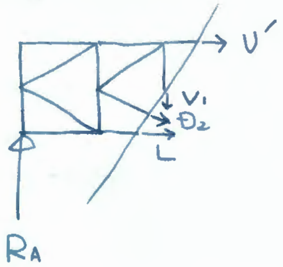
$\therefore V_2 = V_1 + 1 - R_A$.

$0 \leqq x \leqq \lambda$: $V_2 = \frac{x}{2l}$ $\qquad$ $\lambda \leqq x \leqq 2\lambda$: $V_2 = \frac{x}{2l} + \frac{1}{2}$

$2\lambda \leqq x \leqq 6\lambda$: $V_1 - V_2 - R_A = 0$.

$\therefore V_2 = V_1 - R_A = -\frac{1}{2} + \frac{x}{2l}$

(iv)



$0 \leqq x \leqq \lambda$.

$\Sigma V = R_A - 1 - V_1 - D_2 \sin\alpha = 0$

$\therefore V_1 = R_A - 1 - D_2 \sin\alpha$

$= -\frac{x}{l} + \frac{x}{2l} = -\frac{x}{2l}$

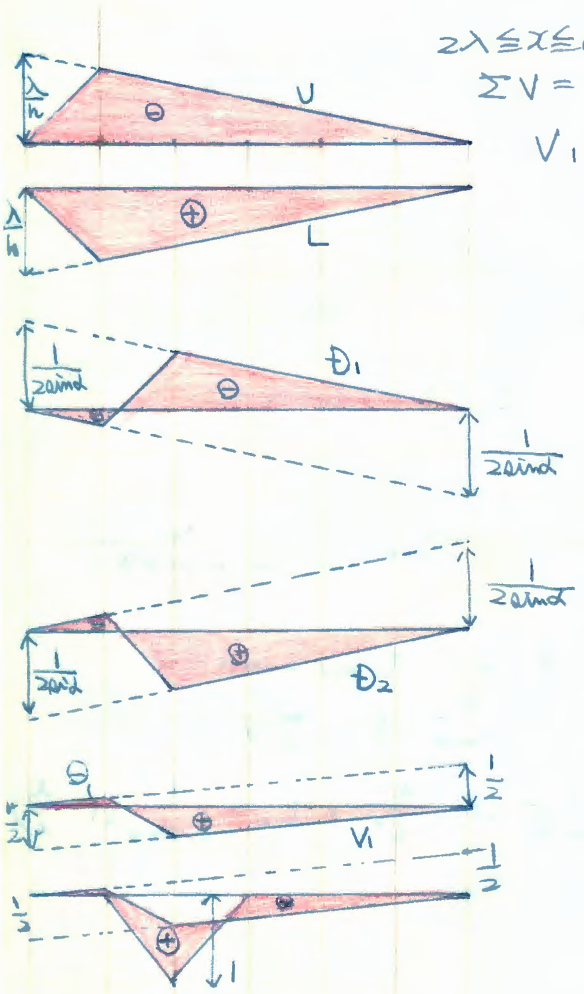
$2\lambda \leqq x \leqq 6\lambda$.

$\Sigma V = R_A - V_1 - D_2 \sin\alpha = 0$

$V_1 = R_A - D_2 \sin\alpha$

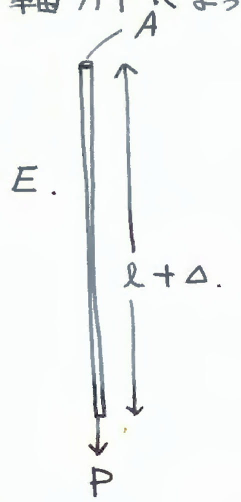
$= 1 - \frac{x}{l} - \frac{1}{2}\left(1 - \frac{x}{l}\right)$.

$= \frac{1}{2} - \frac{x}{2l}$ $\qquad$ $1 - \frac{x}{l}$



$0 \leqq x \leqq \lambda$ $\quad$ $V_2 = V_1 + 1 - R_A = V_1 + \frac{x}{l}$

▨ 軸力Pによってたくわえられるひずみエネルギー.



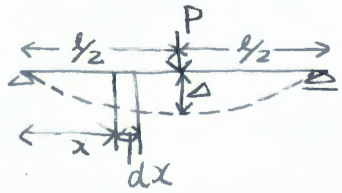
外力仕事 $W = \frac{1}{2} P\Delta$

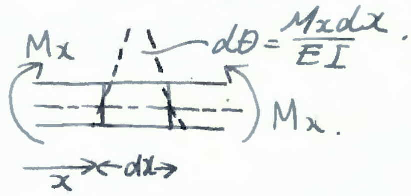
フックの法則より.

$$\frac{P}{A} = E \cdot \frac{\Delta}{\ell} \qquad \therefore \Delta = \frac{P\ell}{EA}.$$

ひずみエネルギー $U = \dfrac{P^2 \ell}{2EA}$

▨ 梁における曲げモーメントによるひずみエネルギー.





M_xによる外力仕事 $dW = \frac{1}{2} M_x d\theta = \dfrac{M_x^2 dx}{2EI} = dU$

$$\therefore U = \int_0^\ell dU = \int_0^\ell \frac{M_x^2 dx}{2EI}.$$

$$M_x = \frac{P}{2}x - \underbrace{\left\{P\left(x - \frac{\ell}{2}\right)\right\}}_{x \leqq \ell/2 \text{ のとき} 0}$$

$$\therefore U = \frac{1}{2EI}\int_0^\ell \left[\frac{P}{2}x - \left\{P\left(x - \frac{\ell}{2}\right)\right\}\right]^2 dx.$$

$$= \frac{1}{2EI}\left[\int_0^\ell \frac{P^2}{4}x^2 dx - \int_{\ell/2}^\ell \left\{P^2 x\left(x - \frac{\ell}{2}\right)\right\} dx + \int_{\ell/2}^\ell P^2\left(x - \frac{\ell}{2}\right)^2 dx\right]$$

$$= \frac{1}{2EI}\left[\frac{P^2\ell^3}{12} - \frac{5P^2\ell^3}{48} + \frac{P^2\ell^3}{24}\right] = \frac{P^2\ell^3}{96EI}$$

$$W = \frac{1}{2}P\Delta = \frac{P^2 l^3}{96EI} = U \text{ より}$$

$$\Delta = \frac{Pl^3}{48EI} \quad //$$

(ex).

$$M_x = M - \frac{M}{l}x \text{ より.}$$

$$U = \int_0^l \frac{M_x^2}{2EI}dx = \frac{M^2}{2EI}\cancel{\left(l - \frac{l}{2}\right)} \cdot \left[x - \frac{x^2}{l} + \frac{x^3}{3l^2}\right]_0^l$$

$$= \frac{M^2 l}{6EI}$$

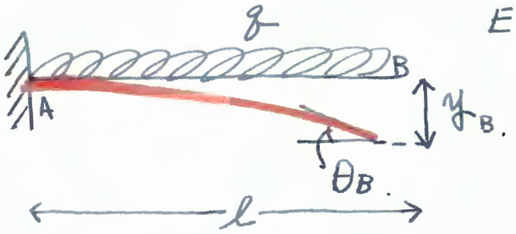
$$\therefore \quad \frac{1}{2}M\theta_A = \frac{M^2 l}{6EI} \qquad \therefore \theta_A = \frac{Ml}{3EI}$$

参 $\begin{cases} U_S = \int_0^l \frac{k S_x^2}{2GA}dx \\ U_N = \int_0^l \frac{N_x^2}{2EA}dx. \end{cases}$ 梁では無視.

単位荷重法. unit load method

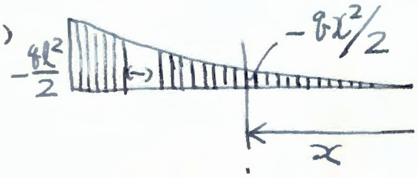
$$\Delta_A = \int_0^l \frac{M_x m_x}{EI} dx.$$

ex) $EI = const.$



y_B, θ_B を求めよ。

(1)



M_x 図.

$$\begin{cases} M_x = -qx^2/2 \\ m_x = -x \end{cases}$$

m_x 系.

$$\therefore y_B = \int_0^l \frac{(-qx^2/2)(-x)}{EI} dx.$$

$$= \frac{q}{2EI} \cdot \frac{l^4}{4} = \frac{ql^4}{8EI}$$

(2) $M_x = -qx^2/2$.

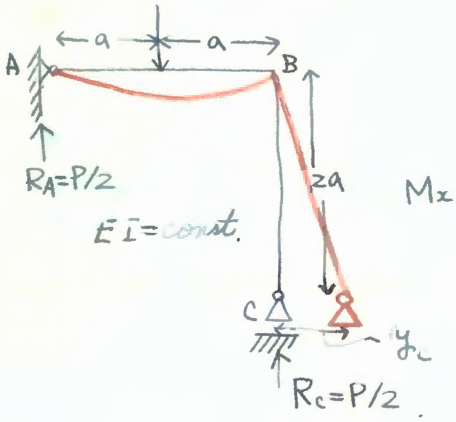
m_x 系.

$m_x = -1$.

$$\therefore \theta_B = \int_0^l \frac{(-qx^2/2)(-1)}{EI} dx$$

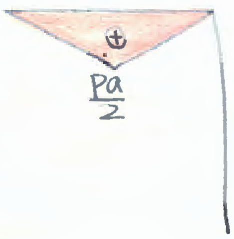
$$= \frac{ql^3}{6EI}$$

(ex)

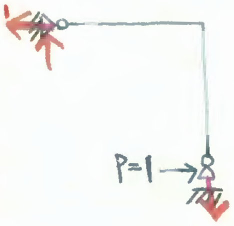


Mx系.

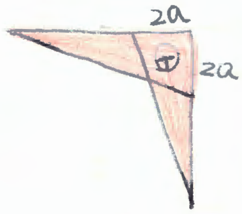
y_c を求めよ。



Mx図.



mx系



mx図.

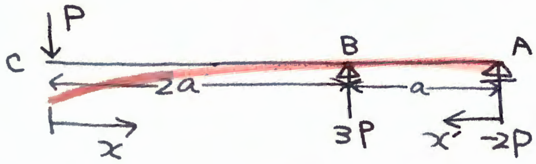
$$\therefore y_c = \int_0^{2a} \frac{M_x m_x}{EI} dx$$

$$= \int_0^{a} \frac{P\frac{x}{2}\cdot x}{EI} dx + \int_a^{2a} \frac{\left(-P\frac{x}{2}+Pa\right)x}{EI} dx + \cancel{\int_0^{2a} \frac{0\cdot x}{EI} dx}$$

$$= \frac{Pa^3}{6EI} + \frac{P}{EI}\left[\frac{ax^2}{2} - \frac{x^3}{6}\right]_a^{2a} = \frac{Pa^3}{2EI}$$

▨ エネルギー不変の法則.

(ex1) C点, 変位を求めよ。



曲げモーメント. $\overline{CB}$ $\quad M_x = -Px$

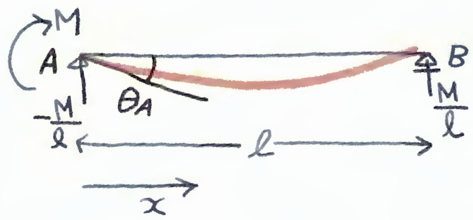
$\overline{AB}$ $\quad M_{x'} = -2Px'$

故に. $$\frac{P\Delta_c}{2} = \int_0^{2a} \frac{M_x^2}{2EI} dx + \int_0^{a} \frac{M_{x'}}{2EI} dx'$$

$$= \int_0^{2a} \frac{P^2 x^2}{2EI} dx + \int_0^{a} \frac{2P^2 x'^2}{EI} dx'$$

$$= \frac{8P^2a^3}{6EI} + \frac{2P^2a^3}{3EI} = \frac{2P^2a^3}{EI} \qquad \therefore \Delta_c = \frac{4Pa^3}{EI}$$

(ex2) A点たわみ角を求めよ。



$$M_x = M - \frac{M}{l}x.$$

ひずみエネルギー

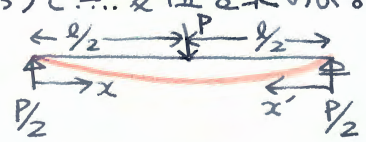
$$U = \int_0^l \frac{M_x^2}{2EI} = \int_0^l \frac{M^2(1-\frac{x}{l})^2}{2EI} dx.$$

$$= \frac{M^2}{2EI}\left[-l\frac{1}{3}(1-\frac{x}{l})^3\right]_0^l = \frac{M^2 l}{6EI}$$

$$\frac{1}{2}M\theta_A = \frac{M^2 l}{6EI}$$

$$\therefore \theta_A = \frac{Ml}{3EI}$$

(ex3) C点. 変位を求めよ。



$$U = \int_0^{l/2} \frac{\frac{P^2}{4}x^2}{2EI} dx + \int_0^{l/2} \frac{\frac{P^2}{4}x'^2}{2EI} dx'$$

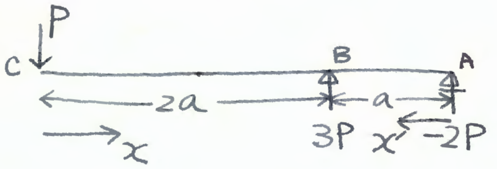
$$= \frac{P^2 l^3}{192EI} \times 2 = \frac{P^2 l^3}{96EI}$$

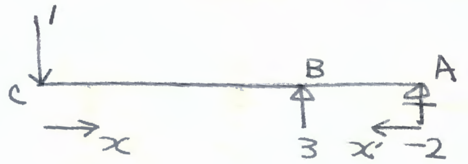
$$\frac{P\Delta_c}{2} = \frac{P^2 l^3}{96EI} \qquad \therefore \Delta_c = \frac{Pl^3}{48EI}$$

変形量を求める。

◎仮想仕事の原理.(単位荷重法)

(ex4) C点変位を求めよ。

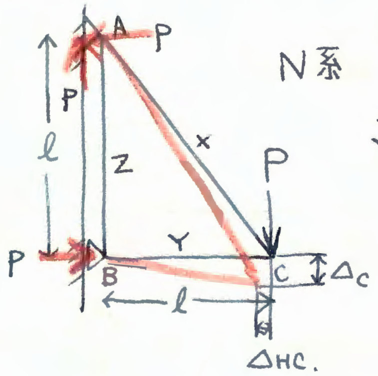




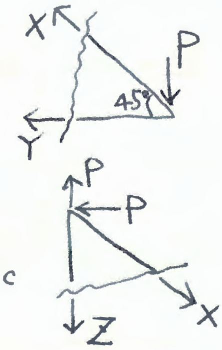
M_x系 $\begin{cases} M_x = -Px \\ M_{x'} = -2Px' \end{cases}$　　m_x系. $\begin{cases} m_x = -x \\ m_{x'} = -2x' \end{cases}$

$$\Delta_c = \int_0^{2a} \frac{M_x m_x}{EI} dx + \int_0^{a} \frac{M_{x'} m_{x'}}{EI} dx'$$
$$= \int_0^{2a} \frac{Px^2}{EI} dx + \int_0^{a} \frac{4Px'^2}{EI} dx'$$
$$= \frac{8Pa^3}{3EI} + \frac{4Pa^3}{3EI} = \frac{4Pa^3}{EI}$$

(ex5)　$EA = const$　(イ) Δ_c　(ロ) Δ_{HC} を求めよ。



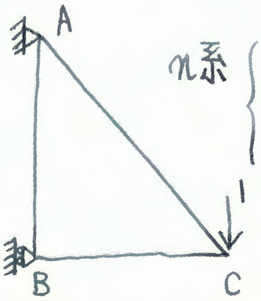
N系



$X \sin 45° = P$　∴ $X = \sqrt{2}P$
$Y + X \cos 45° = 0$　∴ $Y = -P$

$Z = 0$.

(イ)



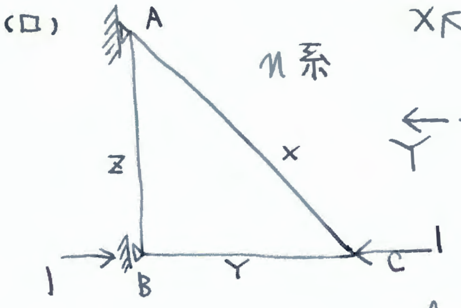
n系 $\begin{cases} X = \sqrt{2} \\ Y = -1 \\ Z = 0 \end{cases}$

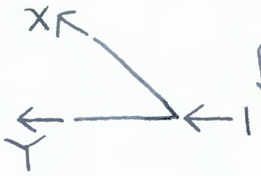
部材	長さ	N	n	Nnl
AB	l	0	0	0
BC	l	$-P$	-1	Pl
CA	$\sqrt{2}l$	$\sqrt{2}P$	$\sqrt{2}$	$2\sqrt{2}Pl$

計 $(2\sqrt{2}+1)Pl$

$$\Delta_c = \Sigma \frac{Nnl}{EA} = \frac{(2\sqrt{2}+1)Pl}{EA}$$

(ロ)





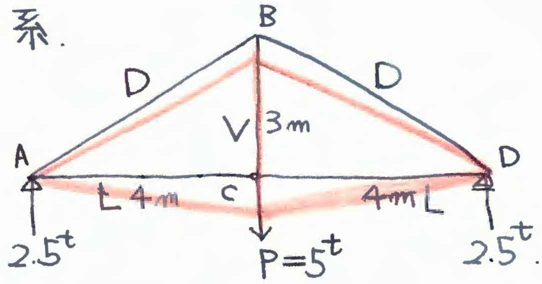
$$\begin{cases} X=0 \\ Y=-1 \\ Z=0. \end{cases}$$

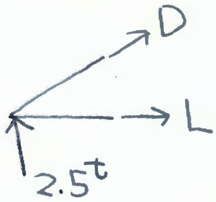
部材	長さ	N	n	Nnl
AB	l	0	0	0
BC	l	$-P$	-1	Pl
CA	$\sqrt{2}l$	$\sqrt{2}P$	0	0
			計.	Pl.

$$\Delta_{Hc} = \frac{Pl}{EA}$$

(ex6) Δ_c を求めよ。

N系.



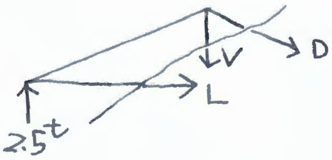


$\frac{3}{5}D + 2.5 = 0$

$\therefore D = -4.17^t.$

$\frac{4}{5}D + L = 0$

$\therefore L = +3.33^t.$



$V + \frac{3}{5}D = 2.5$

$\therefore V = 2.5 - \frac{3}{5}D = 5.00^t.$

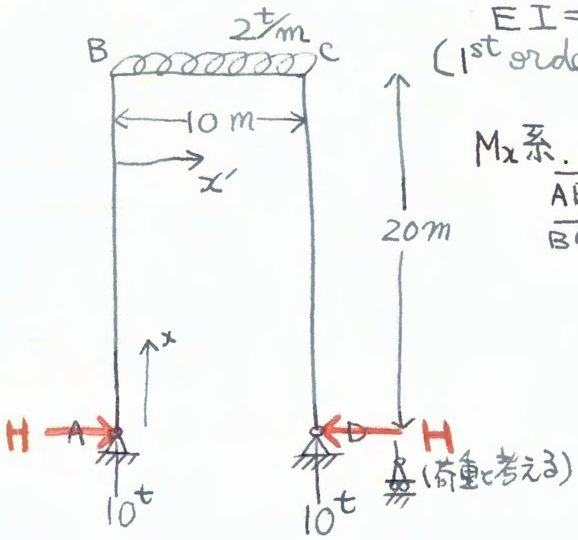
部材	長さ	N	n	Nnl
AB	5	-4.17	-0.83	17.31
AC	4	3.33	0.67	8.92
BC	3	5.00	1.00	15.00
BD	5	-4.17	-0.83	17.31
CD	4	3.33	0.67	8.92
			計.	67.46

$$\therefore \Delta_c = \frac{67.46}{EA} \text{ (m)}$$

(ex7) 水平反力Hを求めよ。

$EI = const.$

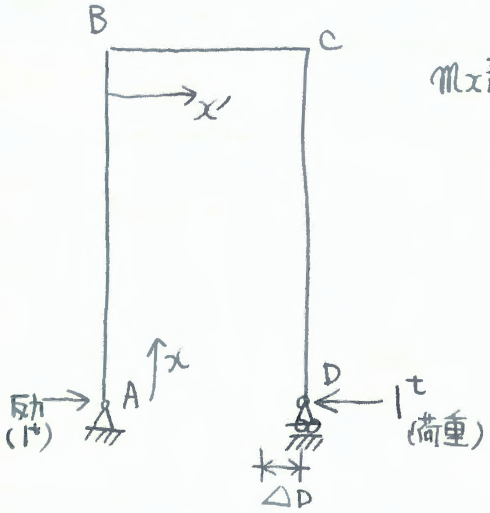
(1st order indeterminate rahmen)



M_x系.

$\overline{AB}$ $M_x = -Hx$

$\overline{BC}$ $M_{x'} = 10x' - 20H - x'^2$



$\mathfrak{M}_x$系

$\overline{AB}$ $\mathfrak{M}_x = -x$

$\overline{BC}$ $\mathfrak{M}_{x'} = -20$

$$\Delta_D = 2\int_0^{20} \frac{Hx^2}{EI} dx + \int_0^{10} \frac{20(x'^2 - 10x' + 20H)}{EI} dx'$$

$$= \frac{16000H}{3EI} + \frac{20}{EI}\left(\frac{1000}{3} - 500 + 200H\right) = \frac{1}{3EI}(28000H - 10000) = 0$$

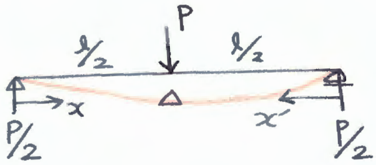
$$\therefore H = 0.357^t$$

たわみ、たわみ角を求める。

カスティリアノの定理（第1）

$$\frac{\partial U}{\partial P_1}=\Delta_1 \quad , \quad \frac{\partial U}{\partial M_1}=\theta_1$$

(ex8)



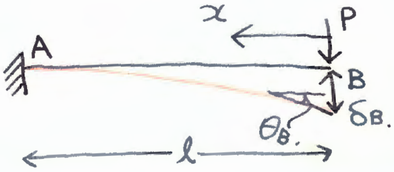
$$U=\int_0^{l/2}\frac{(\frac{P}{2}x)^2}{2EI}+\int_0^{l}\frac{(\frac{P}{2}x')^2}{2EI}dx'$$

$$=2\int_0^{l/2}\frac{P^2x^2}{8EI}=\frac{P^2l^3}{96\cancel{384}EI}$$

$$\frac{\partial U}{\partial P}=\frac{Pl^3}{48EI}=\Delta \quad //$$

(ex9)

(1)



$$M_x=-Px$$

$$U=\int_0^l\frac{P^2x^2}{2EI}dx$$

$$\therefore \frac{\partial U}{\partial P}=\int_0^l\frac{Px^2}{EI}dx=\frac{Pl^3}{3EI}=\delta_B \quad //$$

(2)

$$M_x=-Px-\overline{M}$$

$$U=\int_0^l\frac{(Px+\overline{M})^2}{2EI}dx \quad \therefore \frac{\partial U}{\partial \overline{M}}=\int_0^l\frac{2(Px+\overline{M})}{2EI}dx$$

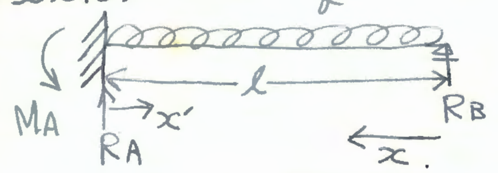
$$=\frac{\cancel{P}1}{EI}\left(\frac{Pl^2}{2}+\overline{M}l\right)$$

ここで $\overline{M}=0$ とおけば

$$\theta_B=\frac{Pl^2}{2EI} \quad //$$

最小仕事の定理. ⟵ 不静定量を求める.

(ex10)



$R_A + R_B = ql$. $R_B = ql - R_A$. ——— ①

$\Sigma M_B = R_A l - \frac{ql^2}{2} - M_A = 0$. $\therefore M_A = R_A l - \frac{ql^2}{2}$ ——— ②

不静定量R_Bとすると

$$M_x = R_B x - \frac{q}{2}x^2 .$$

①より・・・

$$\therefore U = \int \frac{1}{2EI}\left(R_B x - \frac{qx^2}{2}\right)^2 dx .$$

$$\cancel{= \int \frac{1}{2EI}\left\{(ql - R_A)x - \frac{qx^2}{2}\right\} dx}$$

$$\delta_B = \frac{\partial U}{\partial R_B} = \int_0^l \frac{1}{2EI} \cdot 2x\left(R_B x - \frac{qx^2}{2}\right) dx .$$

$$= \frac{1}{EI}\left(\frac{R_B l^3}{3} - \frac{ql^4}{8}\right) = 0 \quad \therefore R_B = \frac{3ql}{8}$$

不静定量R_Aとすると

②より・・・

$$M_x = R_A x' - M_A - \frac{q}{2}x'^2 .$$

$$= R_A x' - \left(R_A l - \frac{ql^2}{2}\right) - \frac{q}{2}x'^2$$

$$\therefore U = \int_0^l \frac{1}{2EI}\left\{R_A x' + \left(\frac{ql^2}{2} - R_A l\right) - \frac{q}{2}x'^2\right\}^2 dx'$$

$$\therefore \frac{\partial U}{\partial R_A} = \int_0^l \frac{x'-l}{EI}\left\{R_A x' + \left(\frac{ql^2}{2} - R_A l\right) - \frac{q}{2}x'^2\right\} dx'$$

$$= \int_0^l \frac{1}{EI}\left\{-\frac{q}{2}x'^3 + \left(R_A + \frac{ql}{2}\right)x'^2 + \left(\frac{ql^2}{2} - 2R_A l\right)x' + R_A l^2 - \frac{ql^3}{2}\right\} dx'$$

$$= \frac{1}{EI}\left\{-\frac{ql^4}{8} + \left(R_A + \frac{ql}{2}\right)\frac{l^3}{3} + \left(\frac{ql^2}{2} - 2R_A l\right)\frac{l^2}{2} + R_A l^3 - \frac{ql^4}{2}\right\}$$

$$= \frac{1}{EI}\left(\frac{l^3}{3}R_A - \frac{5ql^4}{24}\right) = 0 = \delta_A \quad \therefore R_A = \frac{5ql}{8}$$

不静定量M_Aとすると.

$$R_A = \frac{1}{l}\left(M_A + \frac{ql^2}{2}\right)$$

$$M_x = \frac{1}{l}\left(M_A+\frac{ql^2}{2}\right)x' - M_A - \frac{q}{2}x'^2$$

$$= -\frac{q}{2}x'^2 + \left(\frac{ql}{2}+\frac{M_A}{l}\right)x' - M_A$$

$$U=\int_0^l \frac{1}{2EI}\left\{-\frac{q}{2}x'^2+\left(\frac{ql}{2}+\frac{M_A}{l}\right)x'-M_A\right\}^2 dx'.$$

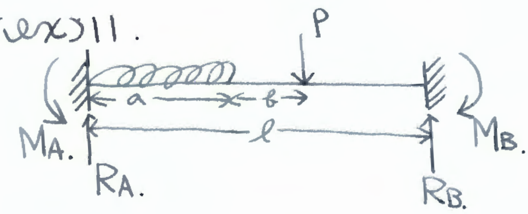
$$\therefore\ \theta_A = \frac{\partial U}{\partial M_A} = \int_0^l \frac{1}{EI}\left\{-\frac{q}{2}x'^2+\left(\frac{ql}{2}+\frac{M_A}{l}\right)x'-M_A\right\}\left(\frac{x'}{l}-1\right)dx'$$

$$=\frac{1}{EI}\int_0^l\left\{-\frac{qx'^3}{2l}+\left(q+\frac{M_A}{l^2}\right)x'^2-\left(\frac{ql}{2}+\frac{2M_A}{l}\right)x'+M_A\right\}dx'$$

$$=\frac{1}{EI}\left\{-\frac{ql^3}{8}+\left(q+\frac{M_A}{l^2}\right)\frac{l^3}{3}-\left(\frac{ql}{2}+\frac{2M_A}{l}\right)\frac{l^2}{2}+M_A l\right\}$$

$$=\frac{1}{EI}\left(\frac{l}{3}M_A-\frac{ql^3}{24}\right)=0 \qquad \therefore\ M_A=\frac{ql^2}{8}$$

(ex)11.



$0 \leqq x \leqq a.\quad M_x = -M_A + R_A x - \frac{q}{2}x^2$

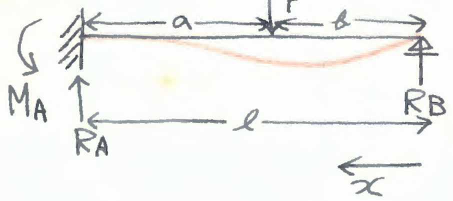
$a \leqq x \leqq a+b\quad M_x = -M_A + R_A x - qa\left(x-\frac{a}{2}\right)$

$a+b \leqq x \leqq l\quad M_x = -M_A + R_A x - qa\left(x-\frac{a}{2}\right) - P\left(x-\overline{a+b}\right)$

$$U=\frac{1}{2EI}\int M_x^2\,dx = f(M_A, R_A)$$

$$\frac{\partial U}{\partial R_A} = \delta_A = 0,\quad \frac{\partial U}{\partial M_A}=\theta_A=0 \longrightarrow \begin{cases} R_A = \\ M_A = \end{cases}$$

(ex.12) 不静定量をR_Bとしてこれを求めよ。



$0 \leqq x \leqq b \qquad M_x = R_B x.$

$b \leqq x \leqq l. \qquad M_x = R_B x - P(x-b)$

$$\therefore\ U = \frac{1}{2EI}\int_0^b R_B^2 x^2 dx + \frac{1}{2EI}\int_b^l \{R_B x - P(x-b)\}^2 dx$$

$$\therefore\ \frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^b R_B x^2 dx + \frac{1}{EI}\int_b^l \{R_B x^2 - P x^2 + P b x\} dx$$

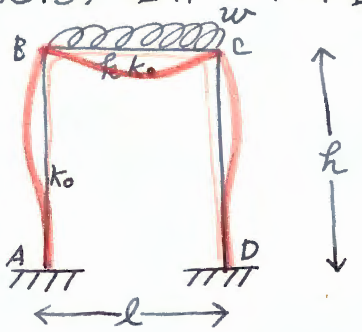
$$= \frac{1}{EI}\left\{\frac{R_B b^3}{3} + \frac{R_B}{3}(l^3 - b^3) - \frac{P}{3}(l^3 - b^3) + \frac{Pb}{2}(l^2 - b^2)\right\}$$

$$= \frac{1}{EI}\left\{\frac{l^3}{3}R_B - \frac{P}{6}(2l^3 + b^3 - 3bl^2)\right\}$$

$$\therefore\ R_B = \frac{P(2l^3 + b^3 - 3bl^2)}{2l^3}$$ //

たわみ角法.

(ex13) 曲げモーメント図をかけ。



$$\psi = 0$$
$$\varphi_A = \varphi_D = 0$$
$$\varphi_C = -\varphi_B \quad (\text{対称})$$

$$M_{AB} = \varphi_B$$
$$M_{BA} = 2\varphi_B$$
$$M_{BC} = k(2\varphi_B - \varphi_B) + C_{BC}$$
$$= k\varphi_B - \frac{wl^2}{12}$$

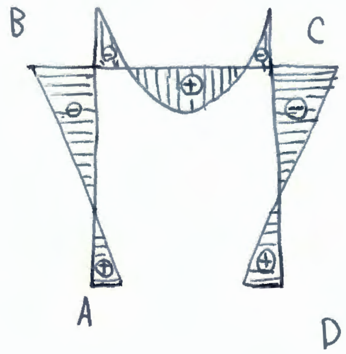
節点方程式.

$$M_{BA} + M_{BC} = 0$$
$$\therefore (2+k)\varphi_B - \frac{wl^2}{12} = 0 \qquad \therefore \varphi_B = \frac{wl^2}{12(2+k)}$$

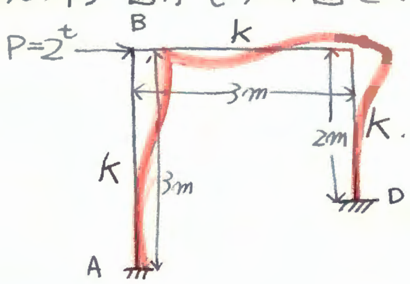
$$M_{AB} = \varphi_B = \frac{wl^2}{12(2+k)}$$
$$M_{BA} = 2\varphi_B = \frac{wl^2}{6(2+k)}$$
$$M_{BC} = \frac{kwl^2}{12(2+k)} - \frac{wl^2}{12} = -\frac{wl^2}{6(2+k)}$$

BCの中点での曲げモーメントM_0

$$M_0 = \frac{wl^2}{8} - \frac{wl^2}{6(2+k)} = \frac{2+3k}{24(2+k)}wl^2$$



(ex14) 曲げモーメント図をかけ。



$\varphi_A = \varphi_D = 0.$

$M_{AB} = \varphi_B + \psi$

$M_{BA} = 2\varphi_B + \psi$

$M_{BC} = 2\varphi_B + \varphi_C$

$M_{CB} = \varphi_B + 2\varphi_C$

$M_{CD} = 2\varphi_C + \psi \cdot \frac{3}{2}$

$M_{DC} = \varphi_C + \psi \cdot \frac{3}{2}$

節点.

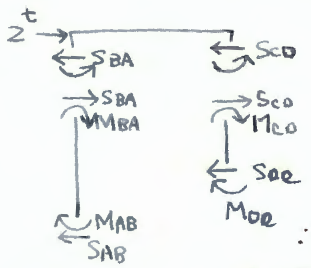
$M_{BA} + M_{BC} = 0$

$\therefore 4\varphi_B + \varphi_C + \psi = 0$ —— ①

$M_{CB} + M_{CD} = 0$

$\therefore \varphi_B + 4\varphi_C + \frac{3}{2}\psi = 0$ —— ②

層方程式.



$S_{BA} + S_{CD} - 2 = 0.$

$M_{AB} + M_{BA} + 3S_{BA} = 0$

$M_{DC} + M_{CD} + 2S_{CD} = 0$

$\therefore 2(M_{AB} + M_{BA}) + 3(M_{DC} + M_{CD}) + 12 = 0$

$\therefore 2(3\varphi_B + 2\psi) + 3(3\varphi_C + 3\psi) + 12$

$= 6\varphi_B + 9\varphi_C + 13\psi + 12 = 0$ —— ③

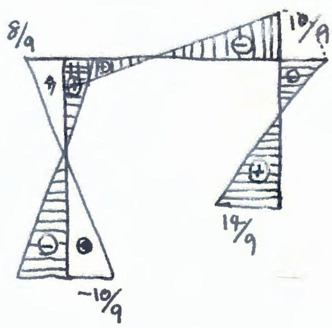
①②より $10\varphi_B - 5\varphi_C = 0$ $\therefore 2\varphi_B = \varphi_C$ —— ④

①③より. $46\varphi_B + 4\varphi_C = 12$

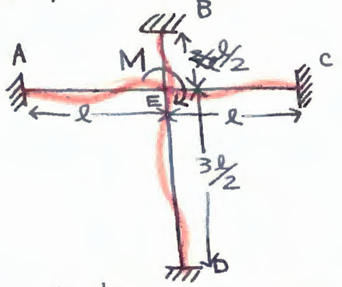
④を代入. $54\varphi_B = 12$ $\therefore \varphi_B = 2/9$ $\varphi_C = 4/9$

$\psi = -\varphi_C - 4\varphi_B = -4/9 - 8/9 = -12/9$

$\therefore \begin{cases} M_{AB} = -10/9 \ ^{tm} \\ M_{BA} = -8/9 \ ^{tm} \\ M_{BC} = 8/9 \ ^{tm} \\ M_{CB} = 10/9 \ ^{tm} \\ M_{CD} = -10/9 \ ^{tm} \\ M_{DC} = -14/9 \ ^{tm} \end{cases}$



(ex15). $EI = const.$



$\varphi = 6EKR$

$\varphi = 2EK\theta$

$K = \frac{I}{\ell}$ //

たわみ角法.

$M_{AE} = \varphi_E$.

$M_{EA} = 2\varphi_E$.

$M_{ED} = \frac{2}{3} \cdot 2\varphi_E$

$M_{DE} = \frac{2}{3} \cdot \varphi_E$

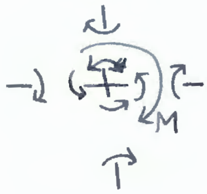
$M_{EC} = 2\varphi_E$

$M_{CE} = \varphi_E$

$M_{EB} = 2 \cdot 2\varphi_E$

$M_{BE} = 2 \cdot \varphi_E$

節点..



$M_{EA} + M_{EB} + M_{EC} + M_{ED} - M = 0$

$\therefore \varphi_E = \frac{3}{28}M$.

$M_{AE} = \frac{3}{28}M$	$M_{CE} = \frac{3}{28}M$
$M_{EA} = \frac{3}{14}M$	$M_{EC} = \frac{3}{14}M$
$M_{ED} = \frac{1}{7}M$	$M_{EB} = \frac{3}{7}M$
$M_{DE} = \frac{1}{14}M$	$M_{BE} = \frac{3}{14}M$

別解.

$M_{EA} = \dfrac{1}{1+1+2+\frac{2}{3}}M = \frac{3}{14}M$

$M_{AE} = \frac{3}{28}M \; (= \frac{1}{2}M_{EA})$

$M_{EB} = M_{EA} \times 2 = \frac{3}{7}M$

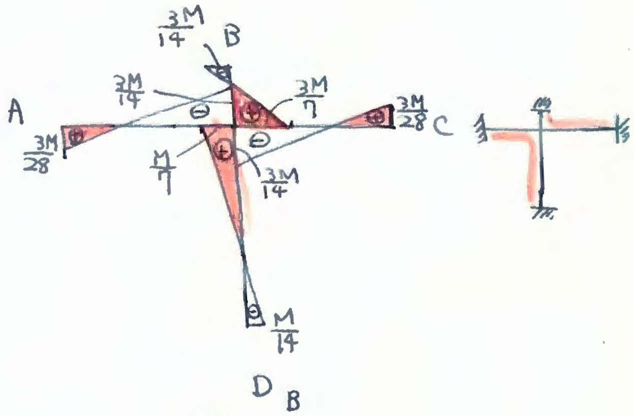
$M_{BE} = \frac{1}{2}M_{EB} = \frac{3}{14}M$.

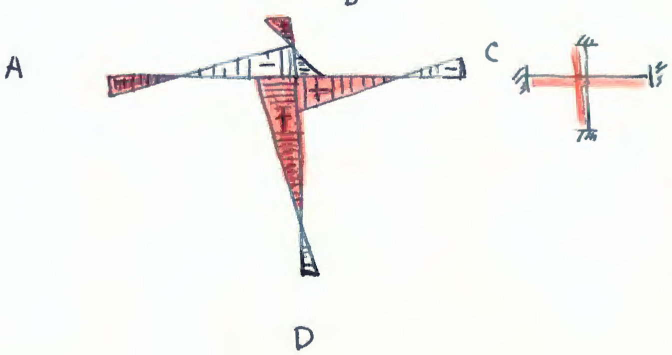
$M_{EC} = M_{EA} = \frac{3}{14}M$.

$M_{CE} = \frac{1}{2}M_{EC} = \frac{3}{28}M$

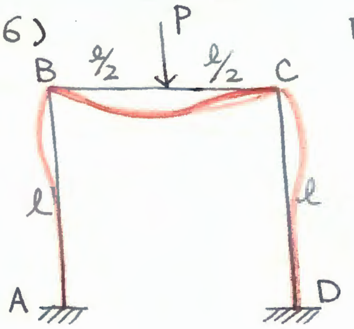
$M_{ED} = \frac{2}{3}M_{EA} = \frac{1}{7}M$

$M_{DE} = \frac{1}{2}M_{ED} = \frac{1}{14}M$





(ex16)



EI = const

$$\begin{cases} \psi = 0 \\ \varphi_A = \varphi_D = 0 \\ \varphi_C = -\varphi_B \end{cases}$$

$M_{AB} = \varphi_B$

$M_{BA} = 2\varphi_B$

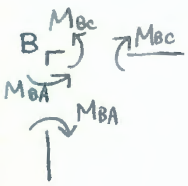
$M_{BC} = 2\varphi_B + \varphi_C - \frac{Pl}{8} = \varphi_B - \frac{Pl}{8}$

$$\left(\begin{aligned} M_{CB} &= \varphi_B + 2\varphi_C + \frac{Pl}{8} = -\varphi_B + \frac{Pl}{8} = -M_{BC} \\ M_{CD} &= 2\varphi_C = -2\varphi_B = -M_{BA} \\ M_{DC} &= \varphi_C = -\varphi_B = -M_{AB} \end{aligned} \right)$$

節点.

$M_{BA} + M_{BC} = 0$

$\therefore\ 3\varphi_B - \frac{Pl}{8} = 0 \quad \therefore\ \varphi_B = \frac{Pl}{24}$



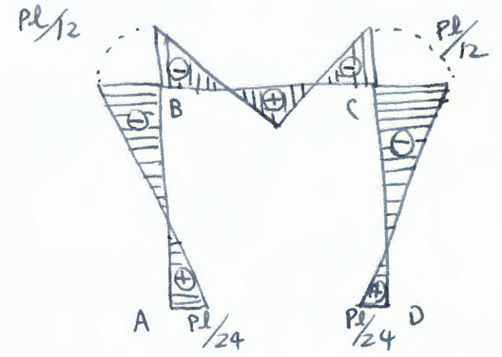
$$\begin{cases} M_{AB} = \frac{Pl}{24} \\ M_{BA} = \frac{Pl}{12} \\ M_{BC} = -\frac{Pl}{12} \\ M_{CB} = \frac{Pl}{12} \\ M_{CD} = -\frac{Pl}{12} \\ M_{DC} = -\frac{Pl}{24} \end{cases}$$

	A	B		C		D
剛比		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	
固定端Mom	0	0	$-\frac{Pl}{8}$	$\frac{Pl}{8}$	0	0
①		$\frac{Pl}{16}$	$\frac{Pl}{16}$	$-\frac{Pl}{16}$	$-\frac{Pl}{16}$	
	$\frac{Pl}{32}$		$-\frac{Pl}{32}$	$\frac{Pl}{32}$		$-\frac{Pl}{32}$
計	$\frac{Pl}{32}$	$\frac{Pl}{16}$	$-\frac{3Pl}{32}$	$\frac{3Pl}{32}$	$-\frac{Pl}{16}$	$-\frac{Pl}{32}$
②		$\frac{Pl}{64}$	$\frac{Pl}{64}$	$-\frac{Pl}{64}$	$-\frac{Pl}{64}$	
	$\frac{Pl}{32}$		$-\frac{Pl}{32}$	$\frac{Pl}{32}$		$-\frac{Pl}{32}$
計	$\frac{Pl}{16}$	$\frac{5Pl}{64}$	$-\frac{7Pl}{64}$	$\frac{7Pl}{64}$	$-\frac{5Pl}{64}$	$-\frac{Pl}{16}$
③		$\frac{Pl}{64}$	$\frac{Pl}{64}$	$-\frac{Pl}{64}$	$-\frac{Pl}{64}$	
	$\frac{Pl}{128}$		$-\frac{Pl}{128}$	$\frac{Pl}{128}$		$-\frac{Pl}{128}$
計	$\frac{9Pl}{128}$	$\frac{3Pl}{32}$	$-\frac{13Pl}{128}$	$\frac{13Pl}{128}$	$-\frac{3Pl}{32}$	$-\frac{9Pl}{128}$
	0.07	0.09	−0.10	0.10	−0.09	−0.07
		.0.005	0.005	−0.005	−0.005	

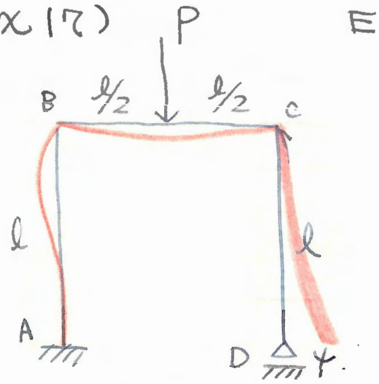
モーメント分配法をつかうと.

	A	B		C		D
剛比		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	
固定端Mo	0	0	−0.125	0.125	0	0
第一回		0.063	0.063	−0.063	−0.063	
	0.031		−0.031	0.031		−0.031
計	0.031	0.063	−0.093	0.093	−0.063	−0.031
第二回		0.015	0.015	−0.015	−0.015	
	0.008		−0.008	0.008		−0.008
計	0.039	0.078	−0.086	0.086	−0.078	−0.039
第3回		0.004	0.004	−0.004	−0.004	
	0.002		−0.002	0.002		
	~~0.041~~	~~0.001~~	~~0.001~~	~~−0.001~~		
	~~0.001~~		~~−0.001~~	~~0.001~~		
計	0.041	0.082	−0.084	0.084	−0.082	−0.041
第4回		0.001	0.001	−0.001	−0.001	
	0.001		−0.001	0.001		−0.001
計	0.042	0.083	−0.084	0.084	−0.084	−0.042
第5回		0.0005	0.0005	−0.0005	−0.0005	
	0		0	0		0
計	0.042	0.084	−0.084	0.084	−0.084	−0.042

$\left(\frac{1}{24} = 0.0417. \quad \frac{1}{12} = 0.0833. \right)$



(ex 17) $\quad EI = \text{const}.$



節点〜

$$M_{BA} + M_{BC} = 4\varphi_B + \varphi_C - \frac{Pl}{8} = 0$$

$$M_{CB} + M_{CD} = \varphi_B + 4\varphi_C + \varphi_D + \psi + \frac{Pl}{8} = 0$$

層〜 $M_{AB} + M_{BA} + M_{CD} = 0$

$$3\varphi_B + 2\varphi_C + \varphi_D + \psi = 0$$

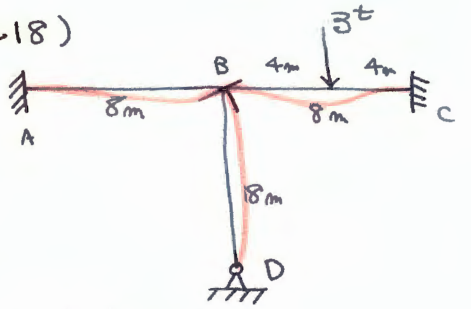
$$5\varphi_B - \frac{Pl}{8} = 0 \qquad \varphi_B = \frac{Pl}{40}$$

$\varphi_B = \varphi_C$?

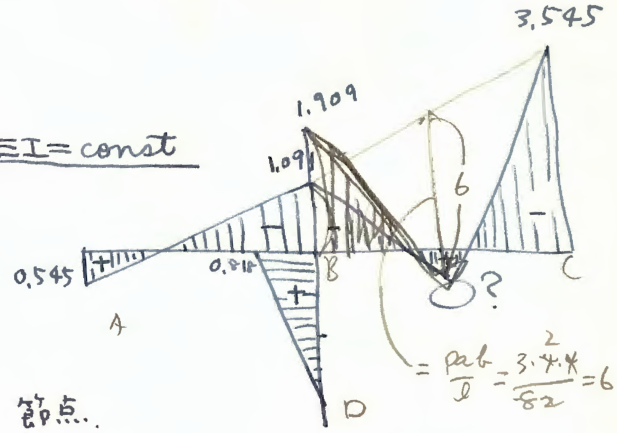
$\varphi_D + \psi =$

$$\begin{cases} M_{AB} = \varphi_B \\ M_{BA} = 2\varphi_B \\ M_{BC} = 2\varphi_B + \varphi_C - \frac{Pl}{8} & = 3\varphi_B - \frac{Pl}{8} \\ M_{CB} = \varphi_B + 2\varphi_C + \frac{Pl}{8} & = 3\varphi_B + \frac{Pl}{8} \\ M_{CD} = 2\varphi_C + \varphi_D + \psi & = 2\varphi_B + \varphi_D + \psi \\ M_{DC} = 0 \end{cases}$$

(ex18)



EI = const



1) タワミ角法

$$\begin{cases} M_{AB} = \varphi_B . \\ M_{BA} = 2\varphi_B . \\ M_{BC} = 2\varphi_B - \frac{3 \cdot 8}{8} = 2\varphi_B - 3 \\ M_{CB} = \varphi_B + 3 . \\ M_{BD} = 1.5\varphi_B . \\ M_{DB} = 0 \end{cases}$$

節点.

$$M_{BA} + M_{BC} + M_{BD} = 5.5\varphi_B - 3 = 0$$

$$\therefore \varphi_B = 6/11$$

$$\therefore \begin{cases} M_{AB} = 6/11 = 0.545 \\ M_{BA} = 12/11 = 1.091 \\ M_{BC} = -21/11 = -1.909 \\ M_{CB} = 39/11 = 3.545 \\ M_{BD} = 9/11 = 0.818 . \end{cases}$$

2) モーメント分配法.

	A	B	B	C
剛比		1/3	1/3	
固定端モーメント	0	0	-3	3
第1回	0.5	1	1	0.5
計	0.5	1	-2	3.5
第2回	0.04	0.08	0.08	0.04
計	0.54	1.08	-1.92	3.54
第3回	0.005	0.01	0.01	0.005
計	0.545	1.09	-1.91	3.545
第4回	0	0.001	0.001	0
計	0.545	1.091	-1.909	3.545

B	D
1/3	
0	0
1 -0.25	0.5 -0.5
0.75	0
0.08 -0.02	0.04 -0.04
0.81	0
0.01 -0.003	0.005 -0.005
0.817	0
0.001	0
0.818	0

$\frac{1-2+0.75}{3} = -0.08$

$\frac{1.08-1.92+0.81}{3} = -0.01$

$\frac{1.09-1.91+0.817}{3} = -0.00$