構造力学(1)及び演習.

4/8

教. ☆構造力学I 大村裕 森北出版 1.500+α

参. 構造力学 伊藤学 〃 1.800

〃 吉田俊弥 朝倉書店 3.300

〃 村上・吉村 コロナ社 2.000

〃 II 能町純雄 朝倉書店 2.300

〃 II 小西・横尾・成岡 丸善 1.500

〃 II 山崎徳也 共立出版 2.400

☆ 〃 入門(3) 高橋武雄 培風館 2,000

構造力学 酒井忠明. 技報堂 ~~5.600~~

○ Structural Analysis Laursen Mcgraw-hill -kogakusha. 2,780

☆ Theory of Structures Timoshenko-Young 〃 2.570

? Strength of materials I. II. 〃 〃

骨組構造解析要覧 日本鋼構造協会 培風館 10,000

THEORY OP ELASTISITY TIMOSHENKO-GOODIER McGRAW HILL-KOGAKSHA 2600

応用弾性学 C.WANG 猪瀬訳. 3500

〃 川本眺万 共立 2700

DYNAMIC 振動
STATIC 静的.

応用弾性学　倉西正嗣　共立

THEORY OF PLATE AND SHELL　TIMOSHENKO-□　McGRAWHILL -KOGAKSHA　2400

薄肉構造の理論 上　小松.

有限要素法への入門書

- マトリックス法による構造物の解法　マーチン 吉識　培風館　2600
- アジア版 INTRODUCTION TO FINITE ELEMENT ANALYSIS　MARTIN　McGRAWHILL　1900
- マトリックス構造解析入門　リブリスレイ　培風館　1800
- 〃 〃 の基礎理論　シェムンスキー　〃　3600

4/9.

力. P, $\mathbb{F} = F_x i + F_y j$　　並進(translation) Δ, $\Delta = \Delta_x i + \Delta_y j$

回転(rotation) θ

モーメント $M = P \cdot l$

$\mathbb{M} = \mathbb{F} \cdot l$

$\mathbb{F}l$.

$$\mathbb{F} = \begin{bmatrix} \mathbb{F}_1 \\ \mathbb{F}_2 \\ \vdots \\ \mathbb{F}_n \end{bmatrix} \qquad \mathbb{F}_1 = F_{1x} i + F_{1y} j$$

$$= \begin{bmatrix} F_{1x} \\ F_{2x} \\ \vdots \\ F_{nx} \end{bmatrix} i + \begin{bmatrix} F_{1y} \\ F_{2y} \\ \vdots \\ F_{ny} \end{bmatrix} j \qquad \mathbb{F} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ \vdots \\ F_{nx} \\ F_{ny} \end{bmatrix} \qquad \mathbb{F} = \begin{bmatrix} \mathbb{F}_1 \\ \mathbb{F}_2 \\ \vdots \\ \mathbb{F}_n \\ \mathbb{M}_1 \\ \mathbb{M}_2 \\ \vdots \\ \mathbb{M}_m \end{bmatrix} \qquad \mathbb{F} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ \vdots \\ F_{nx} \\ F_{ny} \\ M_1 \\ \vdots \\ M_m \end{bmatrix}$$

ベクトル　スカラー

一般力 Generalised Force

一般変位 (Generalized Displacement)

$$\Delta = \begin{bmatrix} \Delta_{1x} \\ \Delta_{1y} \\ \Delta_{2x} \\ \Delta_{2y} \\ \vdots \\ \Delta_{nx} \\ \Delta_{ny} \\ \theta_1 \\ \vdots \\ \theta_m \end{bmatrix}$$

Translation … $\Sigma \mathbb{F}_i = 0$

Rotation …. $\Sigma \mathbb{M}_i = 0$.

$$\begin{cases} \mathbb{F}_i = F_{ix}\,i + F_{iy}\,j \\ \mathbb{M}_i = \mathbb{F}_i \times l_i \end{cases}$$

$$\Sigma \mathbb{F}_i = (F_{1x} + F_{2x} + \cdots F_{nx})\,i + (F_{1y} + F_{2y} + \cdots F_{ny})\,j = 0$$

$$\begin{cases} \Sigma F_{ix} = 0 & \cdots \quad \Sigma H = 0 \\ \Sigma F_{iy} = 0 & \cdots \quad \Sigma V = 0 \end{cases}$$

一点に集中する力のつりあい.

$\Sigma M_i = 0$.

構造物. 支えられるもの・・・支えられるものが動かないようにする.

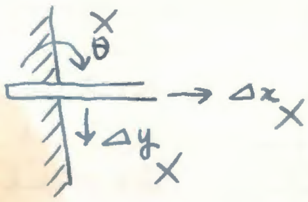
↓

支承（支点）

Δx

Δy

θ

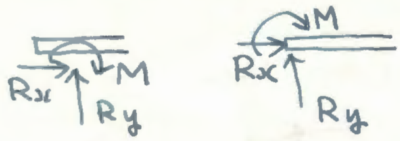


✓ $\Delta x \to R_x$

✓ $\Delta y \to R_y$

✓ $\theta \to M$

固定支点. 拘束3.



ピン

✓ Δx R_x

✓ Δy R_y

○ θ

ヒンジ支点.

2（拘束）

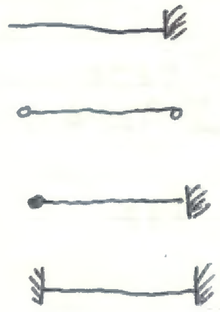
○ Δx

✓ Δy R_y

○ θ

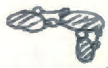
移動支点.

1（〃）

	図	拘束度.	拘束度−3 ‖ (自由度)
m_0		3	0
m_1		4	1
m_2		5	2
m_3		6	3

位置 … 外的安定 … 反力 $r \geqq 3$

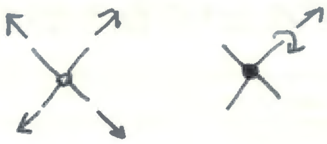
移 … 内的 〃　　↑釣合条件数.



$d-1$

$r \geqq d-1+3.$

$$U_i = 0 \times m_0 + 1 \times m_1 + 2 \times m_2 + 3 \times m_3$$
$$= m_1 + 2m_2 + 3m_3$$



$C_j = 2j_2 + 3j_3 - 3.$

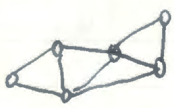
条件式	2	3
	j_2	j_3

$U_i > C_i$ … 不静定 } 安定.

$U_i = C_i$ … 静定 }

$U_i < C_i$ …… 不安定.

トラス.



j　m.

$U_i = 2m$

$C_i = 2j - 3.$

$m > 2j - 3$

$m = 2j - 3$ … 静定トラス.

P193 (1, 2) 構造力学 国民科学社

研究課題

外的　反力成分の数 r（支点拘束度と同じ）

$$\begin{cases} r<3 & \text{：外的不安定} \\ r=3 & \text{：外的安定かつ静定.} \\ r>3 & \text{：外的 〃 かつ不静定.} \end{cases}$$

1　○複数の構面の場合
板の数 d.

$$\begin{cases} r<d+2 & \text{：外的不安定.} \\ r=d+2 & \text{：外的安定かつ静定.} \\ r>d+2 & \text{：外的 〃 かつ不静定.} \end{cases}$$

内的　m：部材数 ——→ 接合点 $m-1$
j_0：節点拘束度総数

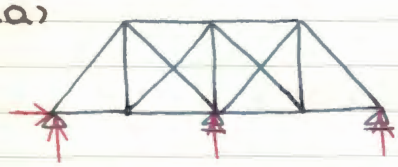
$$\begin{cases} j_0<3(m-1) & \text{内的不安定} & m+3<2j \\ j_0=3(m-1) & \text{〃 安定かつ静定} & m+3=2j \\ j_0>3(m-1) & \text{〃 〃 〃 不静定.} & m+3>2j \end{cases}$$

トラス…すべてヒンジ
$j_0=2(2m-j)$

$$\begin{cases} r+j_0 \fallingdotseq 3m \\ r+m \fallingdotseq 2j & \text{すべてヒンジ（トラス）} \end{cases}$$

NO.

(a)



$r=4$. $r-3=1$

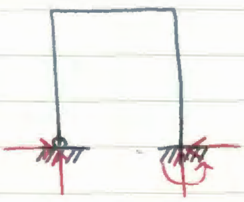
外的1次不静定.

$m=15$. $j=8$

$m+3-2j=2$

内的2次不静定.

(b)



$r=5$. $r-3=2$

外的2次不静定.

$m=3$. $j_0=3+3=6$

$j_0-3(m-1)=0$

内的静定.

(c)

$r=8$ $\therefore r-3=5$

外的5次不静定.

$m=7$, $j_0=18$

$\therefore j_0-3(m-1)=0$

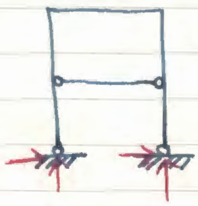
内的静定

(d)

$r=5$, $d=2$ $\therefore r-(d+2)=1$

外的1次不静定.

(e)



$r=4$. $r-3=1$

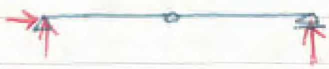
外的1次不静定

$m=6$, $j_0=16$

$j_0-3(m-1)=1$

内的1次不静定.

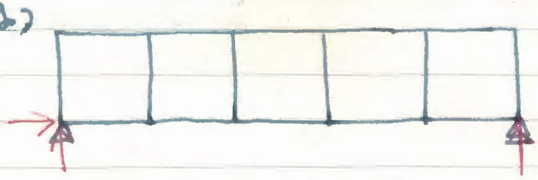
(f)



$r=3$. $d=2$ $\therefore r-(d+2)=-1$

外的1次不安定

(g)



$r=3$ $r-3=0$

外的静定.

$m=16$. $j_0=6\times8+3\times4=60$

$\therefore j_0-3(m-1)=15$

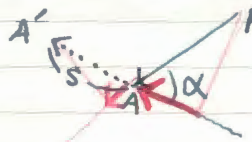
内的15次不静定.

4/15. 第2章. 構造物の弾性変形.

不静定構造物 ------ [変形] ──→ 適合条件.
indeterminate structure
静定構造物
determinate structure

▨ 実仕事（＞弾性変形の仕事）

▨ 仕事. $W = P \cdot S \cdot \cos\alpha$.　　　$W = PS$.



A点の動く方向の成分のカ $P\cos\alpha$ が仕事をする。

又は、A点がPの作用線方向に動いた距離 $S\cos\alpha$ うごかすPの仕事。

ベクトルで表わせば $\left.\begin{matrix}\mathbb{P}\\ \mathbb{S}\end{matrix}\right\} \rightarrow W = \mathbb{P} \cdot \mathbb{S}$ （スカラー）

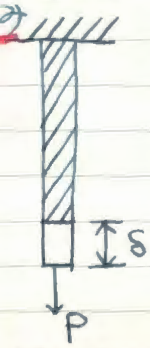
例えば、

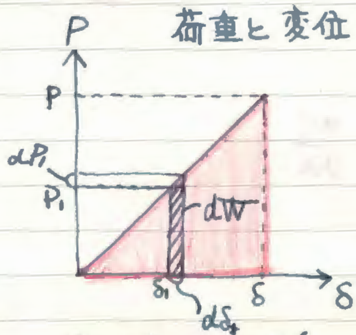
$\mathbb{P} = \begin{bmatrix} \mathbb{P} \\ \mathbb{M} \end{bmatrix}$　　$\mathbb{S} = \begin{bmatrix} \mathbb{S} \\ \theta \end{bmatrix}$ とすると.

$$\mathbb{W} = \mathbb{P} \cdot \mathbb{S} = [\mathbb{P}\ \ \mathbb{M}] \begin{bmatrix} \mathbb{S} \\ \theta \end{bmatrix} = \mathbb{P}\mathbb{S} + \mathbb{M} \cdot \theta$$

▨ 次に弾性変形を考えると.

i) 軸力のみ





$P_1 = C\delta_1 \rightarrow P = C\delta$

荷重と変位が比例 →（変位が徐々に増すに従って荷重も徐々にPに近づく

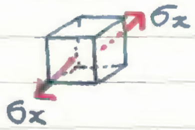
$$d\mathbb{W}\ (U_0) = P_1 \cdot d\delta = c\delta\, d\delta$$

$$\mathbb{W}\ (U_0) = \int_0^{\delta} c\delta\, d\delta = \frac{1}{2}c\delta^2 = \frac{1}{2}P\delta$$

: 外力が弾性体にした仕事!

次に内力が弾性体にする仕事について…

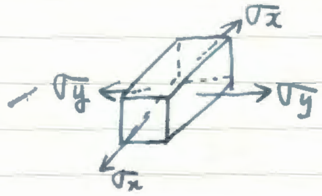
微小正六面体を考えると.



ひずみ $\varepsilon_x = \frac{\sigma_x}{E}$ ∴ $dU_i = \frac{1}{2}\sigma_x \cdot \varepsilon_x$

故に.

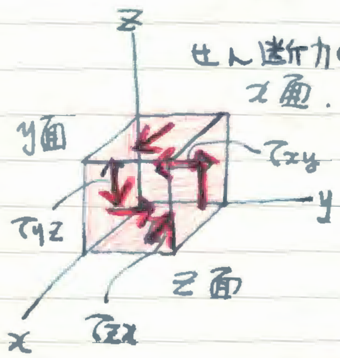
$$U_i = \int \frac{1}{2}\sigma_x \varepsilon_x dV.$$



$U_i = \int \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y)dV$ と仮定する。

一般的に考えると.

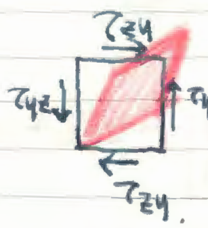
$$U_i = \int \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx} + \tau_{xy}\gamma_{xy})dV$$

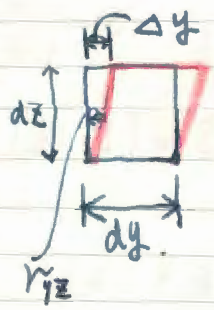


せん断力の方向.

x面.

$\frac{1}{2}\cdot\sigma_x\cdot\varepsilon_x$: σ_x という応力によって生じた ε_x なるひずみ. これによってなされた実仕事.



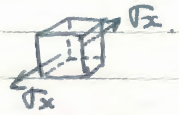


$$\gamma_{yz} = \frac{\Delta y}{dz}$$

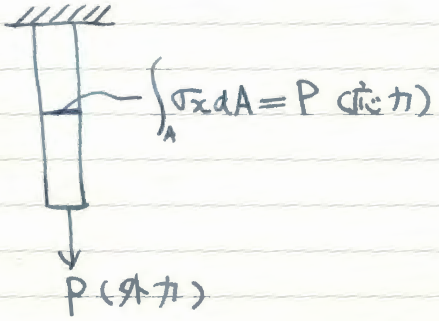
$$\varepsilon_x = \frac{\Delta x}{dx} = \frac{\partial u}{\partial x}$$

$$U_i = \int \frac{1}{2}\sigma_x \varepsilon_x dV$$

$$\sigma_x = \frac{P}{A} \leftarrow \text{応力}$$



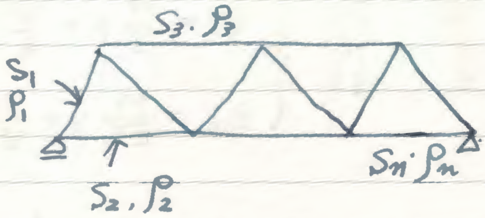
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{P}{EA} = \frac{1}{2}\frac{P}{A}\cdot\frac{P}{EA}\cdot A\cdot l = \frac{1}{2}\cdot\frac{P^2 l}{EA}$$



部材力をN

$$U_i = \frac{1}{2}\cdot\frac{l}{EA}N^2 = \frac{1}{2}N^2\rho$$

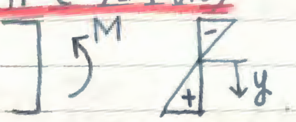
例えば



$$U_i = \frac{1}{2}S_1^2\rho_1 + \frac{1}{2}S_2^2\rho_2 + \cdots + \frac{1}{2}S_n^2\rho_n$$

$$= \frac{1}{2}\sum_{i=1}^{n} S_i^2\rho_i \quad ☆ \qquad (2.4)$$

ii) 曲げモーメントのみ



$$\begin{cases}\sigma_x = \frac{M}{I}y \\ \varepsilon_x = \frac{M}{EI}y\end{cases}$$

純曲げ

pure bending (曲げモーメントconst→せん断力零)の状態とする。

$$U_i = \int \frac{1}{2}\sigma_x \varepsilon_x dV = \frac{1}{2}\int \frac{1}{2}\cdot\frac{M}{I}y\cdot\frac{M}{EI}y\, dV = \frac{1}{2}\cdot\frac{M^2}{EI^2}\int_l\int_A y^2 dA\, dx$$

→次ページへ

$\int_A y^2 dA = I$ より.

$$U_i = \frac{1}{2}\frac{M^2}{EI}\int dx = \boxed{\frac{1}{2}\frac{M^2}{EI}\cdot l}$$

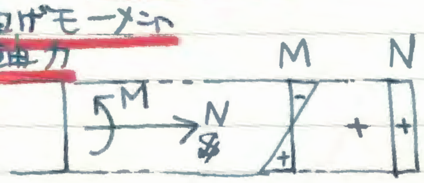
not pure bending のとき.（ただしせん断力は無視する）

$$U_i = \frac{1}{2}\frac{1}{EI^2}\iint M^2 y^2 dA\,dx$$

$$= \frac{1}{2}\int\frac{M^2}{EI^2}\int y^2 dA\,dx$$

$$= \boxed{\frac{1}{2}\int\frac{M^2}{EI}dx}$$

iii) {曲げモーメント
　　{軸力



$$\begin{cases}\sigma_x = \dfrac{M}{I}y + \dfrac{N}{A}\\[2mm] \varepsilon_x = \dfrac{M}{EI}y + \dfrac{N}{EA}\end{cases}$$

$$U_i = \int\frac{1}{2}\sigma_x\varepsilon_x dV = \int\frac{1}{2}\left(\frac{M}{I}y + \frac{N}{A}\right)\left(\frac{M}{EI}y + \frac{N}{EA}\right)dV$$

$$= \frac{1}{2}\int\left(\frac{M^2}{EI^2}y^2 + 2\frac{MN}{EIA}y + \frac{N^2}{EA^2}\right)dV$$

○ { M : pure bending でない.（一定でない）
　{ N : xの方向に一定.　　　　とする.

$$U_i = \frac{1}{2}\int\frac{M^2}{EI^2}\underbrace{\int y^2 dA}_{I}\,dx + \int\frac{MN}{EIA}\underbrace{\int y\,dA}_{\text{零（一次モーメント）}}\,dx + \frac{1}{2}\int\frac{N^2}{EA^2}dV$$

$$= \boxed{\frac{1}{2}\int_0^l\frac{M^2}{EI}dx + \frac{1}{2}\frac{N^2 l}{EA}} \quad (\because V = Al) \qquad (2.9')$$

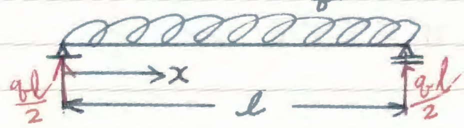
軸力　曲げモーメント　せん断力　ねじり

★ $$U_i = \frac{1}{2}\int\frac{N^2}{EA}dx + \frac{1}{2}\int\frac{M^2}{EI}dx + \frac{1}{2}\int\frac{Q^2}{GA}\kappa dx + \frac{1}{2}\int\frac{T^2}{GC}dx$$

$\tau = k\frac{Q}{A}$ → kを全断面について積分したもの、κとする。

C：ねじり剛性。

(例題2.1)



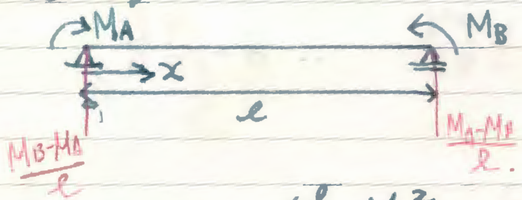
$$M_x = \frac{ql}{2}x - \frac{q}{2}x^2$$

曲げモーメントだけを考えると

$$U_i = \int_0^l \frac{M_x^2}{2EI}dx = \frac{1}{2EI}\cdot\frac{q^2}{4}\int_0^l (lx - x^2)^2 dx$$

$$= \frac{q^2}{8EI}\left(\frac{l^5}{3} - \frac{l^5}{2} + \frac{l^5}{5}\right) = \frac{q^2 l^5}{240EI}$$ //

(例題2.2)



$$M_x = \frac{M_B - M_A}{l}x + M_A$$

$$U_i = \int_0^l \frac{M_x^2}{2EI}dx = \frac{1}{2EI}\int_0^l \left(\frac{M_B - M_A}{l}x + M_A\right)^2 dx$$

$$= \frac{1}{2EI}\left[\frac{(M_B - M_A)^2}{l^2}\frac{l^3}{3} + \frac{M_A(M_B - M_A)}{l}l^2 + M_A^2 l\right]$$

$$= \frac{l}{2EI}\left[\frac{(M_B - M_A)^2}{3} + M_A M_B - M_A^2 + M_A^2\right]$$

$$= \frac{l}{6EI}\left(M_B^2 + M_A^2 + M_A M_B\right)$$ //

(例題2.3)

⇒

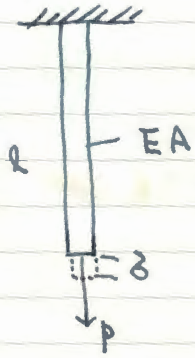
純曲げ $U_i = M^2 l / 2EI$.

$$\theta = \frac{Ml}{EI} \rightarrow M = \frac{\theta EI}{l}$$

$$\therefore U_i = \frac{\theta^2 EI}{2l}$$

$\theta = 2\pi$ より $U_i = 2\pi^2 EI / l$ //

4/22.



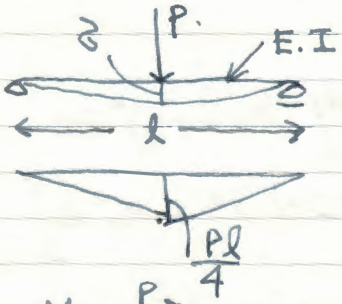
◎エネルギー不滅の原理. CONSERVATION LAW OF ENERGY.

$$U_o = U_i$$

$$U_o = \frac{1}{2} P \delta$$

$$U_i = \frac{1}{2} \frac{P^2 l}{EA}$$

$$\therefore \frac{1}{2} P \delta = \frac{1}{2} \frac{P^2 l}{EA} \qquad \therefore \delta = \frac{Pl}{EA}$$ //



$M_x = \frac{P}{2} x$.

$\left(= \frac{Pl}{4} \cdot \frac{2}{l} \cdot x\right)$

$$U_o = \frac{1}{2} P \delta$$

$$U_i = \frac{1}{2} \int_0^l \frac{M^2}{EI} dx$$

$$= \int_0^{l/2} \frac{1}{EI} \cdot \frac{P^2}{4} x^2 dx = \frac{P^2}{4EI} \cdot \frac{1}{3} \left(\frac{l}{2}\right)^3$$

$$= \frac{1}{2} \cdot \frac{1}{48} \frac{P^2 l^3}{EI}$$

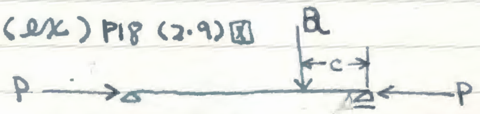
$$\therefore \delta = \frac{Pl^3}{48EI}$$ //

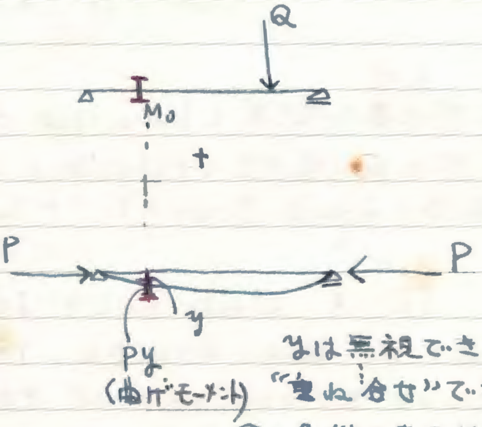
▨ 重ね合せの原理 PRINCIPLE OF SUPERPOSITION

"THE TOTAL EFFECT AT SOME POINT IN A STRUCTURE DUE TO A NUMBER OF LOADS APPLIED SIMULTANEOUSLY IS EQUAL TO THE SOME OF THE EFFECTS FOR THE LOARS APPLIED INDIVIDUALITY"

成立条件. ① 材料が HOOKS LAW に従って挙動する。
$(\sigma = E \cdot e)$
② 変形前の形で釣合方程式をたてても.
変形後の挙動を解明できる.
(微小変位の仮定が成り立つ。)

(ex) P18 (2.9)図





$$EI\frac{d^2y}{dx^2} = -\frac{Qc}{l}x - Py$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = -\frac{1}{EI}\cdot\frac{Qc}{l}x$$

$k = \sqrt{\frac{P}{EI}}$ とおく.

$$\frac{d^2y}{dx^2} + k^2y = -\frac{1}{EI}\frac{Qc}{l}x$$

$$y = \frac{Q\sin kc}{Pk\sin kl}\sin kx - \frac{Qc}{Pl}x.$$

$$y'' = -k^2\frac{Q\sin kc}{Pk\sin kl}\sin kx$$

$$= -\frac{P}{EI}\cdot\frac{Q\sin kc}{Pk\sin kl}\sin kx$$

$$\therefore\ M = -EIy'' = \frac{Q\sin kc}{k\sin kl}\sin kx$$

yは無視できない。
"重ね合せ"できない。
③の条件にあてはまらない.
(∵ Py=0とすると Pが意味なくなる)

Pを2倍にしても yは2倍にならない
(線形でない)

NO.

重ね合せの原理.

$$U_0 = \frac{1}{2}(P_1\delta_1 + P_2\delta_2 + \cdots + P_n\delta_n)$$

$$= \frac{1}{2}\mathbb{P}^*\cdot\delta \qquad \mathbb{P} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{bmatrix}$$

$$\delta = [F]\cdot\mathbb{P}$$

$$U_0 = \frac{1}{2}\mathbb{P}^*\cdot[F]\cdot\mathbb{P} \quad = 二次形式$$

$$\mathbb{P} = [F]^{-1}\cdot\delta = [K]\cdot\delta$$

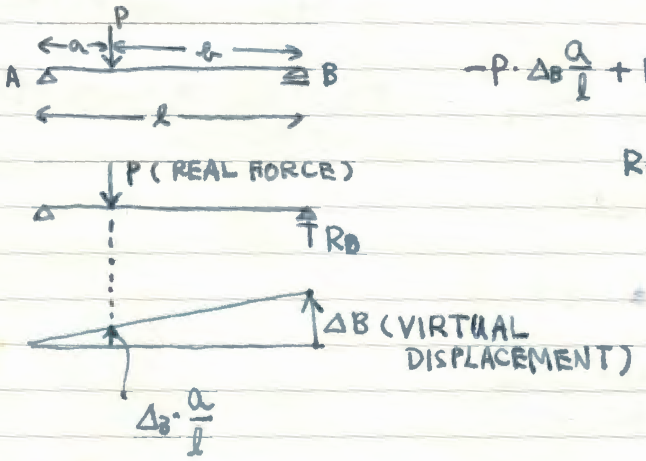
$$U_0 = \frac{1}{2}\delta^*\cdot[K]\cdot\delta$$

◎ 仮想~~仕事~~変位の原理 (PRINCIPLE OF VIRTUAL DISPLACEMENT)

仮想仕事の原理 (PRINCIPLE OF VIRTUAL WORK)

○ 剛体 (RIGID BODY) については.

"IF A RIGID BODY IS IN EQUILIBRIUM (釣合) UNDER A SET OF REAL FORCES, THEN THE VIRTUAL WORK DONE BY THE SET OF FORCES DURING A VIRTUAL DISPLACEMENT IS EQUAL TO ZERO."



$$-P\cdot\Delta_B\frac{a}{l} + R_B\Delta_B = 0$$

$$R_B = \frac{a}{l}P$$

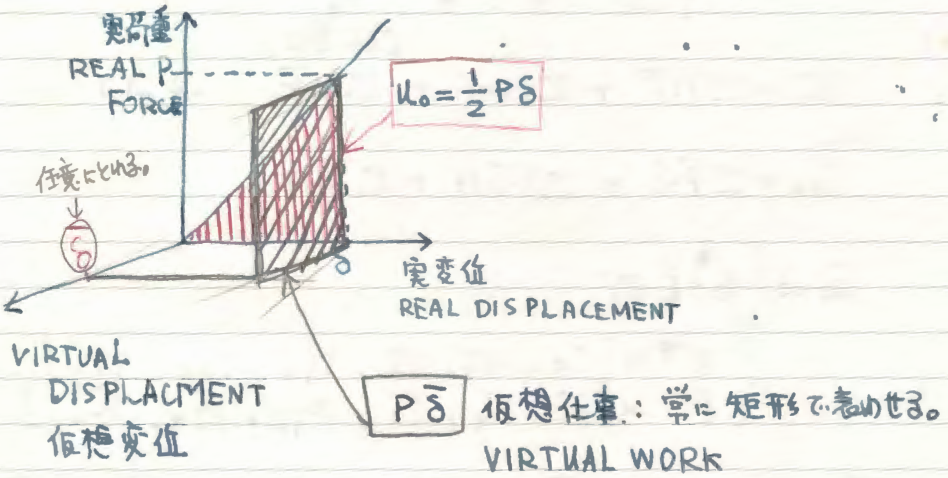
○ 弾性体 (ELASTIC BODY) については、

"IF A STRUCTURE IS IN EQUILIBRIUM UNDER A SET OF FORCES AND IF THE STRUCTURE IS GIVEN A VIRTUAL DISPLACEMENT CONSISTENT WITH THE CONSTRAINTS OF THE STRUCTURE, THEN THE EXTERNAL VIRTUAL WORK DONE IS EQUAL TO THE INTERNAL VIRTUAL WORK DONE"

(CONSISTENT WITH: 〜に適合する)

CONSTRAINTS (拘束条件)

$\begin{cases} y_{x=0}=0 \\ \frac{dy}{dx}_{x=0}=0 \end{cases}$ $y_{x=0}=0$, $y_{x=l}=0$, $\frac{dy}{dx}_{x=0}=0$, $\frac{dy}{dx}_{x=l}=0$ $y_{x=0}=0$, $y_{x=l}=0$



4/23.

エネルギー保存則.(CONSERVATION LAW OF ENERGY)

$$U_o = U_i$$

仮想仕事の原理(PRINCIPLE OF VIRTUAL WORK)

$$\overline{U_o} = \overline{U_i}$$

$$\begin{cases} \overline{U_o} \to W_e \leftarrow \text{external} \\ \overline{U_i} \to -W_i \leftarrow \end{cases}$$

$$W_e + W_i = 0 \qquad (2\cdot19)$$

実仕事.

$$\begin{cases} U_o = \frac{1}{2}\mathbb{P}^* \cdot \delta & (2\cdot1) \\ U_i = \int_V \frac{1}{2}\sigma^* \varepsilon \cdot dV & (2\cdot2) \end{cases}$$

仮想仕事.

$$\overline{U_o} = \mathbb{P}^* \cdot \overline{\delta} \text{ or } \overline{U_o} = \overline{\mathbb{P}^*} \cdot \delta$$

$$\begin{cases} \mathbb{P}^* \cdots P, M, R \\ \overline{\delta} \cdots \overline{\delta}, \overline{\theta}, \overline{r_R} \end{cases}$$

(支点変位)

$$\overline{U_o} = \Sigma P \cdot \overline{\delta} + \Sigma M \cdot \overline{\theta} + \Sigma R \cdot \overline{r} \qquad (2\cdot21)$$

or

$$\overline{U_o} = \Sigma \overline{P} \cdot \delta + \Sigma \overline{M} \cdot \theta + \Sigma \overline{R} \cdot r$$

$$\overline{U_i} = \int_V \sigma^* \cdot \overline{\varepsilon}\, dV$$

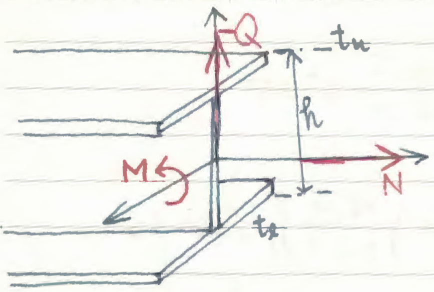
$$\begin{cases} \sigma^* = [\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy}] \\ \overline{\varepsilon}^* = [\overline{\varepsilon_x}, \overline{\varepsilon_y}, \overline{\varepsilon_z}, \overline{\gamma_{yz}}, \overline{\gamma_{zx}}, \overline{\gamma_{xy}}] \end{cases}$$

$$\overline{U_i} = \int_V (\sigma_x \overline{\varepsilon_x} + \sigma_y \overline{\varepsilon_y} + \sigma_z \overline{\varepsilon_z} + \tau_{yz}\overline{\gamma_{yz}} + \tau_{zx}\overline{\gamma_{zx}} + \tau_{xy}\overline{\gamma_{xy}})\, dV$$

or

$$\overline{U_i} = \int_V \overline{\sigma}^* \cdot \varepsilon\, dV$$

$$= \int_V (\overline{\sigma_x}\varepsilon_x + \overline{\sigma_y}\varepsilon_y + \overline{\sigma_z}\varepsilon_z + \overline{\tau_{yz}}\gamma_{yz} + \overline{\tau_{zx}}\gamma_{zx} + \overline{\tau_{xy}} \cdot \gamma_{xy})\, dV$$



Q M N：外力.

1. 軸力のみ受ける場合.

$\sigma = N/A$, $\bar{\varepsilon}_x = \bar{N}/EA$

$\therefore \bar{U}_i = \int_V \sigma \bar{\varepsilon}\, dV = \int_V \frac{N}{A}\cdot\frac{\bar{N}}{EA}\, dV = \frac{N\cdot\bar{N}}{EA^2}\int_V dV = \frac{N\bar{N}}{EA^2}\cdot A\ell$

$= \frac{N\bar{N}}{EA}\ell = N\bar{N}\rho \quad (\rho = \frac{\ell}{EA})$

$\therefore \Sigma P\bar{\delta} + \Sigma R\bar{r} = \Sigma N\bar{N}\rho$

or

$\bar{\sigma} = \bar{N}/A$, $\varepsilon = N/EA + \alpha t$ ← 線膨張係数. (α), 温度上昇 (t)

$\bar{U}_i = \int_V \bar{\sigma}\varepsilon\, dV = \int_V \frac{\bar{N}}{A}(\frac{N}{EA} + \alpha t)\, dV$

$= \frac{N\bar{N}}{EA}\ell + \bar{N}\alpha t \ell$

$= N\bar{N}\rho + \bar{N}\alpha t\ell$

$\therefore \Sigma\bar{P}\cdot\delta + \Sigma\bar{R}\cdot r = \Sigma N\bar{N}\rho + \Sigma\bar{N}\alpha t\ell$.

$t = 0$ とすると

$\Sigma\bar{P}\cdot\delta + \Sigma\bar{R}\cdot r = \Sigma N\bar{N}\rho$

支点変位 $r = 0$ とすると.

$\Sigma\bar{P}\cdot\delta = \Sigma N\bar{N}\rho \quad (\text{or } \Sigma P\cdot\bar{\delta} = \Sigma N\bar{N}\rho)$

2. 曲げモーメントと軸力を受ける場合.

$\begin{cases} \sigma = \frac{N}{A} + \frac{M}{I}y \\ \bar{\sigma} = \frac{\bar{N}}{A} + \frac{\bar{M}}{I}y \end{cases}$ $\begin{cases} \varepsilon = \frac{N}{EA} + \frac{M}{EI}y \;(+\alpha t + \alpha\frac{t_\ell - t_u}{h}) \\ \bar{\varepsilon} = \frac{\bar{N}}{EA} + \frac{\bar{M}}{EI}y \end{cases}$

$\bar{U}_i = \int_V \sigma\bar{\varepsilon}\, dV = \int_V (\frac{N}{A} + \frac{M}{I}y)(\frac{\bar{N}}{EA} + \frac{\bar{M}}{EI}y)\, dV$

次ページへつづく →

NO.

$$=\int_V \frac{N\bar{N}}{EA^2} dV + \int_V \left(\frac{N\bar{M}}{EAI} + \frac{\bar{N}M}{EAI}\right) y\, dV + \int \frac{M\bar{M}}{EI^2} y^2 dV$$

$dV = dA \cdot ds$ とおくと.

$$=\int_l \frac{N\bar{N}}{EA} ds + \left(\frac{N\bar{M}}{EAI} + \frac{\bar{N}\cdot M}{EAI}\right) \iint y\, dA\, ds + \int \frac{M\bar{M}}{EI^2} \iint y^2 dA\, ds$$

($\iint y\, dA = 0$, $\iint y^2 dA = I$)

$$=\int_l \frac{N\bar{N}}{EA} ds + \int \frac{M\bar{M}}{EI} ds$$

$$\therefore \Sigma P\bar{\delta} + \Sigma R\bar{r} = \int \frac{N\bar{N}}{EA} ds + \int \frac{M\bar{M}}{EI} ds$$

$$\left(\text{or} \quad \Sigma \bar{P}\delta + \Sigma \bar{R} r = \int \frac{\bar{N}N}{EA} ds + \int \frac{\bar{M}M}{EI} ds \right)$$

あるいは.

$$\bar{U}_i = \int_V \bar{\sigma}\, \varepsilon\, dV$$

$$=\int_V \left(\frac{\bar{N}}{A} + \frac{\bar{M}}{I} y\right)\left(\frac{N}{EA} + \frac{M}{EI} y + \alpha t + \alpha \frac{t_l - t_u}{h} y\right) dV$$

$$=\int_V \frac{\bar{N}N}{EA^2} dV + \int_V \left(\frac{\bar{N}M}{EAI} + \frac{N\bar{M}}{EAI}\right) y\, dV + \int_V \frac{\bar{M}M}{EI^2} dV$$

$$+\int_V \frac{\bar{N}}{A}\left(\alpha t + \alpha \frac{t_l - t_u}{h} y\right) dV + \int_V \frac{\bar{M}}{I} y \left(\alpha t + \alpha \frac{t_l - t_u}{h} y\right) dV$$

~~$$=\int \frac{\bar{N}N}{EA} ds + \int \frac{\bar{M}M}{EI} ds + \int \bar{N}\left(\alpha t + \alpha \frac{t_l - t_u}{h}\right) ds$$~~

~~$$\therefore \Sigma \bar{P}\delta + \Sigma \bar{R} r = \Sigma\int \frac{\bar{N}N}{EA} ds + \Sigma\int \frac{\bar{M}M}{EI} ds + \Sigma\int \bar{N}\left(\alpha t + \alpha \frac{t_l - t_u}{h}\right) ds$$~~

$$= \int \frac{\bar{N}N}{EA} ds + \int \frac{\bar{M}M}{EI} ds + \int \bar{N}\alpha t\, ds + \int \bar{M}\alpha \frac{t_l - t_u}{h} ds.$$

$$\therefore \Sigma \bar{P}\delta + \Sigma \bar{R} r = \Sigma\int \frac{\bar{N}N}{EA} ds + \Sigma\int \frac{\bar{M}M}{EI} ds$$

$$+ \Sigma\int \bar{N}\alpha t\, ds + \Sigma\int \bar{M}\alpha \frac{t_l - t_u}{h} ds.$$

今、$\gamma = 0$, $t = 0$, $t_l = t_u$ とすると.

☆ $\Sigma \bar{P} \delta = \Sigma \int \frac{N\bar{N}}{EA} ds + \Sigma \int \frac{M\bar{M}}{EI} ds$ (2·26)に準ず。

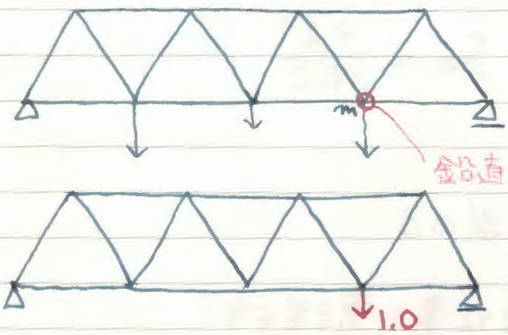
トラス. $\Sigma \bar{P}\delta = \Sigma N\bar{N}\rho$ (or $\Sigma P\bar{\delta} = \Sigma N\bar{N}\rho$)

アーチ
ラーメン
etc.

$\Sigma \bar{P}\delta = \Sigma \int \frac{N\bar{N}}{EA} ds + \Sigma \int \frac{M\bar{M}}{EI} ds \left(+ \Sigma \int \bar{N} \alpha t \, ds + \Sigma \int \bar{M} \alpha \frac{t_l - t_u}{h} ds \right)$

たわみを求める。

1.

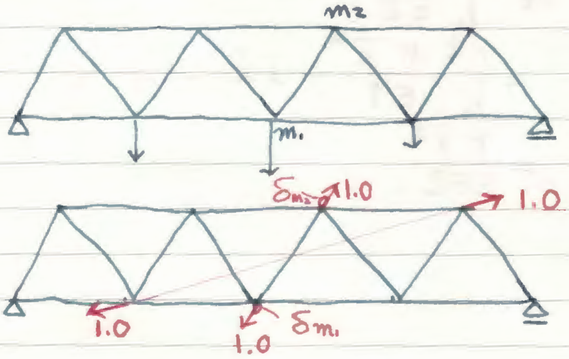


荷重Pから部材力Nが発生.

仮想荷重$\bar{P}$=1.0から仮想の部材力$\bar{N}$が発生.

$1.0 \cdot \delta_m = \Sigma N\bar{N}\rho \quad \therefore \delta_m = \Sigma N\bar{N}\rho$.

2.



$P \longrightarrow N$

$\bar{P} = 1.0 \longrightarrow \bar{N}$

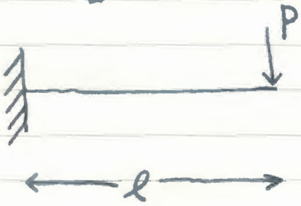
$\Sigma \bar{P}\delta = 1.0 \cdot \delta_{m_1} + 1.0 \cdot \delta_{m_2} = \delta_{m_1} + \delta_{m_2} = \Sigma N\bar{N}\rho$

m_1 と m_2 が
どれだけはなれたか.

NO

4/30.

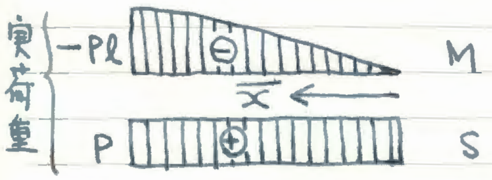
[例2.4]　(1) δ_B (解1) REAL WORK によって求める。



$$U_i = U_e.$$

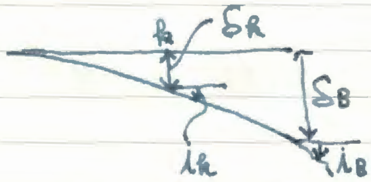
$$U_e = \frac{1}{2} P \delta_B$$

$$U_i = \int_0^l \frac{M^2}{2EI} dl$$



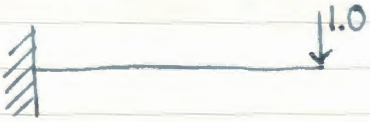
$$M = -P\bar{x}\ ,\ dl = d\bar{x}$$

$$U_i = \int_0^l \frac{P^2 \bar{x}^2}{2EI} d\bar{x} = \frac{P^2 l^3}{6EI}$$



$$\therefore\ \delta_B = \frac{Pl^3}{3EI}$$

(解2) VIRTUAL WORK によって求める.



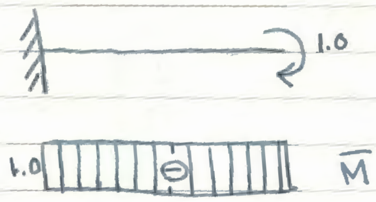
仮想仕事の原理より.

$$\begin{cases} M = -P\bar{x} \\ \bar{M} = -\bar{x} \\ dl = d\bar{x} \end{cases}$$

$$\delta_B = \int_0^l \frac{M\bar{M}}{EI} dl = \int_0^l \frac{P\bar{x}^2}{EI} d\bar{x} = \frac{Pl^3}{3EI}$$

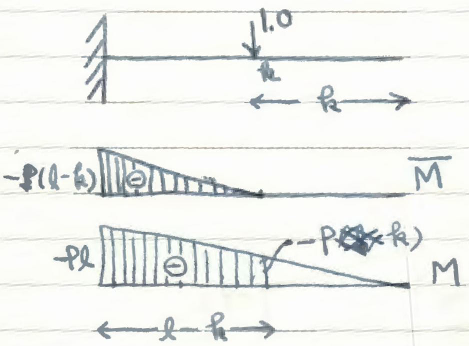
(2) i_B（たわみ角）を求める.

エネルギー保存則は使えない ⟶ VIRTUAL WORK.



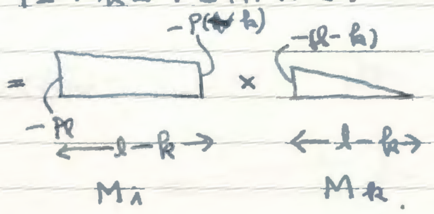
$$i_B = \int_0^l \frac{M\bar{M}}{EI}\,dl = \int_0^l \frac{P\bar{x}}{EI}\,d\bar{x} = \frac{Pl^2}{2EI}$$

(3) δ_k を求める.



$$\delta_k = \int_0^l \frac{M\bar{M}}{EI}\,dl = \int_k^l \frac{M\bar{M}}{EI}\,d\bar{x}.$$

P27 表2.1を用いて.

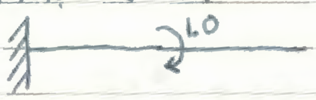


$$\int_k^l \frac{M\bar{M}}{EI}\,dl = \frac{l-k}{6EI}\left\{2(-Pl)\{-(l-k)\} + \{-P(k)\}\{-(l-k)\}\right\}$$

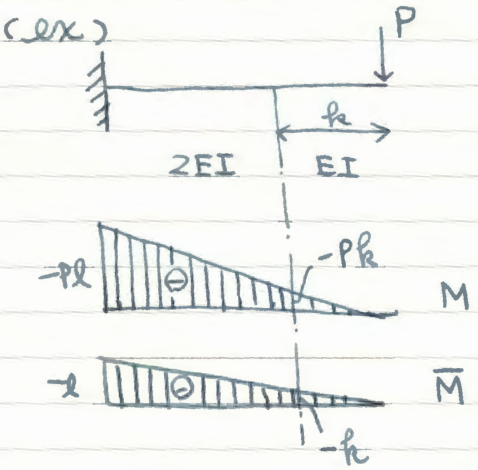
$$= \frac{l-k}{6EI}\left\{2Pl(l-k) + Pk(l-k)\right\}$$

$$= \frac{P(l-k)^2}{6EI}(2l+k)$$

(4) i_k を求める.



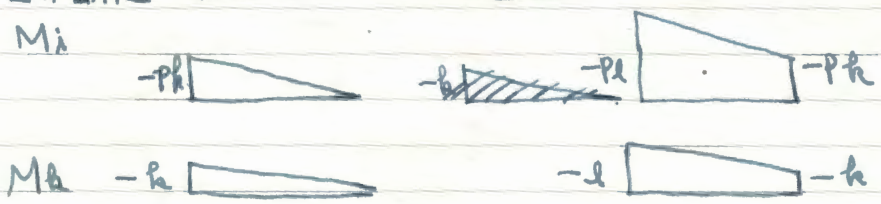
(ex)



$$\delta_B = \int_0^l \frac{M\bar{M}}{EI}\,dl$$

$$= \int_0^k \frac{M\bar{M}}{EI}\,dl + \int_k^l \frac{M\bar{M}}{2EI}\,dl$$

区間	$0 \sim k$	$k \sim l$
区間長	k	$l-k$
曲げ剛性	EI	$2EI$



$\int M_i M_k\,dl$ $\qquad \frac{k^3}{3}P \qquad \frac{l-k}{6}\left[Pl(2l+k)+Pk(2k+l)\right]$

$$= \frac{l-k}{6}(l^2+kl+k^2)\,2P$$

($y_b = y_b' = 0$ とおくと.
$\frac{l}{6}\cdot 2 y_a y_a' = \frac{l}{3} y_a y_a'$)

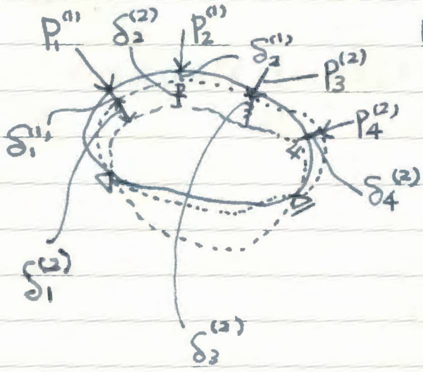
$\int \frac{M_i M_k}{EI}\,dl$ $\qquad \frac{k^3}{3EI}P \qquad \frac{l-k}{6EI}(l^2+kl+k^2)P$

$$\delta_B = \frac{k^3}{3EI}P + \frac{l-k}{6EI}(l^2+kl+k^2)P$$

$$= \frac{Pl^3}{3EI}\left\{\left(\frac{k}{l}\right)^3 + \frac{l-k}{2l}\left(1+\frac{k}{l}+\frac{k^2}{l^2}\right)\right\}$$

$$= \frac{Pl^3}{3EI}\left\{\left(\frac{k}{l}\right)^3 + \frac{1}{2}\left(1-\frac{k}{l}\right)\left(1+\frac{k}{l}+\frac{k^2}{l^2}\right)\right\}$$

相反作用の定理.

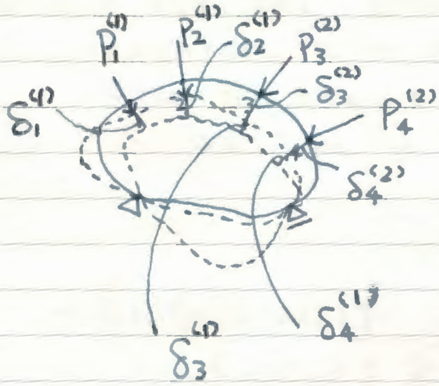


$$U^{(1)} = \frac{1}{2}P_1^{(1)}\delta_1^{(1)} + \frac{1}{2}P_2^{(1)}\delta_2^{(1)}$$

$$U^{(2)} = P_1^{(1)}\delta_1^{(2)} + P_2^{(1)}\delta_2^{(2)} + \frac{1}{2}P_3^{(2)}\delta_3^{(2)} + \frac{1}{2}P_4^{(2)}\delta_4^{(2)}$$

$$P^{(1)} \longrightarrow P^{(2)}$$

$$U = U^{(1)} + U^{(2)}$$



$$U^{(2)'} = \frac{1}{2}P_3^{(2)}\delta_3^{(2)} + \frac{1}{2}P_4^{(2)}\delta_4^{(2)}$$

$$U^{(1)'} = P_3^{(2)}\delta_3^{(1)} + P_4^{(2)}\delta_4^{(1)} + \frac{1}{2}P_1^{(1)}\delta_1^{(1)} + \frac{1}{2}P_2^{(1)}\delta_2^{(1)}$$

$$P^{(2)} \longrightarrow P^{(1)}$$

$$U' = U^{(1)'} + U^{(2)'}$$

$U = U'$ より. (荷重ののせ方によってひずみエネルギーは変らない)

$$P_1^{(1)}\delta_1^{(2)} + P_2^{(1)}\delta_2^{(2)} = P_3^{(2)}\delta_3^{(1)} + P_4^{(2)}\delta_4^{(1)}$$

$$\Downarrow$$

$$P_1^{(1)}\delta_1^{(2)} + P_2^{(1)}\delta_2^{(2)} + P_3^{(1)}\delta_3^{(2)} + P_4^{(1)}\delta_4^{(2)} = P_1^{(2)}\delta_1^{(1)} + P_2^{(2)}\delta_2^{(1)} + P_3^{(2)}\delta_3^{(1)} + P_4^{(2)}\delta_4^{(1)}$$

$$\Downarrow$$

$$\mathbb{P}^{(1)}\boldsymbol{\delta}^{(2)} = \mathbb{P}^{(2)}\boldsymbol{\delta}^{(1)}$$

★ BETTI'S RECIPROCAL THEOREM

FOR A LINEARLY ELASTIC STRUCTURE, THE WORK DONE BY A SET OF EXTERNAL FORCES $\mathbb{P}^{(1)}$, ACTING THROUGH THE DISPLACEMENTS DUE TO A SET OF EXTERNAL FORCES $\mathbb{P}^{(2)}$, IS EQUAL TO THE WORK DONE BY THE FORCES $\mathbb{P}^{(2)}$, ACTING THROUGH THE DISPLACEMENTS DUE TO THE SET OF FORCES, $\mathbb{P}^{(1)}$

$P^{(1)}$ ∝ ···· $P_i = 1.0$

$P^{(2)}$ ····· $P_k = 1.0$.

$\delta^{(1)}$ ···· δ_{ik} (k点に単位荷重が載荷する時のP_i方向のたわみ)

$\delta^{(2)}$ ··· δ_{ki} (i点 〃 〃 P_k 〃 〃)

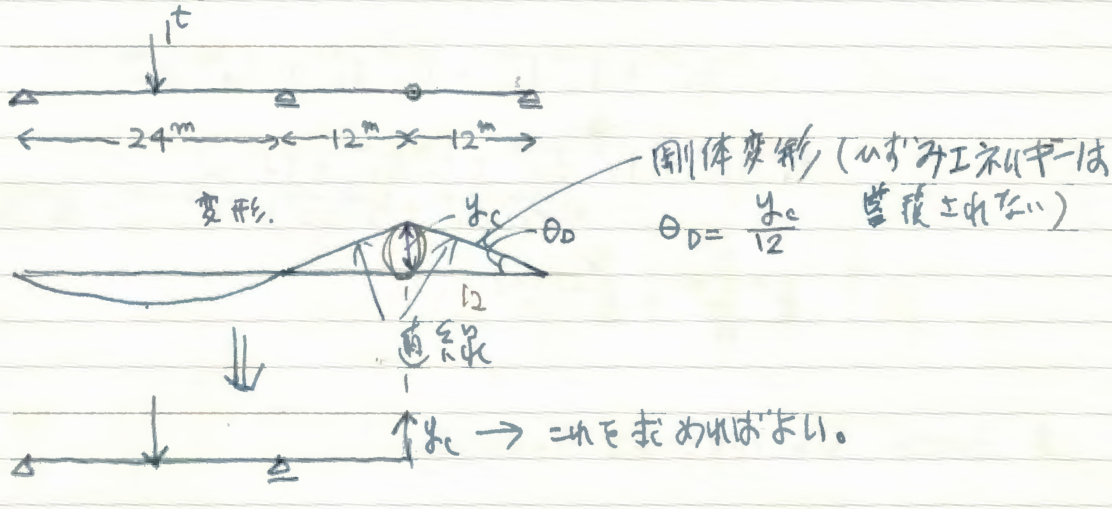
$$\boxed{\delta_{ik} = \delta_{ki}}$$

☆ MAXWELL'S RECIPROCAL THEOREM

FOR A LINEARLY ELASTIC STRUCTURE, THE DEFLECTION COMPONENT ~~FO~~ AT ANY POINT "i" DUE TO A COMPONENT OF FORCE APPLIED AT ANY POINT "k" IS EQUAL TO THE COMPONENT OF DEFLECTION AT "k" DUE TO AN ~~COMPONENT OF~~ EQUAL FORCE APPLIED AT "i".

P41. 研究課題. 1. ～ 6. レポート提出.　　5/6 (金) はじまる時間まで… 電卓もってくる.

5/6. テスト問題.

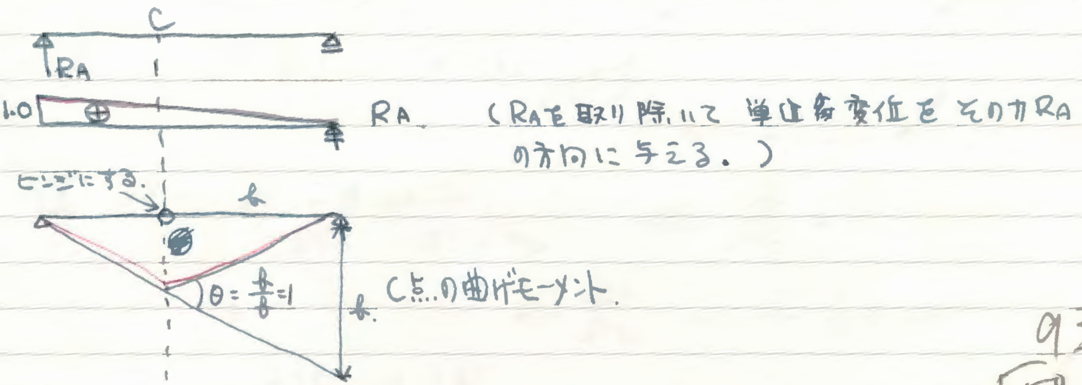


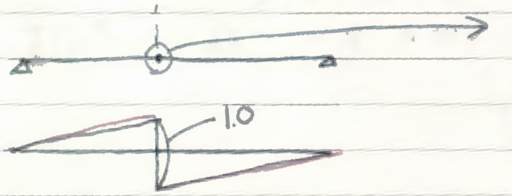
5/6

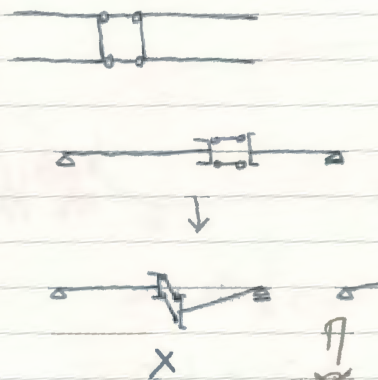
★ MÜLLER-BRESLAU PRINCIPLE

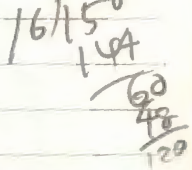
THE INFLUENCE LINE FOR ANY FORCE QUANTITY IN A STRUCTURE IS REPRESENTED TO SOME SCALE BY THE DEFLECTED SHAPE OF THE STRUCTURE RESULTING FROM MOVING THE FORCE QUANTITY UNDER CONSIDERATION THROUGH A SMALL DISPLACEMENT.

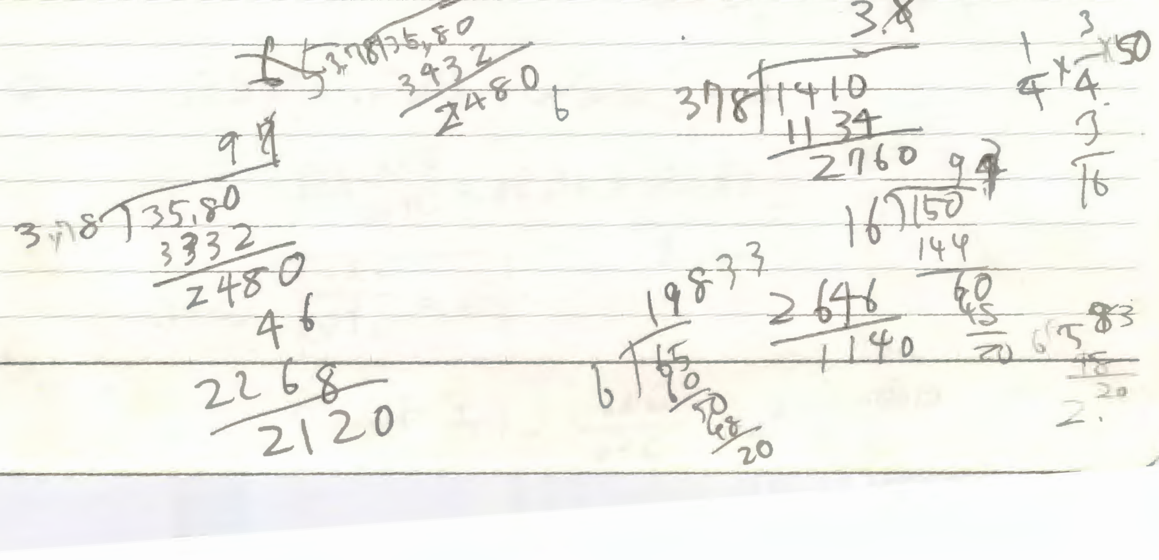
例えば









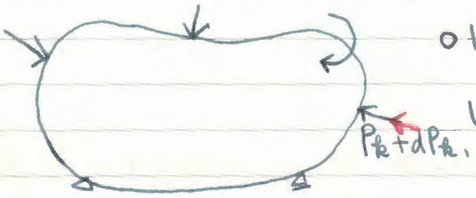


NO

Castigliano's Theorem (The second theorem)

If a linearly elastic structure is subjected to a set of loads, the displacement of any load is equal to the partial delivative of the strain energy with respect to that load.

For instance



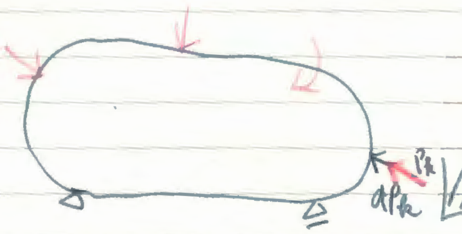
○ $U_0 = \frac{1}{2}\sum_i P_i \delta_i$

$U_i = U_i(P_1, P_2, \cdots, P_n)$

○ $\frac{\partial U_i}{\partial P_k} dP_k$ ⇐ (はじめに $P_1 \sim \cdots P_k \cdots \sim P_n$ 次に dP_k) Ⓟ

$U_0 = \frac{1}{2}\sum P_i \delta_i + \frac{\partial U_i}{\partial P_k} dP_k$ ——— ①

(はじめに dP_k 次に $P_1 \sim \cdots P_k \cdots \sim P_n$) Ⓟ



○ $U_0 = \frac{1}{2} dP_k d\delta_k$

○ $dP_k \delta_k + \frac{1}{2}\sum P_i \delta_i$

$\therefore\ U_0 = \frac{1}{2} dP_k d\delta_k + dP_k \delta_k + \frac{1}{2}\sum P_i \delta_i$ ——— ②

①, ② より $\frac{1}{2} dP_k d\delta_k + dP_k \delta_k = \frac{\partial U_i}{\partial P_k} dP_k$

neglect.

$\therefore\ \boxed{\delta_k = \frac{\partial U_i}{\partial P_k}}$ (2·33)

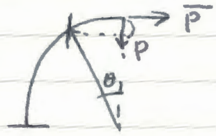
同様に. $P_k = \frac{\partial U_i}{\partial \delta_k}$ (first theorem)

P42.

6. $M_\theta = Pr\sin\theta$ $\quad d\ell = r\,d\theta$.

$$U_i = \int_0^{\pi/2} \frac{P^2 r^2 \sin^2\theta}{2EI} \cdot r\,d\theta = \frac{P^2 r^3}{2EI}\int_0^{\pi/2} \sin^2\theta\,d\theta = \frac{P^2 r^3 \pi}{8EI}$$

$$\therefore \delta_{AV} = \frac{\partial U_i}{\partial P} = \frac{P\pi r^3}{4EI}$$ //



$$M_\theta = Pr\sin\theta + \overline{P}r(1-\cos\theta)$$

$$\delta_{AH} = \frac{\partial U_i}{\partial \overline{P}} = \int \frac{M_\theta}{EI}\frac{\partial M_\theta}{\partial \overline{P}}\, r\cdot d\theta$$

$$= \int_0^{\pi/2} \frac{Pr\sin\theta + \overline{P}r(1-\cos\theta)}{EI} \cdot r(1-\cos\theta)\, r\,d\theta$$

$$= \frac{Pr^3}{EI}\int_0^{\pi/2} \sin\theta(1-\cos\theta)\,d\theta$$

$$= \frac{Pr^3}{EI}\int_1^0 (t-1)\,dt = \frac{Pr^3}{EI}\left[t - \frac{t^2}{2}\right]_0^1 = \frac{Pr^3}{2EI}$$ //

7.

i) $$U_i = \int_0^{1.5} \frac{(\frac{P}{3}x)^2}{2EI_1}dx + \int_{1.5}^{3.0} \frac{(\frac{P}{3}x)^2}{2EI_2}dx + \int_0^{1.5} \frac{(\frac{2}{3}Px)^2}{2EI_1}dx$$

$$\therefore \delta_b = \int_0^{1.5} \frac{Px^2}{9EI_1}dx + \int_{1.5}^{3} \frac{Px^2}{9EI_2}dx + \int_0^{1.5} \frac{4Px^2}{9EI_1}dx$$

$$= \frac{P\cdot(1.5)^3}{3\times 9\times EI_1} + \frac{P\{3^3-(1.5)^3\}}{3\times 9\times EI_2} + \frac{4P\times(1.5)^3}{3\times 9\times EI_1}$$

$$= \frac{5000}{3\times 9\times 2.1\times 10^6}\left\{\frac{(1.5)^3}{12000} + \frac{3^3-(1.5)^3}{20000} + \frac{4\times(1.5)^3}{12000}\right\}\times 100^2$$

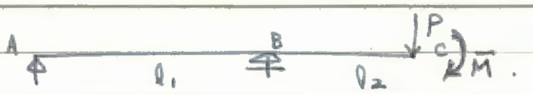
$=$

$$\frac{P\ell_1\ell_2 h}{3EI}$$

0.97

0.97

NO.

10. 

EI = const.

$R_A + R_B = P.$

$\Sigma M_B = l_1 R_A + P l_2 = 0 \quad \therefore \; R_A = -\frac{l_2}{l_1}P. \qquad R_B = \frac{l_1 + l_2}{l_1} P.$

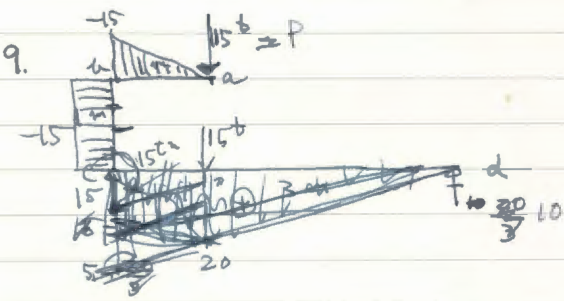
$$\delta_c = \Sigma \int \frac{M_x}{EI}\frac{\partial M_x}{\partial P}\,dx = \int_0^{l_1} \frac{(-\frac{l_2}{l_1}Px)}{EI}\left(-\frac{l_2}{l_1}x\right)dx + \int_0^{l_2}\frac{(-Px')}{EI}(-x')\,dx'$$

$$= \frac{P}{EI}\cdot\frac{l_2^2}{l_1^2}\cdot\frac{l_1^3}{3} + \frac{P}{EI}\cdot\frac{l_2^3}{3} = \frac{P l_2^2}{3EI}(l_1 + l_2)$$ //

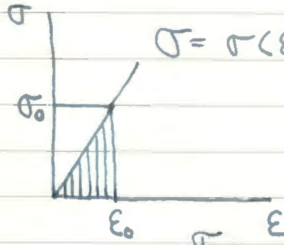
$R_A = -\frac{l_2 P + \bar{M}}{l_1}$

$$\theta_c = \int_0^{l_1}\frac{(-\frac{l_2}{l_1}Px)}{EI}\cdot\left(-\frac{x}{l_1}\right)dx + \int_0^{l_2}\frac{(-Px')}{EI}(-1)\,dx'$$

$$= \frac{P}{EI}\frac{l_2}{l_1^2}\frac{l_1^3}{3} + \frac{P}{EI}\frac{l_2^2}{2} = \frac{P l_2}{EI}\left(\frac{l_1}{3} + \frac{l_2}{2}\right)$$ //

9. 

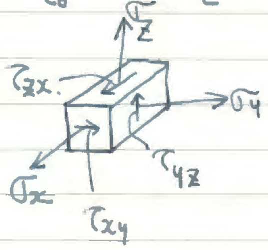
5/7.



$\sigma = \sigma(\varepsilon)$

$dU_i = \sigma \, d\varepsilon$

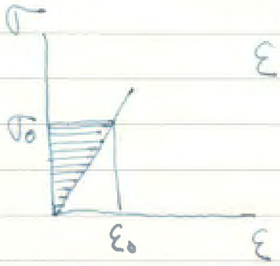
$U_i = \int_0^{\varepsilon_0} \sigma \, d\varepsilon = \frac{1}{2} \sigma_0 \varepsilon_0$



$U_i = \int \sigma^* d\varepsilon$

$\sigma = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{zx} \\ \tau_{xy} \\ \tau_{yz} \end{bmatrix}$ $\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{zx} \\ \gamma_{xy} \\ \gamma_{yz} \end{bmatrix}$

<u>Strain Energy Function</u>



$\varepsilon = \varepsilon(\sigma)$

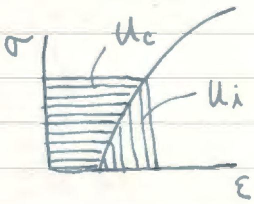
$dU_c = \varepsilon \, d\sigma$

$U_c = \int_0^{\sigma_0} \varepsilon \, d\sigma = \frac{1}{2} \sigma_0 \varepsilon_0$

$U_c = \int \varepsilon^* d\sigma$

Complementary Energy Function (物理的意味をもたない。)

For Linear Material (線形) $U_c = U_i$



First THEOREM : U_i : ひずみの関数.

$\therefore \frac{\partial U_i}{\partial \delta_k} = P_k$.

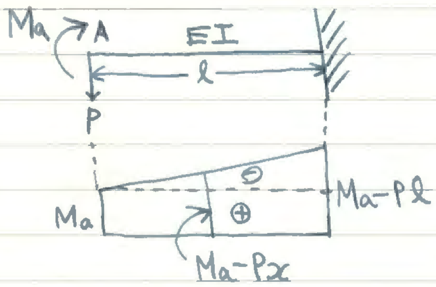
Second THEOREM : U_c : 応力の関数.

$\therefore \frac{\partial U_c}{\partial P_k} = \delta_k$

(Linear であれば $\frac{\partial U_i}{\partial P_k} = \delta_k$ でも可)

$$\boxed{\frac{\partial U_i}{\partial P} = \int \frac{M_x}{EI} \frac{\partial M_x}{\partial P} dx}$$

(FOR INSTANCE)



$$U_i = \frac{1}{2}\int_0^l \frac{M_x^2}{EI}\,dx$$

$$= \frac{1}{2}\frac{1}{EI}\int_0^l (M_a - Px)^2\,dx$$

$$= \frac{1}{2EI}\left(M_a^2 l - M_a P l^2 + \frac{P^2 l^3}{3}\right)$$

$$\delta_A = \frac{\partial U_i}{\partial P} = \frac{1}{2EI}\left(-M_a l^2 + \frac{2Pl^3}{3}\right)$$

$$= -\frac{l^2}{2EI}M_a + \frac{l^3}{3EI}P$$

$$\theta_A = \frac{\partial U_i}{\partial M_a} = \frac{1}{2EI}\left(2M_a l - Pl^2\right)$$

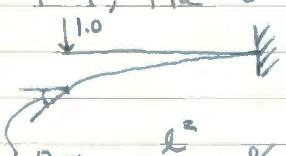
$$= \frac{l}{EI}M_a - \frac{l^2}{2EI}P$$

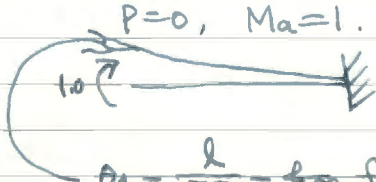
$$\begin{bmatrix}\delta_A \\ \theta_A\end{bmatrix} = \begin{bmatrix}\frac{l^3}{3EI} & -\frac{l^2}{2EI} \\ -\frac{l^2}{2EI} & \frac{l}{EI}\end{bmatrix}\begin{bmatrix}P \\ M_a\end{bmatrix}$$

$P=1,\ M_a=0.$ $\qquad$ $P=0,\ M_a=1$

$\delta_A = \frac{l^2}{3EI} = \sout{k_{11}}\ f_{11}$ $\qquad$ $\delta_A = -\frac{l^2}{2EI} = \sout{k_{12}}\ f_{12}$

$P=1,\ M_a=0$ $\qquad$ $P=0,\ M_a=1.$





$\theta_A = -\frac{l^2}{2EI} = \sout{k_{21}}\ f_{21}$ $\qquad$ $\theta_A = \frac{l}{EI} = \sout{k_{22}}\ f_{22}$

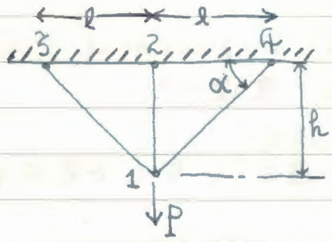
$$\begin{cases} \delta = [\sout{k}\ f]\cdot P \\ P = [f]^{-1}\cdot \delta \\ \quad = [k]\cdot \delta \end{cases}$$

$[\sout{k}\ f]$: ~~stiffness~~ flexibility Matrix.

$[k]$: stiffness Matrix

第3章. 不静定構造物解法概説.

(ex)



部材力 N_{12}, N_{13}, N_{14} （N_{134}） 未知量

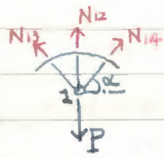
部材長 l_{12}, l_{13}, l_{14} — l_{134}

部材断面積 A_{12} A_{134}

格点変位 U_1 Δ_{12}, Δ_{13}, Δ_{14}

格点1の

I. 釣合条件 (equation of equilibrium) 力のみの関係式



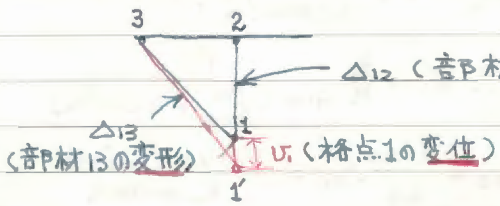
$$\Sigma H = 0. \quad N_{14}\cos\alpha - N_{13}\cos\alpha = 0$$

$$\therefore N_{14} = N_{13} \quad \text{———} ①$$

$$\Sigma V = 0. \quad N_{12} + N_{13}\sin\alpha + N_{14}\sin\alpha - P = 0 \quad \text{———} ②$$

格点1の 位

II. 変形の適合条件 (equation of compatibility) 変位・変形のみの関係式



{ 変形: deformation
{ 変位: displacement

$$l_{12} + \Delta_{12} = h + U_1 \qquad \therefore \Delta_{12} = U_1 \quad \text{———} ③$$

$$(l_{13} + \Delta_{13})^2 = l^2 + (l_{12} + \Delta_{12})^2$$

$$l_{13}^2 * = l^2 + l_{12}^2$$

$$\therefore 2 l_{13}\Delta_{13} + \underbrace{\Delta_{13}^2}_{\text{neglect}} = 2 l_{12}\Delta_{12} + \underbrace{\Delta_{12}^2}_{\text{neglect}}$$

$$l_{13}\Delta_{13} = l_{12}\Delta_{12}$$

$l_{12} = l_{13}\sin\alpha$ より. $\Delta_{13} - \Delta_{12}\sin\alpha = 0$ — ④

III. 部材力・変形関係. (応力・ひずみ関係)

↓

構成方程式 (constitutive equation) $\sigma = E \cdot \varepsilon$

Hook's LAW.

$$\Delta_{12} = \frac{N_{12}}{EA_{12}} l_{12} \quad ⑤ \qquad \Delta_{13} = \frac{N_{13}}{EA_{134}} l_{13} \quad ⑥ \qquad \left(\Delta_{14} = \frac{N_{14}}{EA_{134}} l_{14}\right) \quad ⑦$$

NO

5/13

〈解法〉

1) 応力法（Force Method）

未知数 N_{12}, $N_{13}(=N_{14})$

{釣合条件式 / 変位の適合条件} を部材力で表わす。

適合条件. $\Delta_{13}=\Delta_{12}\sin\alpha$.

$$\frac{N_{13}}{EA_{134}}h\operatorname{cosec}\alpha=\frac{N_{12}}{EA_{12}}h\sin\alpha$$

$$\therefore N_{12}=\frac{A_{12}}{A_{134}}\operatorname{cosec}^2\alpha\cdot N_{13}$$

釣合条件.

$$N_{12}+2N_{13}\sin\alpha-P=0$$

$$\therefore\left(\frac{A_{12}}{A_{134}}\operatorname{cosec}^2\alpha+2\sin\alpha\right)N_{13}=P$$

$$\therefore N_{13}=\frac{\operatorname{cosec}\alpha}{\frac{A_{12}}{A_{134}}\operatorname{cosec}^3\alpha+2}P$$

$$N_{12}=\frac{A_{12}}{A_{134}}\operatorname{cosec}^2\alpha\, N_{13}$$

$$=\frac{A_{12}}{A_{134}}\operatorname{cosec}^2\alpha\,\frac{\operatorname{cosec}\alpha}{\frac{A_{12}}{A_{134}}\operatorname{cosec}^3\alpha+2}P$$

$$\therefore N_{12}=\frac{\operatorname{cosec}^3\alpha}{\operatorname{cosec}^3\alpha+2\frac{A_{134}}{A_{12}}}P$$

$$v_1=\Delta_{12}=\frac{N_{12}}{EA_{12}}l_{12}=\frac{h}{EA_{12}}N_{12}$$

$$=\frac{h}{EA_{12}}\,\frac{\operatorname{cosec}^3\alpha}{\operatorname{cosec}^3\alpha+2\frac{A_{134}}{A_{12}}}P$$

$$\therefore v_1=\frac{h}{E}\cdot\frac{1}{A_{134}\left(\frac{A_{12}}{A_{134}}+2\sin^3\alpha\right)}P$$

2) 変形法（Displacement Method）

未知数 変位 v_1

{釣合条件式 / 構成方程式} を変位で表わす。

$$\begin{cases} v_1\sin\alpha=\dfrac{N_{13}}{EA_{134}}l_{13} \\ v_1=\dfrac{N_{12}}{EA_{12}}l_{12}\end{cases}$$

釣合条件式に代入.

$$\frac{EA_{12}}{l_{12}}v_1+2\frac{EA_{134}\sin\alpha}{l_{13}}\sin\alpha\cdot v_1-P=0$$

$$\left(\frac{EA_{12}}{h}+2\frac{EA_{134}}{h\operatorname{cosec}^3\alpha}\right)v_1=P$$

$$\therefore v_1=\frac{h}{E}\cdot\frac{P}{A_{134}\left(\frac{A_{12}}{A_{134}}+2\sin^3\alpha\right)}$$

$$N_{13}=\frac{EA_{134}}{l_{13}}\Delta_{13}=\frac{EA_{134}v_1\sin\alpha}{h\operatorname{cosec}\alpha}$$

$$=\frac{EA_{134}}{h\operatorname{cosec}^2\alpha}\cdot\frac{h}{EA_{134}}\cdot\frac{P}{\left(\frac{A_{12}}{A_{134}}+2\sin^3\alpha\right)}$$

$$\therefore N_{13}=\frac{\operatorname{cosec}\alpha}{\frac{A_{12}}{A_{134}}\operatorname{cosec}^3\alpha+2}P$$

$$N_{12}=\frac{EA_{12}}{l_{12}}v_1=\frac{EA_{12}}{l_{12}}\cdot\frac{h}{EA_{134}}\cdot\frac{P}{\left(\frac{A_{12}}{A_{134}}+2\sin^3\alpha\right)}$$

$$=\frac{P}{\left(1+2\frac{A_{134}}{A_{12}}\sin^3\alpha\right)}$$

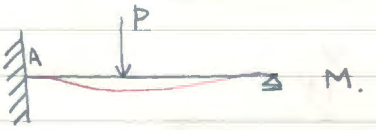
$$N_{12}=\frac{\operatorname{cosec}^3\alpha}{\left(\operatorname{cosec}^3\alpha+2\frac{A_{134}}{A_{12}}\right)}P$$

応力法.	変形法.
未知数　部材力	未知数　変位
↓	↓
構成方程式を使って	構成方程式を使って
↓	↓
適合条件、釣合条件 } を力で表わす	釣合条件を変位で表わす。
余力法	たわみ角法
三連・四連モーメントの定理	直接剛性法.

第4章　弾性方程式による不静定構造物の解法.

(解1)

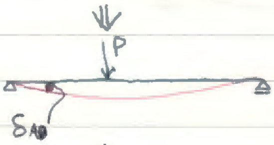
たわみ角は零



一次不静定

⇓

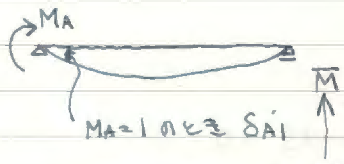
たわみ角が生ずる



釣合条件 ↓ M_0

δ_{A0} ← 一般的な変位(ここではたわみ角)；解析する構造物でないことを示す。↑ 着目点

＋



$M_A=1$ のとき δ_{A1}

$\overline{M}$ ↑ $M_A=1$ のときの曲げモーメント

適合条件. $\delta_{A0} + M_A \delta_{A1} = 0$

$$\delta_{A0} = \int \frac{M_0 \overline{M}}{EI} ds.$$

$$\delta_{A1} = \int \frac{\overline{M}^2}{EI} ds.$$

$$M = M_0 + \overline{M} \cdot M_A$$

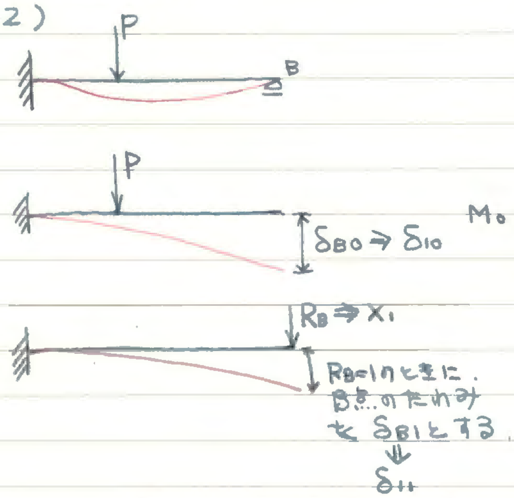
置きかえ　$M_A \Rightarrow$ ~~M_A~~ X_1　　$(A \Rightarrow 1)$

$\overline{M} \Rightarrow M_1$

$$\delta_{10} = \int \frac{M_0 M_1}{EI} dl \ , \quad \delta_{11} = \int \frac{M_1^2}{EI} ds$$

☆ $\delta_{10} + X_1 \delta_{11} = 0 \quad \therefore \delta_{11} X_1 = -\delta_{10}$

（解２）



適合条件. $\delta_{10} + \delta_{11} X_1 = 0$

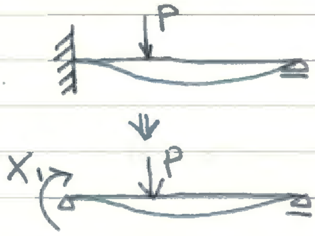
$\therefore \delta_{11} X_1 = -\delta_{10}$

$$\delta_{10} = \int \frac{M_0 M_1}{EI} ds.$$

$$\delta_{11} = \int \frac{M_1^2}{EI} ds.$$

$R_B=1$のときの曲げモーメント.
を M_1とする.

5/14. 前日よりつづく.



$$M = M_0 + M_1 X_1$$

$$U = \int_0^l \frac{M^2}{2EI} ds$$

$$\frac{\partial U}{\partial X_1} = 0$$ ： 適合条件.

$$\int_0^l \frac{M}{EI} \frac{\partial M}{\partial X_1} ds = 0$$

$$\int_0^l \frac{M}{EI} M_1 ds = \int_0^l \frac{(M_0 + M_1 X_1) M_1}{EI} ds$$

$$= \int_0^l \frac{M_0 M_1}{EI} ds + X_1 \int_0^l \frac{M_1^2}{EI} ds = 0$$

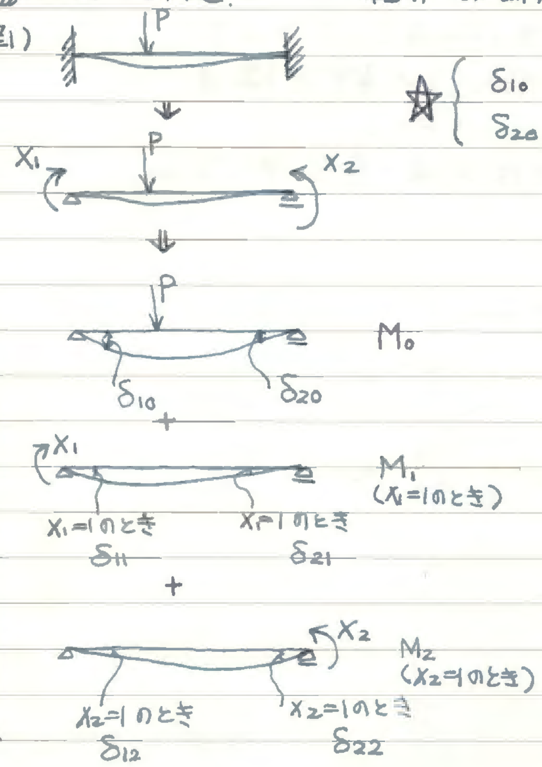
$$\begin{cases} \delta_{10} = \int_0^l \frac{M_0 M_1}{EI} ds \\ \delta_{11} = \int_0^l \frac{M_1^2}{EI} ds \end{cases}$$ とおくと

$$X_1 = -\frac{\delta_{10}}{\delta_{11}}$$ ： 弾性方程式

$\delta_{11}X_1 + \delta_{10}$ …

◎ 二次不静定.　　軸力. せん断力無視.

(解1)



☆ $\begin{cases} \delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \\ \delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \end{cases}$

$$\delta_{10} = \int_0^l \frac{M_0 M_1}{EI} ds \qquad \delta_{20} = \int_0^l \frac{M_0 M_2}{EI} ds$$

$$\delta_{11} = \int_0^l \frac{M_1^2}{EI} ds \qquad \delta_{21} = \int_0^l \frac{M_1 M_2}{EI} ds$$

$$\delta_{12} = \int_0^l \frac{M_2 M_1}{EI} ds \qquad \delta_{22} = \int_0^l \frac{M_2^2}{EI} ds$$

これより X_1, X_2 は求まる。

$$M = M_0 + M_1 X_1 + M_2 X_2$$

$$u = \int_0^l \frac{M^2}{2EI} ds$$

つりあっているとき. X_1 X_2 は u を最小にする (最小仕事の定理)

$$\begin{cases} \dfrac{\partial u}{\partial X_1} = \displaystyle\int_0^l \frac{M}{EI}\frac{\partial M}{\partial X_1} ds = 0 \\ \dfrac{\partial u}{\partial X_2} = \displaystyle\int_0^l \frac{M}{EI}\frac{\partial M}{\partial X_2} ds = 0 \end{cases}$$

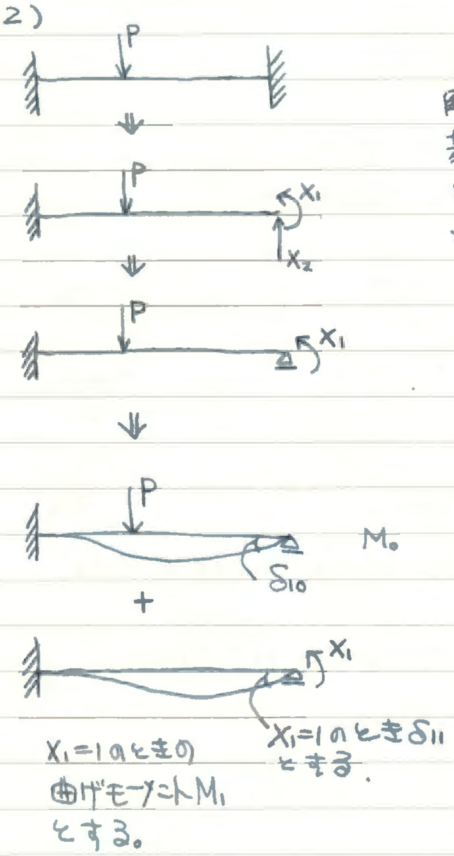
$$\begin{cases} \dfrac{\partial u}{\partial X_1} = \displaystyle\int_0^l \frac{(M_0 + M_1X_1 + M_2X_2)}{EI} M_1 ds = \int_0^l \frac{M_0M_1}{EI} ds + X_1\int_0^l \frac{M_1^2}{EI} ds + X_2 \int_0^l \frac{M_1M_2}{EI} ds \\ \dfrac{\partial u}{\partial X_2} = \displaystyle\int_0^l \frac{(M_0 + M_1X_1 + M_2X_2)}{EI} M_2 ds = \int_0^l \frac{M_0M_2}{EI} ds + X_1\int_0^l \frac{M_1M_2}{EI} ds + X_2 \int_0^l \frac{M_2^2}{EI} ds \end{cases}$$

$$\delta_{10} = \int_0^l \frac{M_0M_1}{EI} ds \qquad \delta_{20} = \int_0^l \frac{M_0M_2}{EI} ds \qquad \delta_{12} = \delta_{21} = \int_0^l \frac{M_1M_2}{EI} ds.$$

$$\delta_{11} = \int_0^l \frac{M_1^2}{EI} ds \qquad \delta_{22} = \int_0^l \frac{M_2^2}{EI} ds$$ とおくと.

$$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0, \quad \delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0$$

(解2)



解の求まっているものであれば基本の形として不静定構造物をとってもよい。
そうすれば式は一層簡単になる。

$$\delta_{10} + \delta_{11} X_1 = 0.$$

$$\begin{cases} \delta_{10} = \int_0^l \frac{M_0 M_1}{EI} ds \\ \delta_{11} = \int_0^l \frac{M_1^2}{EI} ds \end{cases}$$

◎ 弾性方程の一般化.

n次不静定構造物
- 不静定力 $X_1, X_2, \cdots, X_n$
- 支点変位 r_k
- 温度変化 t°, $\Delta t = t^l - t^u$

注)(1〜nとkは一致しない)

第i不静定力の作用点の相対変位 δ_i

$$\delta_i = \delta_{i0} + \delta_{ir} + \delta_{it} + \delta_{i1}X_1 + \delta_{i2}X_2 + \cdots + \delta_{in}X_n$$

荷重による　支点変位による　温度変化による　各不静定力による

$$\delta_{i1}X_1 + \delta_{i2}X_2 + \cdots + \delta_{in}X_n = \delta_i - (\delta_{i0} + \delta_{ir}) - \delta_{it}$$

$1 \cdot \delta_{ir} + \Sigma \bar{R}_k r_k = 0$　　$\delta_{ir} = -\Sigma \bar{R}_k r_k$　　$\bar{R}_k$: i点に1の荷重がかかったときの各反力.

第i不静定力…支点反力とする　支点変位 $r_i = \delta_i$

$$r_i - \delta_{ir} - (\delta_{i0} + \delta_{it}) = r_i + \Sigma \bar{R}_k r_k - (\delta_{i0} + \delta_{it})$$

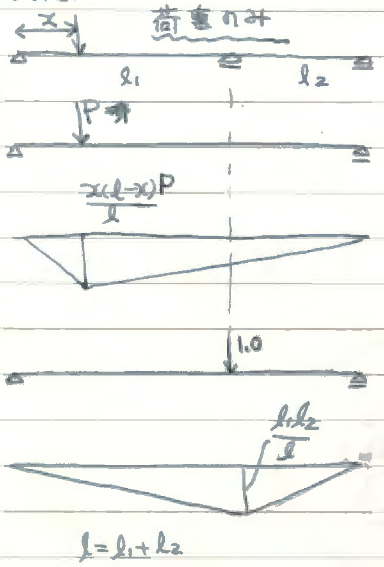
$\boxed{\delta_{ir}}$ …… 教科書P52.

$$\delta_{i1}X_1 + \delta_{i2}X_2 + \cdots + \delta_{in}X_n = -(\delta_{i0} + \delta_{it}) + \boxed{\delta_{ir}}$$

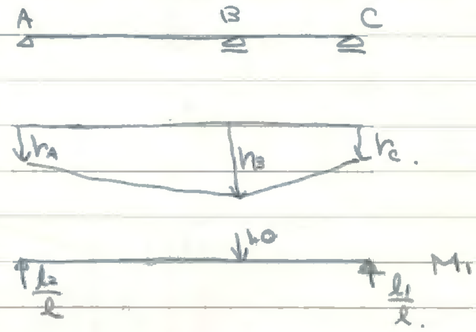
連続梁.

荷重・温度変化なし. 支点変位のみ

(ex1) 荷重のみ



(ex2)



$$\delta_{ir} = 1 \cdot r_B - \frac{l_2}{l} r_A - \frac{l_1}{l} r_B$$

$$\delta_{11} X_1 = \delta_{ir} = r_B - \frac{1}{l}(l_2 r_A + l_1 r_C)$$

$$M = M_0 + M_1 X_1 = M_1 X_1$$

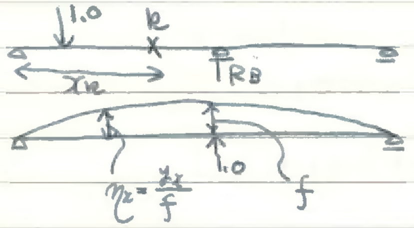
(ex3) 温度変化のみ

(4・7)より

$$\delta_{it} = \int N_i \alpha t \, ds + \int M_i \alpha \frac{t_l - t_u}{h} ds$$

$$= \frac{\alpha \Delta t}{h} \int M_i \, ds = \frac{\alpha \Delta t}{h} \frac{l_1 l_2}{2} \;//$$

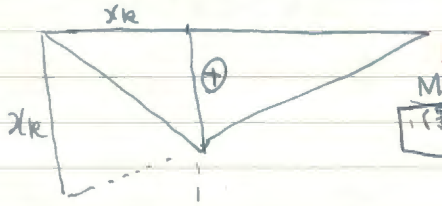
○ 影響線.



$\underline{M_{k,i}} = \underline{M_{0,k,i}} + M_{1,k} \cdot \underline{X_{1,i}}$

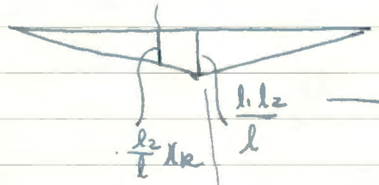
（kでの曲げモーメント）

~~$M_k = M_{0,k} + M_{1,k} \cdot X_1$~~

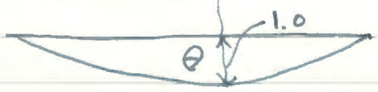


$M_{0,k,0}$

$M_{0,k,i}$ （影響線）



M_1 ⟶ 影響線ではない

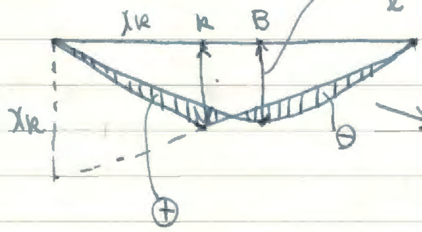


$X_{1,i}$ 反力R_B (~~支力R_B~~)

（影響線）

$M_{1,k}$

$M_{1,k} = \frac{l_2}{l} x_k$



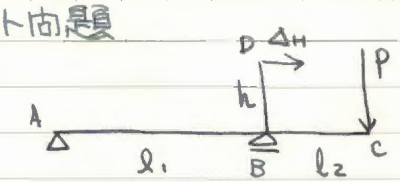
$\to = M_{0,k,i} + \underline{M_{1,k}} \cdot X_{1,i}$

$M_{1,k} = \frac{l_2}{l} x_k$ → const.

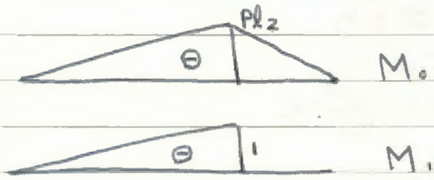
5/20

テスト問題

(1)



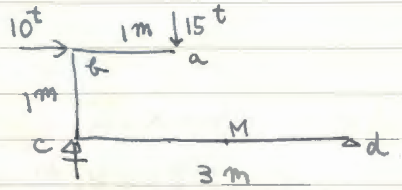
Δ_H を求める。



$$\theta = \int_0^{l_1} \frac{M_0 M_1}{EI} ds = \frac{l_1}{3} P l_2$$

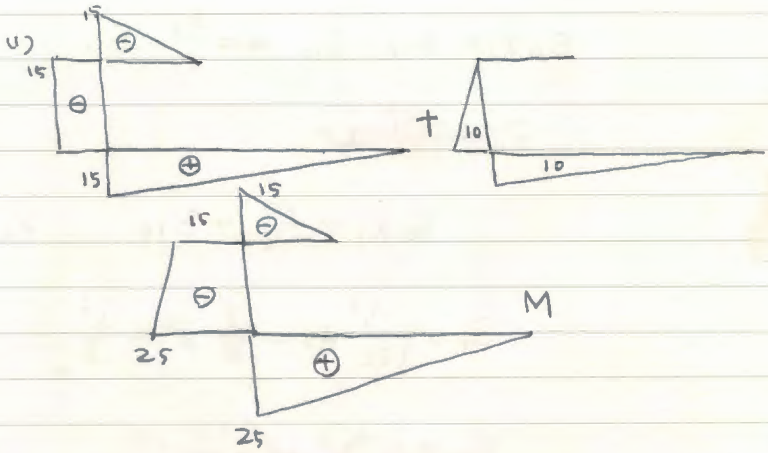
$$\therefore \Delta_H = \frac{P l_1 l_2 h}{3EI}$$

(2)

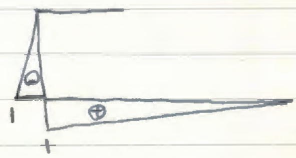


b点の水平変位 (1)
M点の鉛直変位 (2)

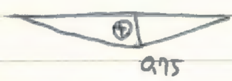
(1)

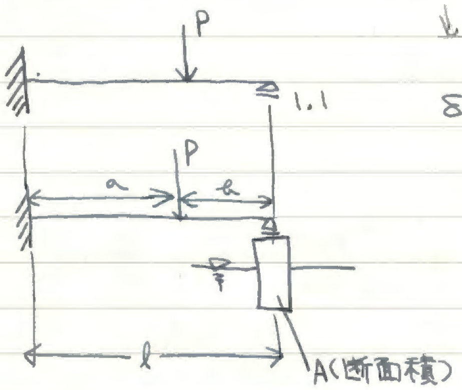


(1)



(2)





$$\delta_{11} X_1 = \delta_{1r} - \delta_{10}$$

$$\delta_{1r} = \sum \bar{R} r$$

$$\delta_{10} = \int_0^l \frac{M_0 M_1}{EI} ds$$

$$W = A d.$$

$$W + X_1 = A(d + r) \quad \therefore X_1 = A r \quad \therefore r = X_1 / A$$

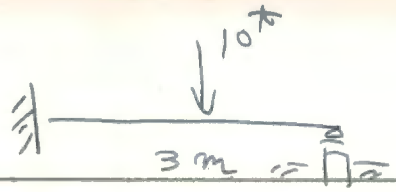
$$\delta_{11} X_1 = \delta_{1r} - \delta_{10} = \frac{X_1}{A} - \delta_{10}. \quad \therefore \delta_{1r} = \sum \bar{R} r = R_B \cdot \frac{X_1}{A} = \frac{X_1}{A}$$

~~$\delta_{11} = \int \frac{M_1^2}{EI} ds =$~~

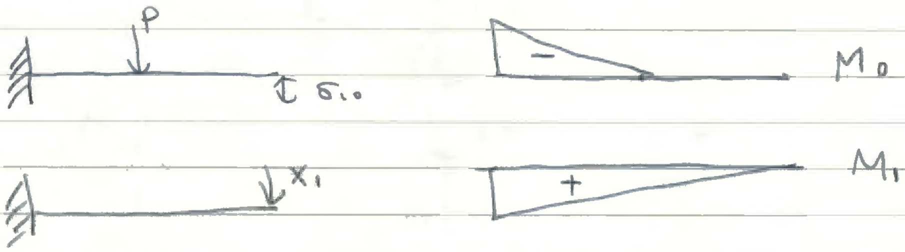
$$\delta_{11} X_1 = \frac{X_1}{A} - \delta_{10} \quad \therefore X_1 = \frac{\delta_{10}}{\frac{1}{A} - \delta_{11}} = \frac{A\delta_{10}}{1 - A\delta_{11}}$$

$$\delta_{11} = \int \frac{M_1^2}{EI} ds = \frac{l}{3} \cdot l^2 = \frac{l^3}{3}$$

$$\delta_{10} = \int \frac{M_0 M_1}{EI} ds = \frac{}{6}$$



左の題



$$\delta_{11} X_1 + \frac{X_1}{A} + \delta_{10} = 0$$

$$\delta_{11} X_1 = \delta_{1r} - \delta_{10}$$

$$\delta_{1r} = \Sigma R_r = -r = -\frac{X_1}{A}$$

$$\delta_{11} X_1 + \frac{X_1}{A} = -\delta_{10}$$

$$X_1 = \frac{\delta_{10}}{\delta_{11} + \frac{1}{A}}$$

$$\delta_{10} = \int_0^{1.5} \frac{\triangle \square}{EI} ds = -\frac{2.8125 \times 10^{-2}}{3.78}$$

$$\delta_{11} = \frac{3^2}{3} \times 3 \times \frac{1}{EI} = \frac{9}{3.78} \times 10^{-3}$$

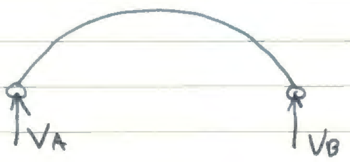
$$X_1 = \frac{2.8125}{3.989} \times 10^{-2}$$

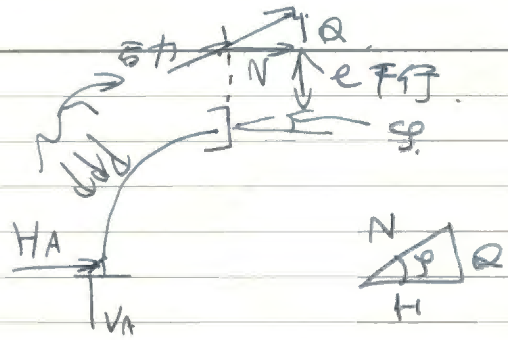
$$r = \frac{X_1}{A} = 0.774\ \text{cm}.$$

NO

5/21.

P62. アーチ.





☆ $N = H\cos\varphi + V\sin\varphi$
$Q = V\cos\varphi + H\sin\varphi$

$M = N \cdot l$

$V_A x - H_A y - \Sigma P_V (x-a) + \Sigma P_H (y-c) - M = 0$
$\therefore M = V_A x - \Sigma P_V (x-a) - \{H_A y - \Sigma P_H (y-c)\}$

荷重が鉛直ならば

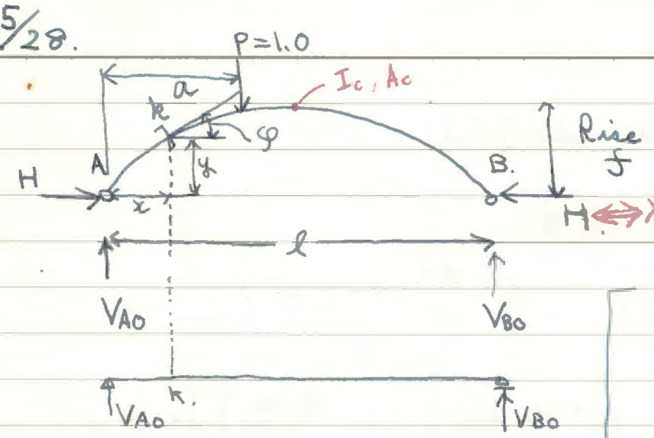
$M = V_A x - \Sigma P (x-a) - H_A y$
$M = M_0 - H_A y$.

☆ $M = M_0 - H_A y$ $(M_0 = V_A x - \Sigma P (x-a))$

$\Sigma M = 0$ about "B"
$V_A l - \Sigma P (l-a) = 0$
$V_A = \frac{\Sigma P (l-a)}{l} = V_{A0}$.

$M = M_0 - H_A y$.

5/28.



鉛直方向の荷重のみ。

H ⇔ X_1

$$M_k = M_{k,0} - H \cdot y$$

$$\left[\begin{array}{l} N_k = H_k \cos\varphi + V_k \sin\varphi \\ V_k = V_{A0} - \sum_0^{k-1} P_i - \int_0^x p\,dx \\ H_k = H + \sum_0^{k-1} P_i^H + \int_0^x p^H\,dx \end{array}\right]$$

鉛直方向のみの荷重のとき

$$N_k = H\cos\varphi + V_k \sin\varphi$$

$$\left\{\begin{array}{l} M_k = M_{k,0} + X_1 \cdot M_{k,1} \\ N_k = N_{k,0} + X_1 \cdot N_{k,1} \end{array}\right.$$

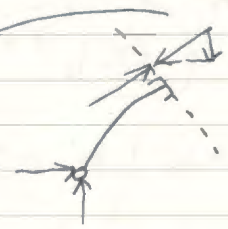
温度変化がないとき.

$$X_1 = -\frac{\delta_{10}}{\delta_{11}}$$

$$\delta_{11} = \int \frac{M_1^2}{EI} ds + \int \frac{N_1^2}{EA} ds + \int \frac{kQ_1^2}{GA} ds.$$ P62 (4.22)

$M_1 = -y$, $N_1 = \cos\varphi$

$N_1 = -\cos\varphi$.



圧縮が正.

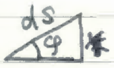
H=0

$$M_0 = M_{k,0}$$

$$N_{k,0} = -V_k \sin\varphi.$$

$$\delta_{10} = \int \frac{M_0 M_1}{EI} ds + \int \frac{N_0 M_1}{EA} ds + \cdots$$

(解)



$dx = ds\cos\varphi \qquad ds = \sec\varphi\, dx$

アーチを二次曲線と定める。

アーチ軸線 ‥‥ $y = \frac{4f}{l^2}(lx - x^2)$

$I = I_c \sec\varphi$, $A = A_c \sec\varphi$ と仮定する。

$$\delta_{11} = \int_0^l \frac{y^2 \sec\varphi\, dx}{EI_c \sec\varphi} + \int_0^l \frac{\cos^2\varphi \sec\varphi\, dx}{EA_c \sec\varphi}$$

$$= \frac{1}{EI_c}\int_0^l y^2 dx + \frac{1}{EA_c}\int_0^l \cos^2\varphi\, dx$$

$$= \frac{1}{EI_c}\int_0^l y^2 dx \left(1 + \frac{I_c}{A_c}\int_0^l \cos^2\varphi\, dx \Big/ \int_0^l y^2 dx\right)$$

$\frac{1}{\nu_A} = \left(1 + \frac{I_c}{A_c}\frac{\int_0^l \cos^2\varphi\, dx}{\int_0^l y^2 dx}\right)$ とおくと

$$X_{*1} = \frac{\frac{1}{EI_c}\int_0^l M_0 y\, dx}{\frac{1}{EI_c}\int_0^l y^2 dx}\nu_A = \frac{\int_0^l M_0 y\, dx}{\int_0^l y^2 dx}\nu_A$$

−δ10

= M1

→ δ11

Rise f が小さいとき $\cos\varphi \fallingdotseq 1$

$$\begin{cases} \int_0^l \cos^2\varphi\, dx \fallingdotseq l \\ \int_0^l y^2 dx = \frac{8}{15}\cdot\frac{l}{f^2} \end{cases}$$

$\nu_A \fallingdotseq 0.98$

NO

◎ X の影響線

$$X = \frac{\int_0^l M_0 y\,dx}{\int_0^l y^2\,dx}\,\nu_A \,, \quad \int_0^l y^2\,dx = \frac{8}{15}\frac{l}{f^2}$$

$$M_0 = \frac{l-a}{l}x \qquad x \leqq a$$

$$M_0 = \frac{l-x}{l}a \qquad a \leqq x$$

$$\int_0^l M_0 y\,dx = \int_0^a \frac{l-a}{l} x y\,dx + \int_a^l \frac{l-x}{l} a y\,dx$$

$$= \frac{af}{3l^2}(l^3 - 2a^2 l + a^3)$$

$$\therefore X = \frac{\frac{af}{3l^2}(l^3-2a^2l+a^3)}{\frac{8}{15}\frac{l}{f^2}} = \frac{5l}{8f}\left\{\frac{a}{l} - 2\left(\frac{a}{l}\right)^3 + \left(\frac{a}{l}\right)^4\right\}\nu_A$$

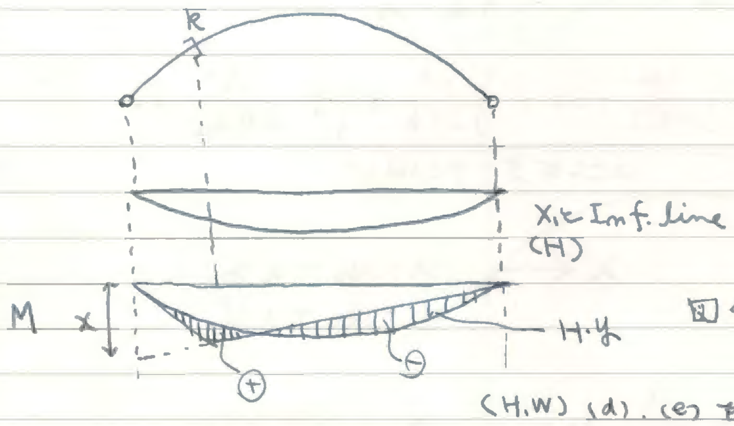
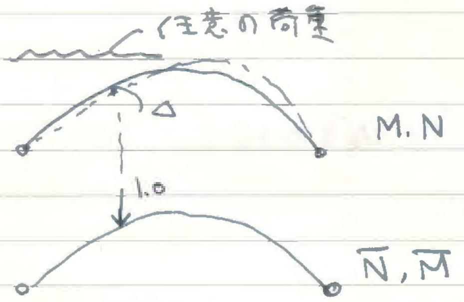


図4.15 (b)

(H.W) (d), (e) を求めておく。

◎ たわみ.



$$\Delta = \int \frac{M\overline{M}}{EI}ds + \int \frac{N\overline{N}}{EA}ds$$

(H.W) もっと簡単な式を求めよ

タイドアーチ

$$M = M_0 - (H^P + H^{\Delta l})y$$
$$= M_0 - H^P y - H^{\Delta l} y$$
$$= M + M^{\Delta l}$$

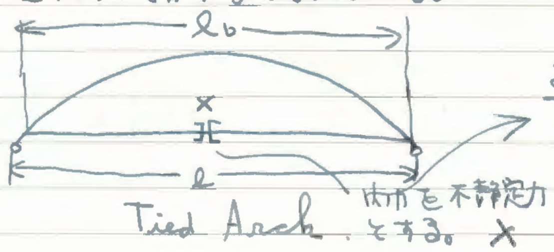
アーチでは支点変位がおこっては、B.Mが増加する。
⇒ アーチでは支点変位のない所でなければならない。

~~M + M~~ $M = M_0 - H^P y + M^{\Delta l}$

アーチによって曲げモーメントが減じた。

↓

支点が動かないようにする。



$$\frac{\partial W}{\partial X} = 0$$

$$W = \int \frac{M^2}{2EI} ds + \int \frac{N^2}{2EA} ds + \frac{X^2}{2EA_b} l_b$$

（最初の2項）2ヒンジアーチと同じ

$$M = M_0 - X y$$
$$N = N_0 + X N_1$$

$$\frac{\partial}{\partial X}\left(\frac{X^2}{2EA_b} l_b\right) = \frac{X}{EA_b} l_b \longrightarrow (4.34)$$

$$X = \frac{\int M_0 y\, ds}{\int_0^l y^2 dx} \nu_B .$$

ν_B を求めると、

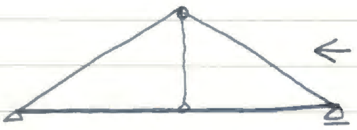
$\nu_B =$

$\nu_B = 0.94 \sim 0.96$ ⟶ 2ヒンジアーチよりXは小さくなる。

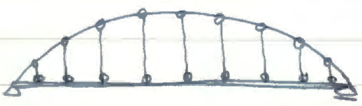
6/3.

たわみを減らしたい。

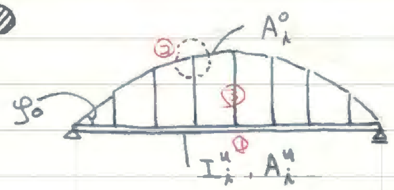
⇓

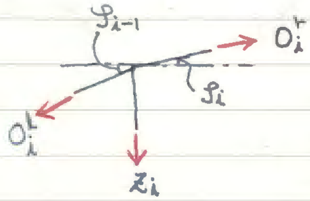


← 一次不静定



ランガーけた.





部材力の水平成分を不静定力 X とすると.
X は一定となる

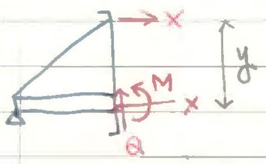
$$O_i^l \cos\varphi_{i-1} - O_i^r \cos\varphi_i = 0$$

$$X\sec\varphi_{i-1}\cos\varphi_{i-1} - X\sec\varphi_i\cos\varphi_i = 0$$

$$z_i + O_i^l \sin\varphi_{i-1} - O_i^r \sin\varphi_i = 0$$

$$z_i + X(\tan\varphi_{i-1} - \tan\varphi_i) = 0$$

$$\therefore \quad z_i = X(\tan\varphi_i - \tan\varphi_{i-1})$$



$$M = M_0 + Xy$$

$$W = W_G + W_A + W_z$$

$$= \underbrace{\int_0^l \frac{M^2}{2EI}dx + \int_0^l \frac{N^2}{2EA}dx}_{\text{けたの部分}} + \Sigma(X\sec\varphi_i)^2\rho_i + \Sigma z_i^2\rho_i$$

$$\frac{\partial M}{\partial X} = y, \quad \frac{\partial N_G}{\partial X} = -1, \quad \frac{\partial z_i}{\partial X} = \tan\varphi_i - \tan\varphi_{i-1}$$

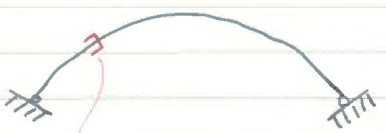
$$\rho_i = \frac{l_i}{E_i A_i}$$

$$\int_0^l \frac{M_0 y}{EI} dx + X\int \frac{y^2}{EI} dx + X\int \frac{dx}{EA} + X\sum \sec^2\varphi_i \rho_i + X\sum(\tan\varphi_i - \tan\varphi_{i-1})^2 \rho_i = 0$$

$$X = -\frac{\delta_{10}}{\delta_{11}}$$

$$\begin{cases} \delta_{10} = \int_0^l \frac{M_0 y}{EI} dx \\ \delta_{11} = \int \frac{y^2}{EI} dx + \int \frac{dx}{EA} + \sum \sec^2\varphi_i \rho_i + \sum(\tan\varphi_i - \tan\varphi_{i-1})\rho_i \end{cases}$$

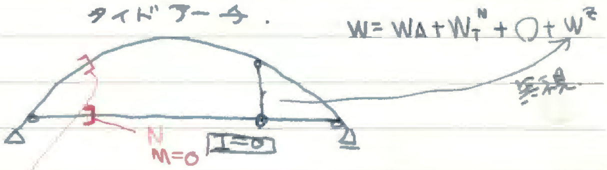
2ヒンジアーチ.



$$M = M_0 - Hy$$

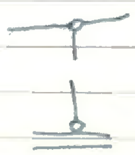
$$H = \frac{\int \frac{M_0 y}{EI} ds}{\int \frac{y^2}{EI} ds} \nu_A$$

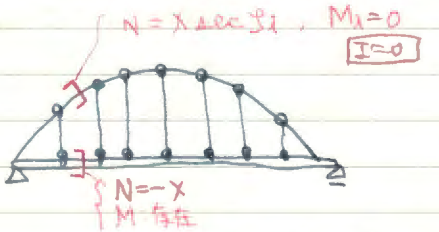
タイドアーチ.



$$M = M_0 - Hy.$$

$$H = \frac{\int \frac{M_0 y}{EI} ds}{\int \frac{y^2}{EI} ds} \nu_B.$$

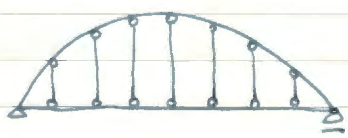




$$M = M_0 + Xy = M_0 - Hy$$

$$H = \frac{\int \frac{M_0 y}{EI} dx}{\int \frac{y^2}{EI} dx} \nu_L$$

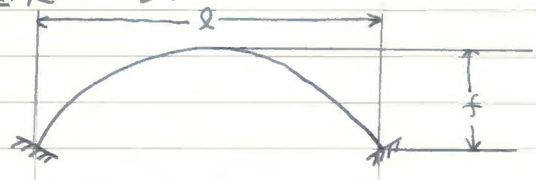
↑ ランガー.



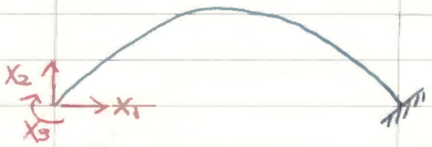
補剛アーチ

◎ 固定アーチ.

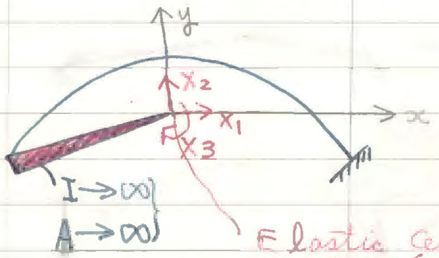
高畫による。



$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 = k_1 = -\delta_{10} \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 = k_2 = -\delta_{20} \\ \delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 = k_3 = -\delta_{30}. \end{cases}$$



$$\begin{cases} M = M_0 + X_1M_1 + X_2M_2 + X_3M_3 \\ N = N_0 + X_1N_1 + X_2N_2 + X_3N_3 \\ Q = Q_0 + X_1Q_1 + X_2Q_2 + X_3Q_3 \end{cases}$$



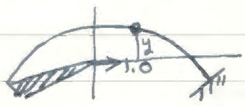
$$W^M = \int \frac{M^2}{2EI} ds$$

$$= \int \frac{M^2}{2EI} ds + \int \frac{M^2}{2EI} ds$$

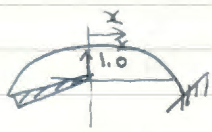
$$\Downarrow$$

$$0.$$

$$\int \frac{x}{EI} ds = 0 \qquad \int \frac{y}{EI} = 0 \qquad \int \frac{xy}{EI} ds = 0$$

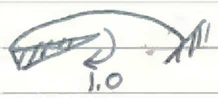


$M_1 = -y$, $(N_1 \quad Q_1)$



$M_2 = x$, $(N_2 \quad Q_2)$

$M_3 = 1.0$, $(N_3 \quad Q_3)$



$$\delta_{12} = \int \frac{M_1M_2}{EI} ds = -\int \frac{xy}{EI} = 0$$

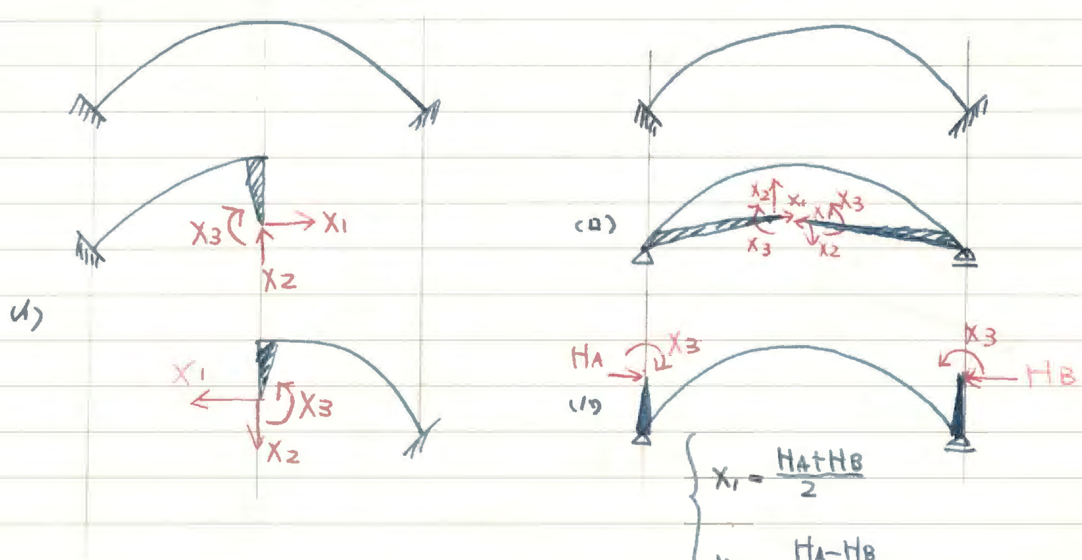
$$\delta_{13} = \int \frac{M_1M_3}{EI} ds = -\int \frac{y}{EI} = 0$$

$$\delta_{21} = \delta_{12} = 0 \qquad \delta_{23} = \int \frac{M_2M_3}{EI} ds = \int \frac{x}{EI} ds = 0.$$

$$\delta_{31} = \delta_{13} = 0 \qquad \delta_{32} = \delta_{23} = 0$$

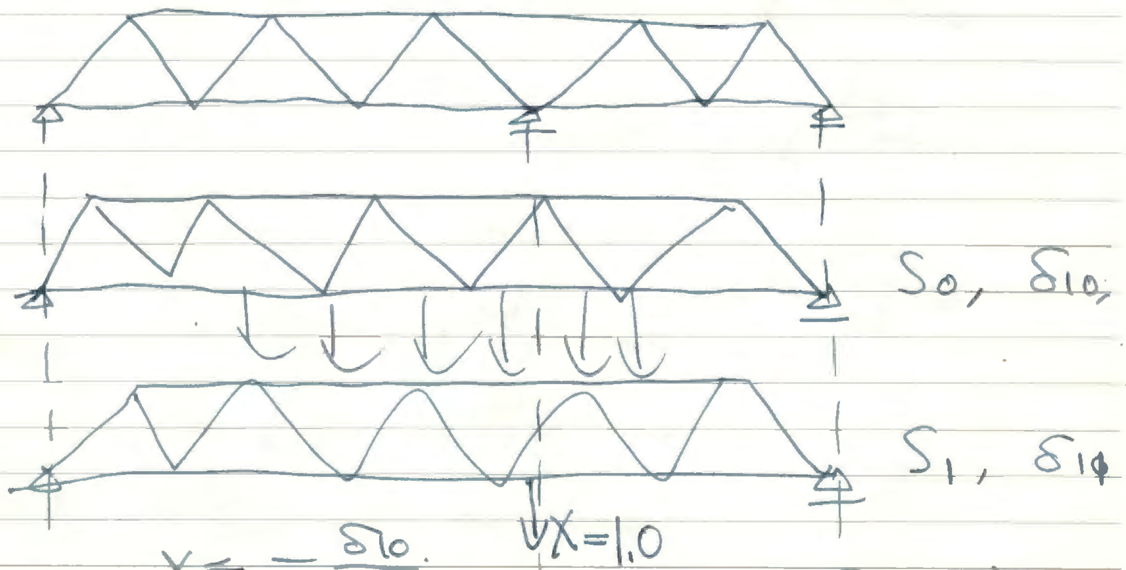
NO

$$\therefore \begin{cases} \delta_{11} X_1 = k_1 \\ \delta_{22} X_2 = k_2 \\ \delta_{33} X_3 = k_3 \end{cases} \qquad (4.43)$$



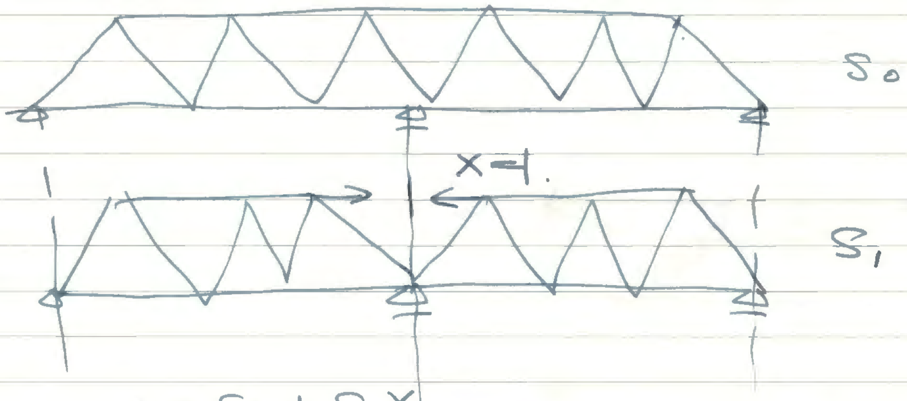
$$\begin{cases} X_1 = \dfrac{H_A + H_B}{2} \\ X_2 = \dfrac{H_A - H_B}{2} \end{cases}$$

6/4. 不静定トラス.



$$X = -\frac{\delta_{10}}{\delta_{11}} \qquad S = S_0 + S_1 X$$

別解.



$$s = S_0 + S_1 X$$

$$\delta_{11} = \Sigma S_1^2 \rho \quad , \quad \rho = \frac{l}{EA}$$

$$= \rho_c \Sigma S_1^2 \frac{\rho}{\rho_c}$$

部材	S_1	ρ/ρ_c	S_0	$S_0 S_1$	S_1^2	$S_1^2 \rho$	$S_0 S_1 \rho$

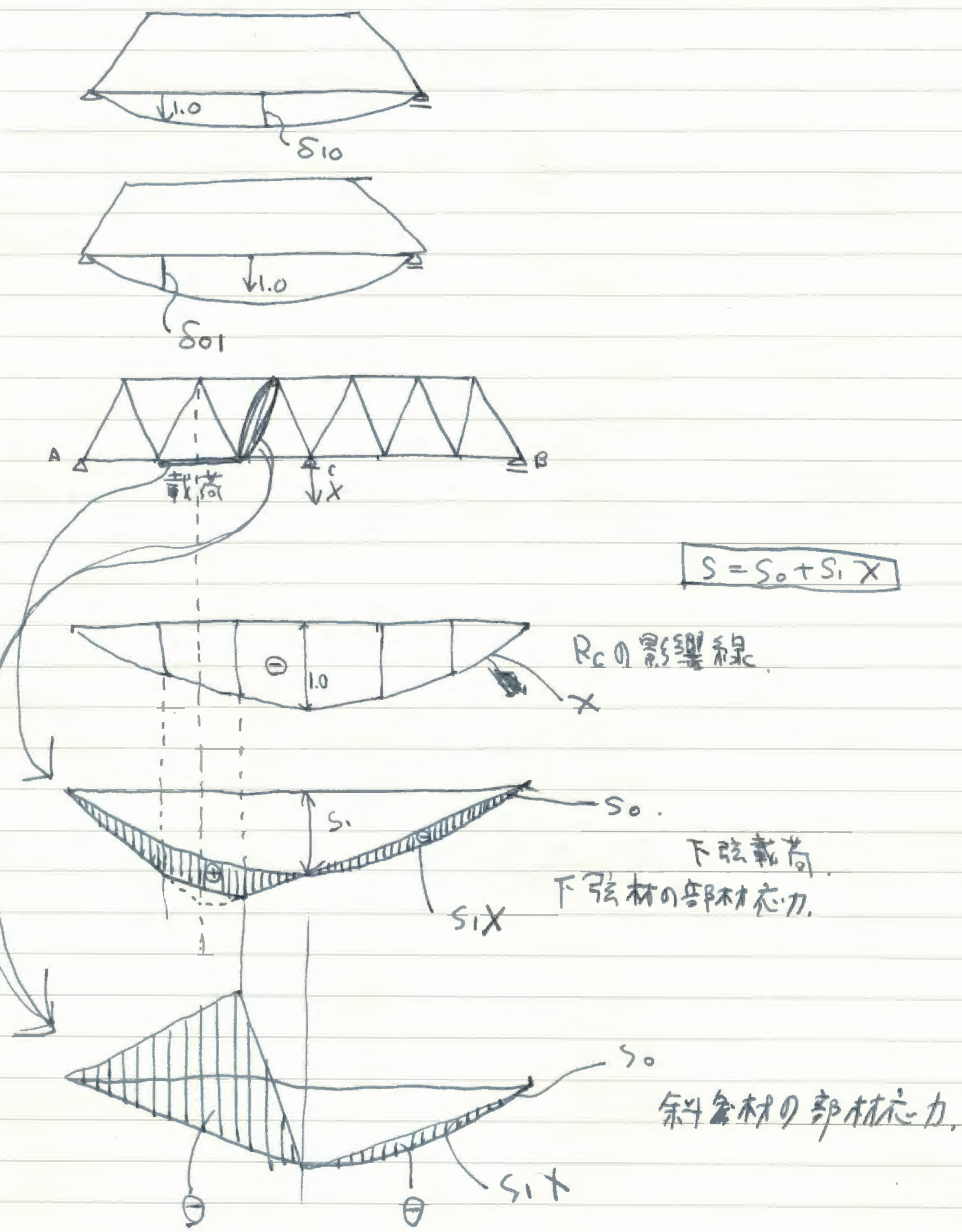
$$\delta_{10} = \Sigma S_0 S_1 \rho = \rho_c \Sigma S_0 S_1 \frac{\rho}{\rho_c}$$

$$X = - \frac{\Sigma S_0 S_1 \frac{\rho}{\rho_c}}{\Sigma S_1^2 \frac{\rho}{\rho_c}}$$

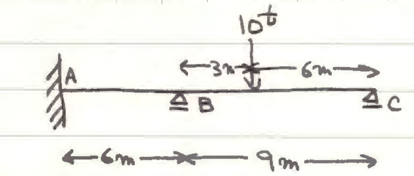
NO.

影響線.

$$X = -\frac{\delta_{10}}{\delta_{11}} = -\frac{\delta_{01}}{\delta_{11}}$$

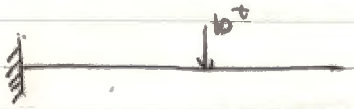


NO

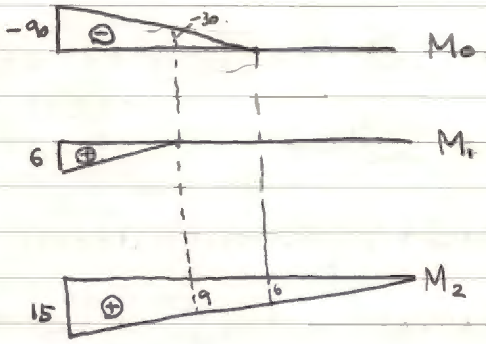


$$\begin{cases} R_B = X_1 \\ R_C = X_2 \end{cases}$$

$$M = M_0 + M_1 X_1 + M_2 X_2$$



$$\begin{cases} \delta_{11} X_1 + \delta_{12} X_2 = -\delta_{10} \\ \delta_{21} X_1 + \delta_{22} X_2 = -\delta_{20} \end{cases}$$



$$\delta_{11} = \int \frac{M_1^2}{EI} ds = \frac{1}{EI} \cdot \frac{6}{3} \times 6 \times 6 = \frac{72}{EI}$$

$$\delta_{12} = \delta_{21} = \int \frac{M_1 M_2}{EI} ds = \frac{1}{EI} \cdot \frac{6}{6} \times 6 \times (2 \times 15 + 9)$$

$$= \frac{234}{EI}$$

$$\delta_{22} = \int \frac{M_2^2}{EI} ds = \frac{15}{3EI} \cdot 15^2 = \frac{1125}{EI}$$

$$\delta_{10} = \int \frac{M_0 M_1}{EI} ds = \frac{6}{6EI} \times 6 \times (-90 \times 2 - 30)$$

$$= \frac{-1260}{EI}$$

$$\delta_{20} = \int \frac{M_0 M_2}{EI} ds = \frac{9}{6EI} \times (-90) \times (15 \times 2 + 6)$$

$$= \frac{-4860}{EI}$$

$$\therefore \begin{cases} 72 X_1 + 234 X_2 = 1260 \\ 234 X_1 + 1125 X_2 = 4860 \end{cases}$$

$X_1 + 3.25 X_2 = 17.5$

~~0.21~~ $X_1 +$ $X_2 = 4.32$
0.208

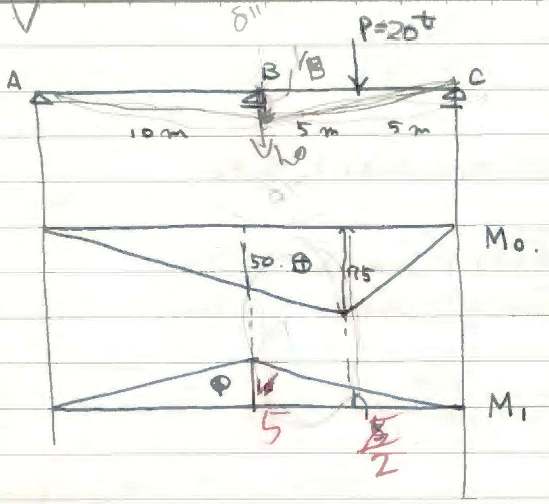
1	3.25	17.50
0.21	1	4.32
1	3.25	17.50
0	0.32	~~3.86~~ 0.65

$\therefore X_2 =$

1	3.25	17.5
0.208	1	4.32
1	3.25	17.5
0	0.324	0.68

$$\therefore X_2 = 2.1^t$$

$$X_1 = 17.5 - 3.25 X_2 = 10.7^t$$



$R_B = X_1$

$\delta_{11} X_1 = \delta_{1r} - \delta_{10}$

$\delta_{11} = \int \frac{M_1^2}{EI} ds = \frac{10}{3EI} \cdot 10^2 \times 2 = \frac{2000}{3EI}$

$= 0.031$

$\delta_{10} = \int \frac{M_0 M_1}{EI} ds$

$= \frac{+1}{EI}\left[\frac{10}{3} \times 50 \times 5 + \frac{5}{6}\left\{50\left(10+\frac{5}{2}\right) + 75\left(5+5\right)\right\} + \frac{5}{3} \times 75 \times \frac{5}{2}\right] = +\frac{10312.5}{3EI} = +0.1075$

$= +0.161$

$\therefore \delta_{11} = 0.031$

$\delta_{10} = -0.215$

$\delta_{1r} = 0.02$

$\therefore\ 0.031 X_1 = 0.02 + 0.215$

$(0.031 - 0.02) X_1 = 0.215 \quad \therefore X_1 =$

$\delta_{1r} = \Sigma P_1 \cdot r = 1 \cdot r_B = r_B = 0.02\ m$

$X_1 = -\frac{\delta_{10}}{\delta_{11}} + \frac{\delta_{1r}}{\delta_{11}} = \frac{0.215}{0.031} + \frac{0.02}{0.031} = 7.6t\ ?$

$EI = 21350$

$\delta_{11} X = \delta_{1r} - \delta_{10}$

$\therefore X = \frac{\delta_{1r} - \delta_{10}}{\delta_{11}}$

$X = \frac{0.02 + 0.161}{0.008}$

$\frac{0.181}{0.008}$

0.161
21350) 3437.5
21350
130250
128100
21500

$0.008X + 0.02 - 0.1075 = 0$

$0.008 X = 0.02 + 0.108$

10312.5

3437.5

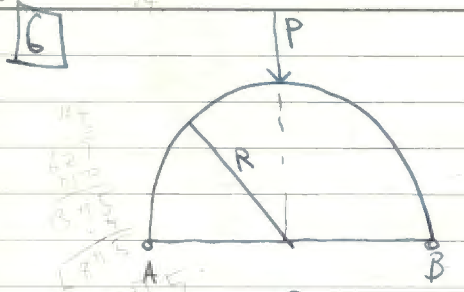
$\delta_{11} X_1 = \delta_{1r} - \delta_{10}$

$0.008 X = 0.02 + 0.108$

δ_{10}	0.161	◎
δ_{1r}	0.02	NO
δ_{11}	0.008	◎

0.088 $\quad \frac{0.161 - 0.02}{0.008} = \frac{0.141}{0.008}$

6



$EI = const$

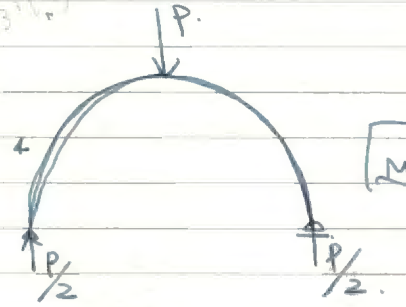
$0.161 + 0.008 X_1 + 0.02 = 0.008 X + 0.02$

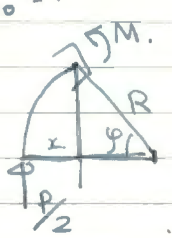
$\delta_{11} X_1 = -\delta_{10}$

$0.161 = 0.008 X + 0.02$

$\delta X = \delta_{1r} - \delta_{10}$

$M_0 =$

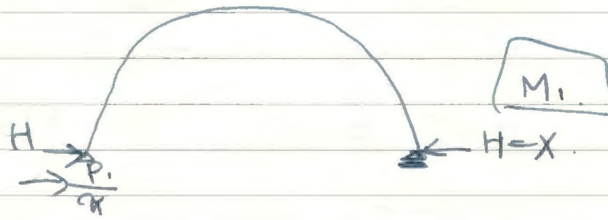


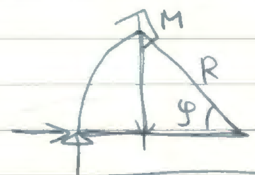


$\delta_{11} X_1 + \delta_{1r} - \delta_{10}$

$x = R - R\cos\varphi$
$= R(1-\cos\varphi)$

$\therefore M_0 = \frac{PR}{2}(1-\cos\varphi)$





$M_1 = R\sin\varphi$

$$M = \frac{PR}{2}(1-\cos\varphi) - \frac{PR}{\pi}\sin\varphi$$

$R\,d\theta = dS$

$$\delta_{10} = \int \frac{M_0 M_1}{EI} dS = 2\int_0^{\frac{\pi}{2}} \frac{\frac{PR^2}{2}\sin\theta(1-\cos\theta)}{EI} R\,d\theta$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2} \sin\theta(1-\cos\theta)\,d\theta$$

$\cos\theta = t \qquad -\sin\theta\,d\theta = dt$

$$\delta_{10} = \frac{PR^3}{EI}\int_0^1 (1-t)\,dt = \frac{PR^3}{2EI}$$

$\frac{1-\cos 2\theta}{2}$

$$\delta_{11} = \int \frac{M_1^2}{EI} dS = \frac{2R^3}{EI}\int_0^{\pi/2} \sin^2\theta\,d\theta = \frac{2R^3}{EI}\left[\frac{\theta - \frac{1}{2}\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{\pi R^3}{2EI}$$

$$X = -\frac{\delta_{10}}{\delta_{11}} = -\frac{P}{\pi} \;//$$

NO

18
144
324
216
108

5

部材	S_0	S_1	ρ	S_0S_1	S_1^2
AC	$-36\sqrt{2}$	$\sqrt{2}$	2.68×10^{-5}	-72	2
AD	$18\sqrt{5}$	$-\sqrt{5}$	2.31×10^{-5}	-90	5
BC	$-36\sqrt{2}$	$\sqrt{2}$	2.69×10^{-5}	-72	2
BD	$18\sqrt{5}$	$-\sqrt{5}$	2.31×10^{-5}	-90	5
CD	108	2	0.95×10^{-5}	216	4
				-108	18

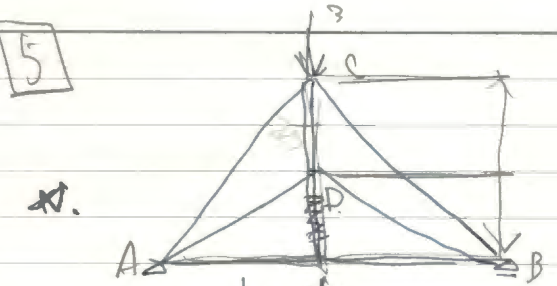
$S_0S_1\rho$	$S_1^2\rho$
-193.68	10.76
-207.90	23.1
-193.68	3.8
-207.90	
-597.96×10^{-5}	37.66×10^{-5}

t 15.88

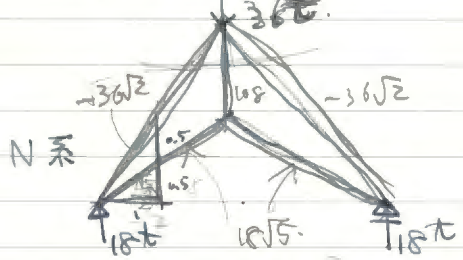
$$\frac{15880\,kg}{20\,m} = 794\,kg/cm$$

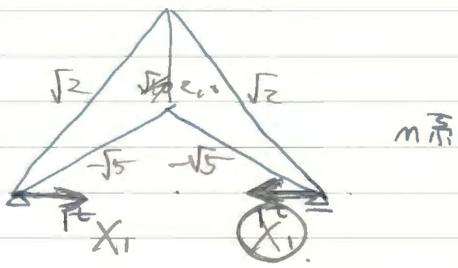
NO

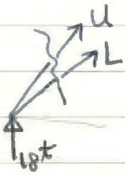
5

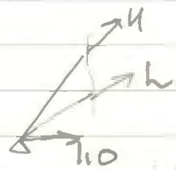


$$\delta_{11}X_1+\delta_{10}=0.$$









$$\left.\begin{aligned}\frac{1}{\sqrt5}L+\frac{1}{\sqrt2}U&=0\\ \frac{1}{2\sqrt5}L+\frac{1}{\sqrt2}U&=-18\end{aligned}\right\}$$

$$\frac{1}{2\sqrt5}L=18 \qquad L=80.5$$

$$U=-50.9$$

$$\frac{1}{\sqrt5}L+\frac{1}{\sqrt2}U=0$$

$$\frac{2}{\sqrt5}L+\frac{1}{\sqrt2}U=-1.0$$

$$-\frac{1}{\sqrt5}L=1.0$$

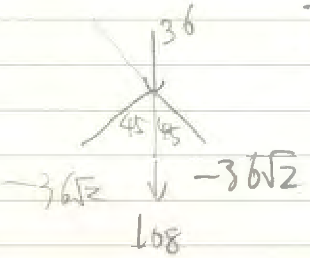
$$\left.\begin{aligned}L&=-\sqrt5\\ U&=\sqrt2\end{aligned}\right\}$$

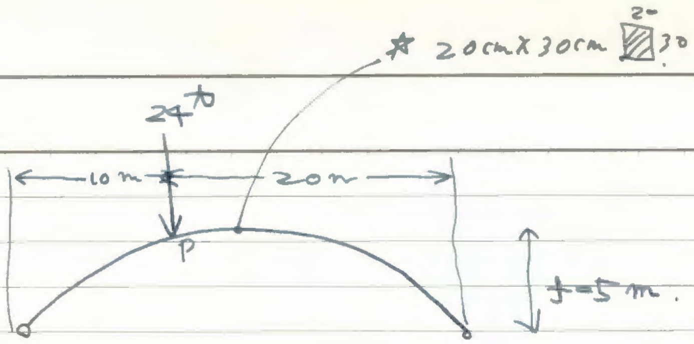
$$\frac{\sqrt2}{\sqrt2}\times2=2.0$$

$$\left\{\begin{aligned}\frac{1}{\sqrt2}U+\frac{1}{\sqrt5}L&=-18\\ \frac{1}{\sqrt2}U+\frac{2}{\sqrt5}L&=0\end{aligned}\right.$$

$$\frac{1}{\sqrt5}L=18$$

$$\left.\begin{aligned}L&=18\sqrt5\\ U&=-36\sqrt2\end{aligned}\right\}$$





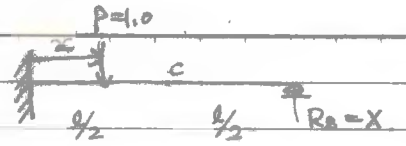
水平反力H,　24.4t
P点のMx　50.tm

$N_x = -20t$　$-25t$

$Q_x = 10t$　$-13t$

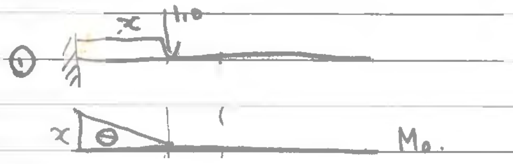
NO.

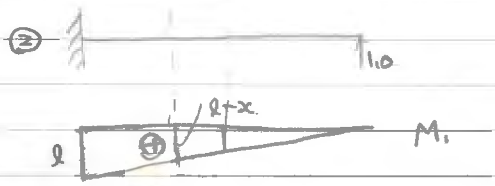
12.



M_c の Inf. line

$R_B(=X)$ の Inf. line を求める。

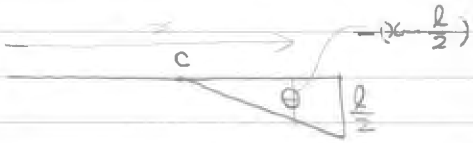




$$\begin{cases} \delta_{10} = \dfrac{-x}{6} \cdot x\,(2l + l - x) \\ \quad = \dfrac{-x^2}{6}(3l - x) \\ \delta_{11} = \dfrac{l}{3} \cdot l^2 = \dfrac{l^3}{3} \end{cases}$$

$$\therefore X_1 = R_B = \frac{-x^2}{2l^3}(x - 3l)$$

① のときの C 点の曲げモーメントの Inf. line



② のときの、C点の曲げモーメントを求めると

$\frac{l}{2}$

故に

$0 \leqq x \leqq l/2$ のとき、

$$M_c = \frac{-x^2}{2l^3}(x - 3l) \times \frac{l}{2}$$

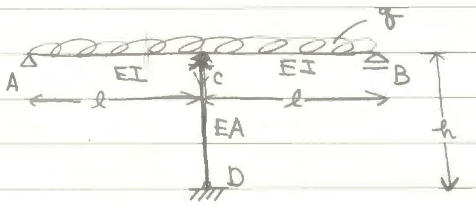
$$= \frac{-x^2}{4l^2}(x - 3l) = -\left(\frac{x}{l}\right)^3 \frac{l}{4} + \left(\frac{x}{l}\right)^2 \frac{3l}{4}$$ //

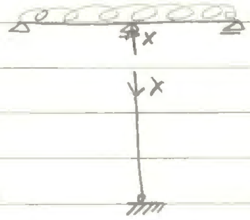
$\frac{l}{2} \leqq x \leqq l$ のとき

$$M_c = -\left(\frac{x}{l}\right)^3 \frac{l}{4} + \left(\frac{x}{l}\right)^2 \frac{3l}{4} - \left(x - \frac{l}{2}\right)$$ //

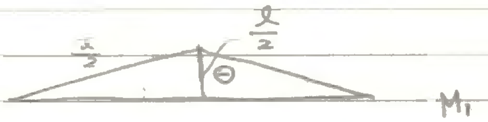
NO.

4.





$$\delta = \frac{h}{EA} P = \frac{h}{EA} X$$



$$\delta_{10} = 2\int_0^l \frac{M_0 M_1}{EI} dx = \frac{-2}{EI}\int_0^l \left(qlx - \frac{qx^2}{2}\right)\frac{x}{2} dx = \frac{-2}{EI}\left[\frac{ql}{6}x^3 - \frac{qx^4}{16}\right]_0^l = \frac{-5ql^4}{24EI}$$

$$\delta_{11} = \frac{1}{EI}\cdot 2 \times \frac{l}{3}\cdot\left(\frac{l}{2}\right)^2 = \frac{l^3}{6EI}$$

$$\delta = \frac{h}{EA} X$$

~~$\delta_{11} X = \delta - \delta_{10}$~~ $\delta_{11} X + \delta + \delta_{10} = 0.$

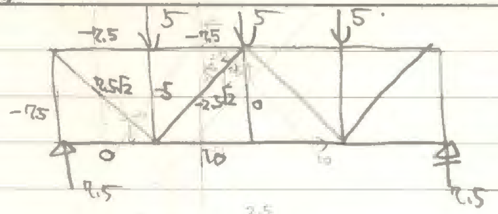
$$\therefore \frac{l^3}{6EI} X + \frac{h}{EA} X - \frac{5ql^4}{24EI} = 0$$

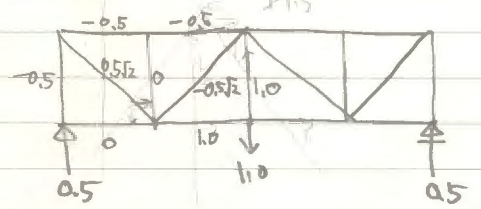
$$\therefore X = \frac{5ql^4}{24EI} \Big/ \left(\frac{l^3}{6EI} + \frac{h}{EA}\right)$$

//

NO.

11.4 √





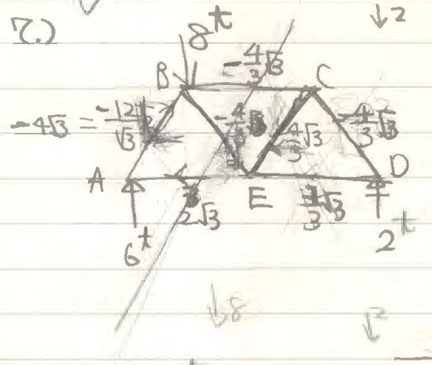
部材	個	S_0	S_1	$\frac{A}{A}\frac{l}{l_c}$	$S_0 S_1 \frac{l}{A}$	$S_1^2 \frac{l}{A}$
Aa	2	-7.5	-0.5	1	3.75	0.25
ad	2	-7.5	-0.5	1	3.75	0.25
aD	2	$7.5\sqrt{2}$	$0.5\sqrt{2}$	$\frac{2\sqrt{2}}{3}$	7.07	0.471
~~[illegible]~~	2					
AD	2	0	0	1	/	/
Dd	2	-5	0	1	/	/
dC	2	-7.5	-0.5	1	3.75	0.25
DC	2	$-2.5\sqrt{2}$	$-0.5\sqrt{2}$	$\frac{2\sqrt{2}}{3}$	2.357	0.471
DC	2	10	1.0	1	10	1.0
Cc	1	0	1.0	1	/	1.0
					61.354	6.384

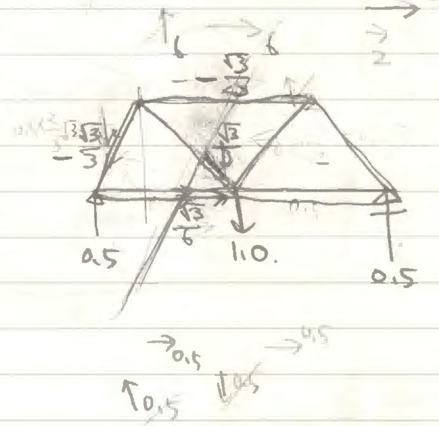
$$X = \frac{-61.354}{6.384} = -9.61 \ t$$

$\uparrow R_B = 9.61 \ t$

NO.

(2)





	S_0	S_1	S_0S_1	S_1^2
AB	$-4\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	4	1/3
BC	$-\frac{4}{3}\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	4/3	1/3
CD	$-\frac{4}{3}\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	4/3	1/3
DE	$\frac{2}{3}\sqrt{3}$	$\frac{\sqrt{3}}{6}$	1/3	1/12
EA	$2\sqrt{3}$	$\frac{\sqrt{3}}{6}$	1	1/12
BE	$-\frac{4}{3}\sqrt{3}$	$\frac{\sqrt{3}}{3}$	−4/3	1/3
EC	$\frac{4}{3}\sqrt{3}$	$\frac{\sqrt{3}}{3}$	4/3	1/3
			8	$\frac{22}{12}$

$1\frac{10}{12}$

$X = -4.36$

Ans

$S_{AB} = -4.41$ ✓

$S_{BC} = 0.21$ ✓

$S_{CD} = 0.21$ ✓

$S_{DE} = -0.10$ ✓

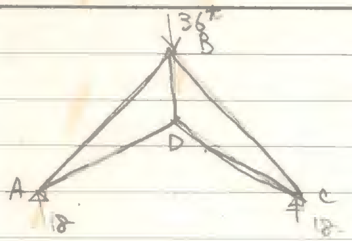
$S_{EA} = 2.21$ ✓

$S_{BE} = -4.83$ ✓

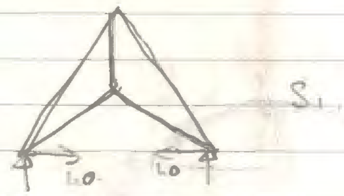
$S_{EC} = -0.21$ ✓

NO

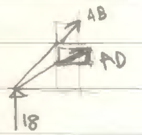
5.



S_0



S_1



$$\frac{AB}{\sqrt{2}} + \frac{2}{\sqrt{5}}AD = 0$$

$$\frac{AB}{\sqrt{2}} + \frac{1}{\sqrt{5}}AD = -18$$

$$\frac{1}{\sqrt{5}}AD = 18$$

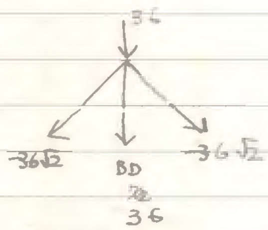
$$AD = 18\sqrt{5} \quad AB = -36\sqrt{2}$$

$$\frac{AB}{\sqrt{2}} + \frac{2}{\sqrt{5}}AD = -1.0$$

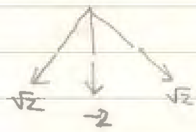
$$\frac{AB}{\sqrt{2}} + \frac{1}{\sqrt{5}}AD = 0$$

$$\frac{1}{\sqrt{5}}AD = -1.0$$

$$AD = -\sqrt{5}$$
$$AB = \sqrt{2}$$



$$\rho = \frac{l}{EA}$$



部材	S_0	S_1	ρ	$S_0 S_1 \rho$	$S_1^2 \rho$
AB	$-36\sqrt{2}$	$\sqrt{2}$	2.69×10^{-5}	-193.68	5.38
AD	$18\sqrt{5}$	$-\sqrt{5}$	2.31×10^{-5}	-207.90	11.55
BD	36	-2	0.95×10^{-5}	-68.40	3.8
DC	$18\sqrt{5}$	$-\sqrt{5}$	2.31×10^{-5}	-207.90	11.58
BC	$-36\sqrt{2}$	$\sqrt{2}$	2.69×10^{-5}	-193.68	5.38
				-871.56	37.66

$$X = -\frac{\delta_{10}}{\delta_{11}} = \frac{37.66}{871.56}$$
$$= 23.14$$

$$\frac{871.56 \times EA}{l}$$

193.95	5.39
205.37	11.57
68.57	3.81
208.33	11.57
193.95	5.39
~~208.13~~	37.73
-873.13	

t

23.14.

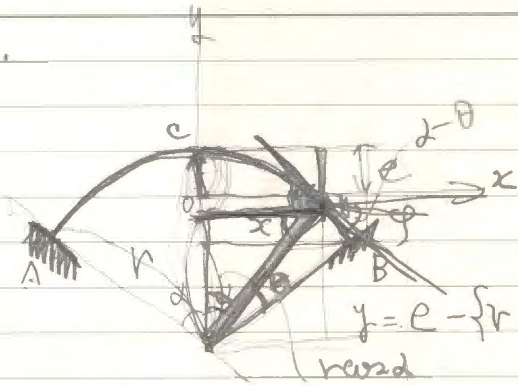
$20\ cm^2$

2.1×10^6

23.14 //

NO

9.



$ds = r\,d\theta$

E, A, I const.

$y = e - \{r - r\cos(\alpha-\theta)\}$

$x = r\sin(\alpha-\theta)$

$\dfrac{2r\sin\alpha\, I}{A}$

$2\int_0^{\alpha} \frac{y}{EI} ds = 2\frac{1}{EI}\int_0^{\alpha}\{e-r+r\cos(\alpha-\theta)\} r\,d\theta$

$= \frac{2}{EI}\left[(e-r)\theta + r\sin(\alpha-\theta)\right]_0^{\alpha}$

$\dfrac{Il}{A}$

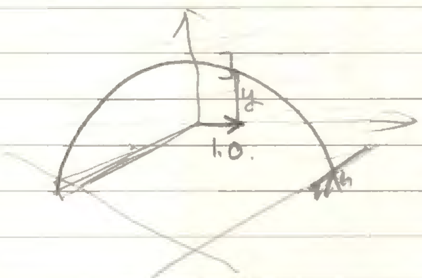
$= \frac{2}{EI}\{(e-r)\alpha + r\sin\alpha\} = 0$

$\therefore (e-r)\alpha = -r\sin\alpha \quad \therefore e = \frac{-r\sin\alpha}{\alpha} + r$

$= \frac{r}{\alpha}(\alpha - \sin\alpha)$

$\delta_{1t} = -\alpha_t t l$

$\delta_{11} X = \alpha_t t l \quad \therefore X = \frac{\alpha_t t l}{\delta_{11}} \quad (l = 2r\sin\alpha)$



$+\frac{l}{EA}$

$\therefore M_1 = -y_1$

~~$\therefore \delta_{11} = \int_0^{2\alpha} \frac{\{x - \frac{r\alpha\theta}{\alpha} - x + r\cos(\alpha-\theta)\}}{EI} r\,d\theta$~~

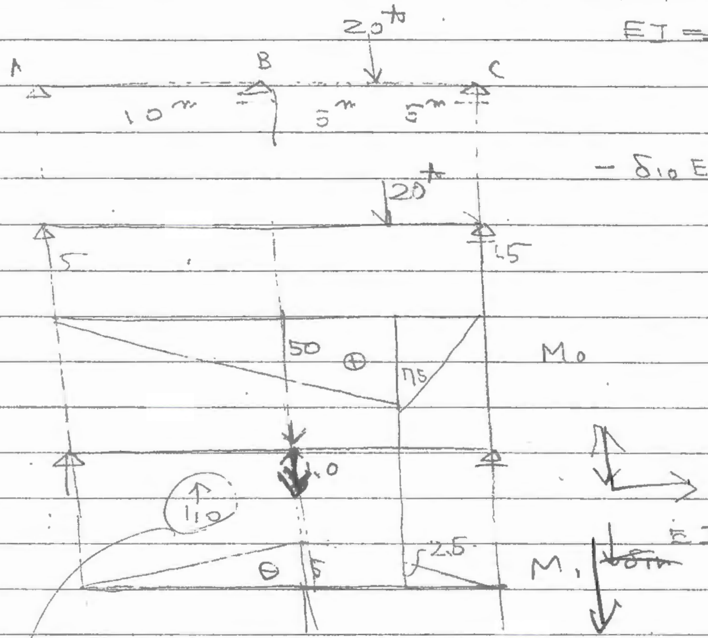
~~$= \frac{1}{EI}\left[\frac{r\cos\theta}{\alpha} - r^2\sin(\alpha-\theta)\right]_0^{2\alpha}$~~

~~$= \frac{1}{EI}\left\{\frac{r\cos 2\alpha}{\alpha} + r^2\sin\alpha - \frac{r}{\alpha} + r^2\sin\alpha\right\}$~~

NO.

δ_{1r}

2.



$EI = 21350$ t·m

$-\delta_{10} EI = \frac{10}{3} \times 5 \times 50$
$+ \frac{5}{6}\{50(125) + 75(10)\}$
$+ \frac{5}{3} \times 75 \times 2.5$
$= \frac{1}{3}(2500 + 3437.5 + 937.5)$
$= 2291.7$

$EI\delta_{11} = 2 \times \frac{10}{3} \times 5^2 = 166.7$

$\delta_1 = 0.02$

$\delta_{10} = -0.107$

$\delta_{11} = 0.00878$

$\delta_{10} = -0.107$

$\delta_{11} X_1 + \delta_{10} = \delta_{1r}$

$X_1 = -\frac{0.02 + 0.107}{0.0078} = 11.2$ t

$\delta_{11} X_1 + \delta_{10} + \delta_{1r} = 0$

$\delta_{11} = -\frac{\delta_{10} + \delta_{1r}}{\delta_{11}}$

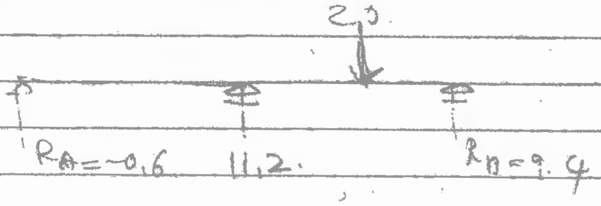
$= -\frac{-0.107 + 0.02}{0.0078} = \frac{0.087}{0.0078}$

$= 11.2$ t

$\boxed{\delta_{11} X_1 + \delta_{10} = \delta_1}$

$\therefore X_1 = \frac{\delta_1 - \delta_{10}}{\delta_{11}}$

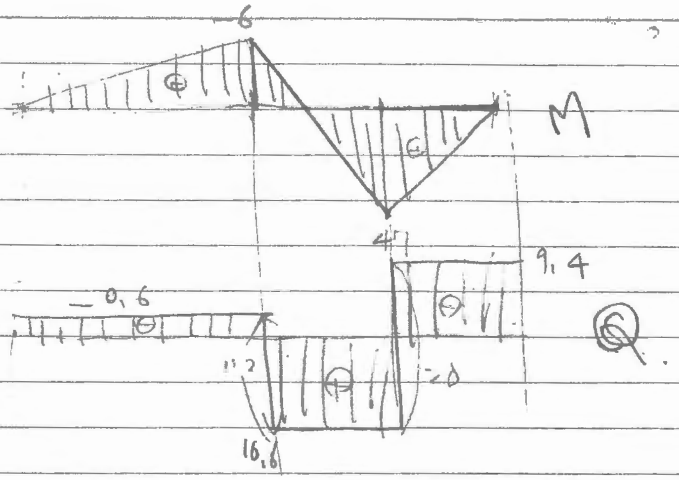
$X_1 = -11.2$



$$\Sigma M_B = 20R_A + 112 - 100 = 0$$

$$R_A = -\frac{12}{20} = -0.6^t$$

$$R_B = 9.4^t$$



伝達マトリックス

① 各区間の常数 α, β　　$\beta_i = \frac{l_i}{l_0}$　　$\alpha_i = \frac{EI_i}{100P_0 l_0^2}$　($\alpha_i = 1$ と P_0 をきめておく)

② 各区間の格間マトリックス

$$[F] = \begin{bmatrix} 1 & \beta & \frac{\beta^2}{2\alpha} & \frac{\beta^3}{6\alpha} & \overline{y}^* \\ 0 & 1 & \frac{\beta}{\alpha} & \frac{\beta^2}{2\alpha} & \overline{\varphi}^* \\ 0 & 0 & 1 & \beta & \overline{M}^* \\ 0 & 0 & 0 & 1 & \overline{Q}^* \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

③ 各節点マトリックス

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & \Gamma & 1 & 0 & 0 \\ -r & 0 & 0 & 1 & Q \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$Q \to R_i$

$\Gamma = k_{\theta i} / 100 P_0 l_0$

$r = k_{yi} l_0 / 100 P_0$

④ 始点Aと終点Bの Z_A, Z_B

⑤ 積計算

⑥ 境界

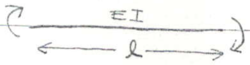
⑦ $\varphi_A =$　$Q_A =$

⑨ 任意点に対する力学量

NO

6/17. 応力法

応力法



曲げモーメントの影響線

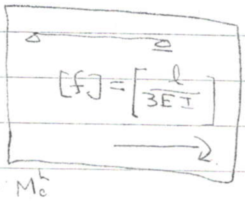
$M = [B_0] P$

$[f] = \begin{bmatrix} \frac{l}{3EI} & -\frac{l}{6EI} \\ -\frac{l}{6EI} & \frac{l}{3EI} \end{bmatrix}$ flexibility matrix

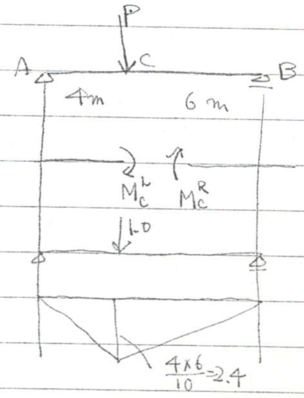
$[F] = \begin{bmatrix} [f]_1 & & \\ & [f]_2 & \\ & & \ddots \end{bmatrix}$

$W = \frac{1}{2} M^* [F] M$

$\Delta = [G_{00}] P \qquad [G_{00}] = [B_0]^* [F] [B_0]$



(例)



列の数:

$[B_0] = \begin{bmatrix} 0 & -2.4 & 0 \\ 0 & 2.4 & 0 \end{bmatrix} \begin{matrix} \leftarrow M_c^L \\ \leftarrow M_c^R \end{matrix}$

↑ A点 ↑ C点 ↑ B点 にのったときの M_c^L, M_c^R

$[F] = \begin{bmatrix} \frac{4}{3EI} & \\ & \frac{6}{3EI} \end{bmatrix} = \frac{2}{3EI} \begin{bmatrix} 2 & \\ & 3 \end{bmatrix}$

$[G_{00}] = \frac{2}{3EI} \begin{bmatrix} 0 & 0 \\ -2.4 & 2.4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & -2.4 & 0 \\ 0 & 2.4 & 0 \end{bmatrix} = \frac{2}{3EI} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 28.8 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

↑ $[B_0]^*$ ↑ $[F]$ ↑ $[B_0]$

$\therefore \Delta = \frac{2}{3EI} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 28.8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ P \\ 0 \end{bmatrix} = \frac{2P}{3EI} \begin{bmatrix} 0 \\ 28.8 \\ 0 \end{bmatrix}$

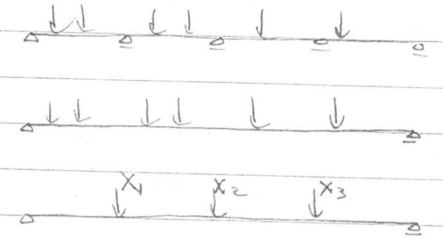
12
24
48
14
15.8

$\begin{bmatrix} 0 & 0 \\ -4.8 & 7.2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2.4 & 0 \\ 0 & 2.4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 28.8 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6/18 17

NO.

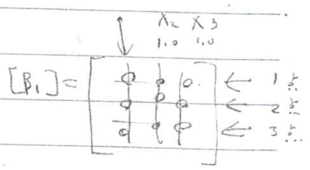


$\mathbb{P}$, $\mathbb{M}$

$[B_0]\mathbb{P}$

$\mathbb{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ $[B_1]\mathbb{X}$

3点の曲げモーメント.
$X_1=1$の時の



$$\mathbb{M} = [B_0]\mathbb{P} + [B_1]\mathbb{X}$$

$$W = \frac{1}{2}\mathbb{M}^*[F]\mathbb{M} \qquad \frac{\partial W}{\partial \mathbb{X}} = 0$$

$$\frac{\partial W}{\partial \mathbb{X}} = \frac{\partial \mathbb{M}^*}{\partial \mathbb{X}}[F]\mathbb{M}$$

$$= [B_1]^*[F]\left([B_0]\mathbb{P} + [B_1]\mathbb{X}\right)$$

$$= [B_1]^*[F][B_0]\mathbb{P} + [B_1]^*[F][B_1]\mathbb{X}$$

$$= [G_{10}]\mathbb{P} + [G_{11}]\mathbb{X} = 0$$

$$\therefore \mathbb{X} = -[G_{11}]^{-1}[G_{10}]\mathbb{P}$$

$$X = -\delta_{11}^{-1}\delta_{10}$$

$$\begin{cases} \delta_{10} = \int \frac{M_0 M_1}{EI} ds, & [G_{10}] = [B_1]^*[F][B_0] \quad (M_1 \to [B_1]^*,\ \frac{ds}{EI} \to [F],\ M_0 \to [B_0]) \\ \delta_{11} = \int \frac{M_1 M_1}{EI} ds, & [G_{11}] = [B_1]^*[F][B_1] \end{cases}$$

$$\mathbb{M} = [B_0]\mathbb{P} + [B_1]\mathbb{X}$$

$$= [B_0]\mathbb{P} - [B_1][G_{11}]^{-1}[G_{10}]\mathbb{P}$$

$$= \left([B_0] - [B_1][G_{11}]^{-1}[G_{10}]\right)\mathbb{P}$$

$$= [B]\mathbb{P}$$

$$[B] = [B_0] - [B_1][G_{11}]^{-1}[G_{10}]$$

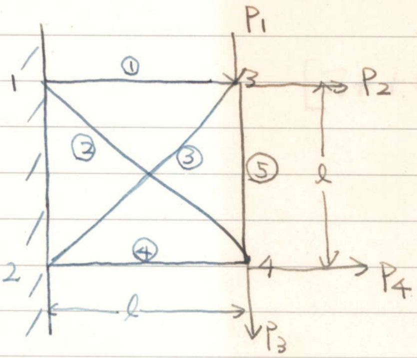
たわみ $\Delta = \frac{\partial W}{\partial \mathbb{P}} = \frac{\partial \mathbb{M}^*}{\partial \mathbb{P}}[F]\mathbb{M}$

~~$= [B][F]\mathbb{M}$~~

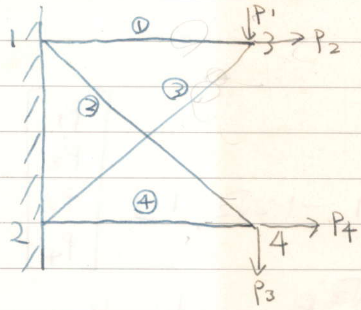
$$= [B]^*[F][B]\mathbb{P} \Rightarrow [B_0]^*[F][B]\mathbb{P}.$$

$$= [B]^*[F][B_0]\mathbb{P}.$$

NO.



$$\mathbb{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \qquad \mathbb{X} = [S_5]$$



$P_1=1$, $P_2 \sim P_4=0$ のとき

①	1
②	0
③	$-\sqrt{2}$
④	0
⑤	0

$P_2=1$ 他0のとき

①	1
②	0
③	0
④	0
⑤	0

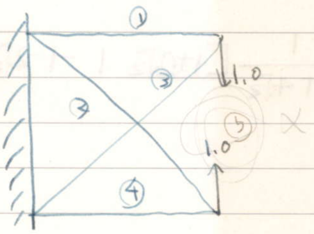
$P_3=1$, 他零

①	0
②	$\sqrt{2}$
③	0
④	−1
⑤	0

$P_4=1$, 他0.

①	0
②	0
③	0
④	1
⑤	0

$$[B_0] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ -\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[B_1] = \begin{bmatrix} 1 \\ -\sqrt{2} \\ -\sqrt{2} \\ 1 \\ 1 \end{bmatrix} \qquad [F] = \begin{bmatrix} 1 & & & & \\ & \sqrt{2} & & 0 & \\ & & \sqrt{2} & & \\ & 0 & & 1 & \\ & & & & 1 \end{bmatrix} \frac{l}{EA}$$

$$[D_{10}] = [B_1]^* [F][B_0]$$

$$= [1 \ \ -\sqrt{2} \ \ -\sqrt{2} \ \ 1 \ \ 1] \begin{bmatrix} 1 & & & & \\ & \sqrt{2} & & & \\ & & \sqrt{2} & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ -\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{l}{EA}$$

$$= \frac{l}{EA} [1 \ \ -2 \ \ -2 \ \ 1 \ \ 1] \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ -\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{l}{EA} [1+2\sqrt{2}, \ 1, \ -1-2\sqrt{2}, \ 1]$$

$-2\sqrt{2}-1$

NO.

$$[D_{11}] = [B_1]^*[F][B_0]$$

$$= \frac{l}{EA}\begin{bmatrix}1 & -2 & -2 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\-\sqrt{2}\\-\sqrt{2}\\1\\1\end{bmatrix} = \frac{l}{EA}\begin{bmatrix}3+4\sqrt{2}\end{bmatrix}$$

$$\therefore [D_{11}]^{-1} = \frac{EA}{l}\cdot\left[\frac{1}{3+4\sqrt{2}}\right]$$

$$\therefore \mathbb{X} = -[D_{11}]^{-1}[D_{10}]\mathbb{P}$$

$$= -\frac{EA}{l}\left[\frac{1}{3+4\sqrt{2}}\right]\cdot\frac{l}{EA}\begin{bmatrix}1+2\sqrt{2}, & 1, & -1-2\sqrt{2}, & 1\end{bmatrix}\begin{bmatrix}P_1\\P_2\\P_3\\P_4\end{bmatrix}$$

$$= -\frac{1+2\sqrt{2}}{3+4\sqrt{2}}P_1 - \frac{1}{3+4\sqrt{2}}P_2 + \frac{1+2\sqrt{2}}{3+4\sqrt{2}}P_3 - \frac{1}{3+4\sqrt{2}}P_4$$

$$= -\frac{1}{3+4\sqrt{2}}\left\{(1+2\sqrt{2})P_1 + P_2 - (1+2\sqrt{2})P_3 + P_4\right\} \quad //$$

$$\mathbb{S} = [B_0]\mathbb{P} + [B_1]\mathbb{X}$$

$$= \begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & \sqrt{2} & 0\\\sqrt{2} & 0 & 0 & 0\\0 & 0 & -1 & 1\\0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}P_1\\P_2\\P_3\\P_4\end{bmatrix} - \begin{bmatrix}1\\-\sqrt{2}\\-\sqrt{2}\\1\\1\end{bmatrix}\frac{1}{3+4\sqrt{2}}\begin{bmatrix}1+2\sqrt{2} & 1 & -1-2\sqrt{2} & 1\end{bmatrix}\begin{bmatrix}P_1\\P_2\\P_3\\P_4\end{bmatrix}$$

$$= \left\{\begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & \sqrt{2} & 0\\-\sqrt{2} & 0 & 0 & 0\\0 & 0 & -1 & 1\\0 & 0 & 0 & 0\end{bmatrix} - \frac{1}{3+4\sqrt{2}}\begin{bmatrix}1+2\sqrt{2} & 1 & -(1+2\sqrt{2}) & 1\\-(4+\sqrt{2}) & -\sqrt{2} & 4+\sqrt{2} & -\sqrt{2}\\-(4+\sqrt{2}) & -\sqrt{2} & 4+\sqrt{2} & -\sqrt{2}\\1+2\sqrt{2} & 1 & -(1+2\sqrt{2}) & 1\\1+2\sqrt{2} & 1 & -(1+2\sqrt{2}) & 1\end{bmatrix}\right\}\begin{bmatrix}P_1\\P_2\\P_3\\P_4\end{bmatrix}$$

NO.

$$= \frac{1}{3+4\sqrt{2}} \left\{ \begin{bmatrix} 3+4\sqrt{2} & 3+4\sqrt{2} & 0 & 0 \\ 0 & 0 & 8+3\sqrt{2} & 0 \\ -(8+3\sqrt{2}) & 0 & 0 & 0 \\ 0 & 0 & -(3+4\sqrt{2}) & 3+4\sqrt{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1+2\sqrt{2} & 1 & -(1+2\sqrt{2}) & 1 \\ -(4+\sqrt{2}) & -\sqrt{2} & 4+\sqrt{2} & -\sqrt{2} \\ -(4+\sqrt{2}) & -\sqrt{2} & 4+\sqrt{2} & -\sqrt{2} \\ 1+2\sqrt{2} & 1 & -(1+2\sqrt{2}) & 1 \\ 1+2\sqrt{2} & 1 & -(1+2\sqrt{2}) & 1 \end{bmatrix} \right\} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$= \frac{1}{3+4\sqrt{2}} \begin{bmatrix} 2+2\sqrt{2} & 2+4\sqrt{2} & 1+2\sqrt{2} & -1 \\ 4+\sqrt{2} & \sqrt{2} & 4+2\sqrt{2} & \sqrt{2} \\ -(4+2\sqrt{2}) & \sqrt{2} & -(4+\sqrt{2}) & \sqrt{2} \\ -(1+2\sqrt{2}) & -1 & -(2+2\sqrt{2}) & 2+4\sqrt{2} \\ -(1+2\sqrt{2}) & -1 & 1+2\sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \quad //$$

$P_1 = P_2 = P_4 = 0$、$P_3 = P$ とすると、

$$\delta = \frac{P}{3+4\sqrt{2}} \begin{bmatrix} 1+2\sqrt{2} \\ 4+2\sqrt{2} \\ -(4+\sqrt{2}) \\ -(2+2\sqrt{2}) \\ 1+2\sqrt{2} \end{bmatrix} \quad //$$

$$u = [B_0]^* [F] [B] \mathbb{P}$$

$$= \begin{bmatrix} 1 & 0 & -\sqrt{2} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & & & & 0 \\ & \sqrt{2} & & & \\ & & \sqrt{2} & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} \frac{1}{3+4\sqrt{2}} \frac{l}{EA}$$

$$\times \begin{bmatrix} 2+2\sqrt{2} & 2+4\sqrt{2} & 1+2\sqrt{2} & -1 \\ 4+\sqrt{2} & \sqrt{2} & 4+2\sqrt{2} & \sqrt{2} \\ -(4+2\sqrt{2}) & \sqrt{2} & -(4+\sqrt{2}) & \sqrt{2} \\ -(1+2\sqrt{2}) & -1 & -(2+2\sqrt{2}) & 2+4\sqrt{2} \\ -(1+2\sqrt{2}) & -1 & 1+2\sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

NO.

$$= \frac{l}{EA} \times \frac{1}{3+4\sqrt{2}} \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2+2\sqrt{2} & 2+4\sqrt{2} & 1+2\sqrt{2} & -1 \\ 4+\sqrt{2} & \sqrt{2} & 4+2\sqrt{2} & \sqrt{2} \\ -(4+2\sqrt{2}) & \sqrt{2} & -(4+\sqrt{2}) & \sqrt{2} \\ -(1+2\sqrt{2}) & -1 & -(2+2\sqrt{2}) & 2+4\sqrt{2} \\ -(1+2\sqrt{2}) & -1 & 1+2\sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

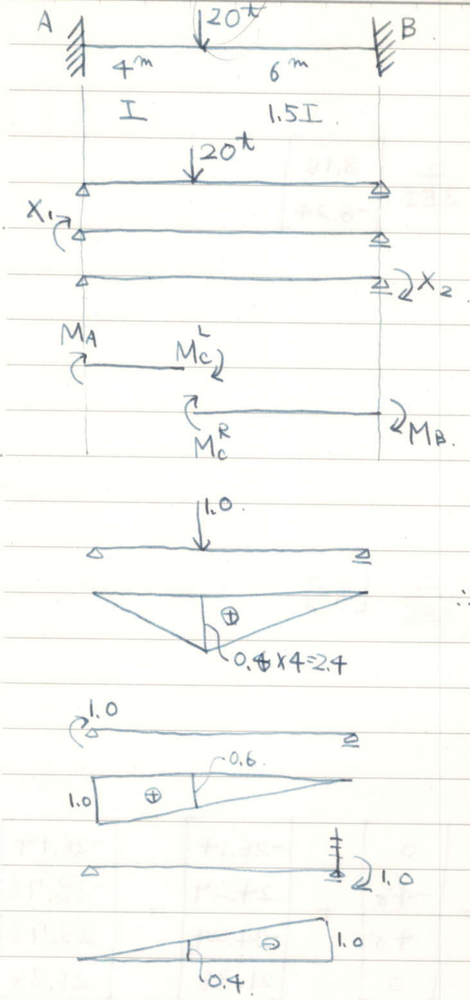
$$= \frac{l}{(3+4\sqrt{2})EA} \begin{bmatrix} 10+6\sqrt{2} & 2+2\sqrt{2} & 9+4\sqrt{2} & -(1+2\sqrt{2}) \\ 2+2\sqrt{2} & 2+4\sqrt{2} & 1+2\sqrt{2} & -1 \\ 9+4\sqrt{2} & 1+2\sqrt{2} & 10+6\sqrt{2} & -(2+2\sqrt{2}) \\ -(1+2\sqrt{2}) & -1 & -(2+2\sqrt{2}) & 2+4\sqrt{2} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$ //

($P_1 = P_2 = P_4 = 0$, $P_3 = P$ のとき

$$u = \frac{lP}{EA(3+4\sqrt{2})} \begin{bmatrix} 9+4\sqrt{2} \\ 1+2\sqrt{2} \\ 10+6\sqrt{2} \\ -(2+2\sqrt{2}) \end{bmatrix}$$ //)

NO.

(応力法)



曲げモーメント図

$$\mathbb{P} = [20]$$

$$[M] = \begin{bmatrix} M_A \\ M_C^L \\ M_C^R \\ M_B \end{bmatrix}, \quad \mathbb{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$[F] = \begin{bmatrix} \frac{4}{3EI} & -\frac{4}{6EI} & 0 & 0 \\ -\frac{4}{6EI} & \frac{4}{3EI} & 0 & 0 \\ 0 & 0 & \frac{6}{4.5EI} & -\frac{6}{9EI} \\ 0 & 0 & -\frac{6}{9EI} & \frac{6}{4.5EI} \end{bmatrix}$$

$$= \frac{2}{3EI} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\therefore [B_0] = \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \\ 0 \end{bmatrix}$$

$$\therefore [B_1] = \begin{bmatrix} 1.0 & 0 \\ -0.6 & 0.4 \\ 0.6 & -0.4 \\ 0 & 1.0 \end{bmatrix}$$

$$[G_{11}] = [B_1]^* [F] [B_1]$$

$$= \begin{bmatrix} 1.0 & -0.6 & 0.6 & 0 \\ 0 & 0.4 & -0.4 & 1.0 \end{bmatrix} \frac{2}{3EI} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1.0 & 0 \\ -0.6 & 0.4 \\ 0.6 & -0.4 \\ 0 & 1.0 \end{bmatrix}$$

$$= \frac{2}{3EI} \begin{bmatrix} 2.6 & -2.2 & 1.2 & -0.6 \\ -0.4 & 0.8 & -1.8 & 2.4 \end{bmatrix} \begin{bmatrix} 1.0 & 0 \\ -0.6 & 0.4 \\ 0.6 & -0.4 \\ 0 & 1.0 \end{bmatrix} = \frac{2}{3EI} \begin{bmatrix} 4.64 & -1.96 \\ -1.96 & \sout{6.32}\ 3.44 \end{bmatrix}$$

NO.

$$[G_{10}] = [B_1]^*[F][B_0]$$

$$= \frac{2}{3EI}\begin{bmatrix} 2.6 & -2.2 & 1.2 & -0.6 \\ -0.4 & 0.8 & -1.8 & 2.4 \end{bmatrix}\begin{bmatrix} 0 \\ -2.4 \\ 2.4 \\ 0 \end{bmatrix} = \frac{2}{3EI}\begin{bmatrix} 8.16 \\ -6.24 \end{bmatrix}$$

$$[G_{11}]^{-1} = \frac{3EI}{2}\cdot\frac{1}{12.12}\begin{bmatrix} 3.44 & +1.96 \\ 1.96 & 4.64 \end{bmatrix}$$

101

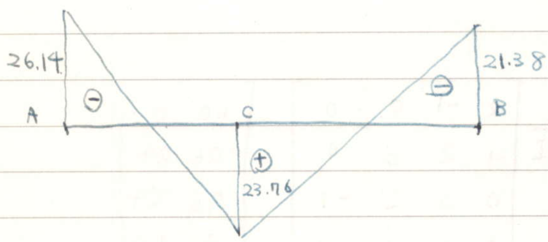
$$\therefore \mathbb{X} = -[G_{11}]^{-1}[G_{10}]\mathbb{P}$$

$$= -\frac{3EI}{2}\cdot\frac{1}{12.12}\begin{bmatrix} 3.44 & 1.96 \\ 1.96 & 4.64 \end{bmatrix}\begin{bmatrix} 8.16 \\ -6.24 \end{bmatrix}\frac{2}{3EI}[20]$$

$$= -\begin{bmatrix} 1.307 \\ -1.069 \end{bmatrix}[20] = \begin{bmatrix} -26.14 \\ 21.38 \end{bmatrix} \quad //$$

$$\mathbb{M} = [B_0]\mathbb{P} + [B_1]\mathbb{X} \quad \left(= -B_1\cdot G_{11}^{-1}G_{10}\mathbb{P}\right)$$

$$= \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \\ 0 \end{bmatrix}[20] + \begin{bmatrix} 1.0 & 0 \\ -0.6 & 0.4 \\ 0.6 & -0.4 \\ 0 & 1.0 \end{bmatrix}\begin{bmatrix} -26.14 \\ 21.38 \end{bmatrix} = \begin{bmatrix} 0 \\ -48 \\ 48 \\ 0 \end{bmatrix} + \begin{bmatrix} -26.14 \\ 24.24 \\ -24.24 \\ 21.38 \end{bmatrix} = \begin{bmatrix} -26.14 \\ -23.76 \\ 23.76 \\ 21.38 \end{bmatrix}$$

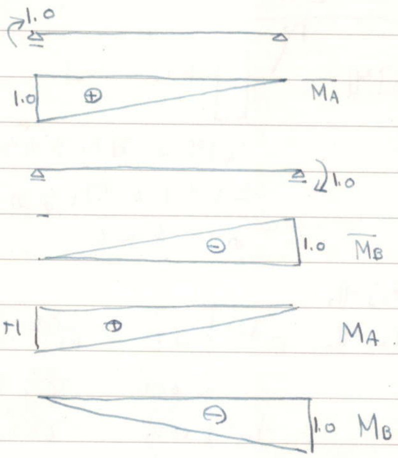


$$[B] = [B_0] - [B_1][G_{11}]^{-1}[G_{10}] \quad \left(~~= \mathbb{M}/20~~\right) = \mathbb{P}^{-1}\mathbb{M}$$

$$= \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.307 \\ -1.212 \\ 1.212 \\ -1.069 \end{bmatrix} = \begin{bmatrix} -1.307 \\ -1.188 \\ 1.188 \\ 1.069 \end{bmatrix}$$

6月11日.

$$\begin{cases}\theta_A = M_A\theta_{AA} + M_B\theta_{AB}\\ \theta_B = M_A\theta_{BA} + M_B\theta_{BB}\end{cases}$$



$$\theta_{AA} = \int \frac{M_A \overline{M_A}}{EI}dx \qquad \theta_{AA} = \frac{l}{3EI}$$

$$EI\,\theta_{AA} = \frac{l}{3}\times 1\times 1 = \frac{l}{3}$$

$$\theta_{BB} = \frac{l}{3EI}$$

$$EI\,\theta_{BB} = \frac{l}{3}\times(-1)\times(-1) = \frac{l}{3}.$$

$$\theta_{AB} = -\frac{l}{6EI}$$

$$EI\,\theta_{AB} = \frac{l}{6}\times(1)\times(-1) = -\frac{l}{6} = EI\,\theta_{BA}. \qquad \theta_{BA} = -\frac{l}{6EI}$$

$$\begin{bmatrix}\theta_A\\ \theta_B\end{bmatrix} = \begin{bmatrix}\theta_{AA} & \theta_{AB}\\ \theta_{BA} & \theta_{BB}\end{bmatrix}\begin{bmatrix}M_A\\ M_B\end{bmatrix} = \begin{bmatrix}\frac{l}{3EI} & -\frac{l}{6EI}\\ -\frac{l}{6EI} & \frac{l}{3EI}\end{bmatrix}\begin{bmatrix}M_A\\ M_B\end{bmatrix}$$

$$\Theta = \begin{bmatrix}\theta_A\\ \theta_B\end{bmatrix} \qquad \mathbb{M} = \begin{bmatrix}M_A\\ M_B\end{bmatrix} \qquad \Theta = [f]\,\mathbb{M}$$

$$[f] = \begin{bmatrix}\frac{l}{3EI} & -\frac{l}{6EI}\\ -\frac{l}{6EI} & \frac{l}{3EI}\end{bmatrix}$$

flexibility matrix
(たわみ性マトリックス
柔性マトリックス

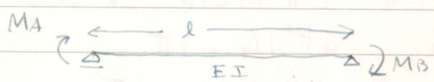
$$\mathbb{M} = [f]^{-1}\Theta = [k]\,\Theta$$

Stiffness matrix
剛性マトリクス.

@ 応力-歪み関係

$$\begin{cases}\sigma = E\varepsilon\\ \varepsilon = \frac{1}{E}\sigma\end{cases}$$

転置 Transpose 行と列をおきかえる。(*, T, ′)



$$W = \int_0^l \frac{M_x^2}{2EI}dx = \frac{1}{2}M_A\theta_A + \frac{1}{2}M_B\theta_B = \frac{1}{2}(M_A\theta_A + M_B\theta_B)$$

$$= \frac{1}{2}[M_A\ M_B]\begin{bmatrix}\theta_A\\ \theta_B\end{bmatrix} = \frac{1}{2}\mathbb{M}^*\Theta = \frac{1}{2}\mathbb{M}^*\underbrace{[f]\mathbb{M}}_{\Theta}$$

NO.

Castigliano の定理.

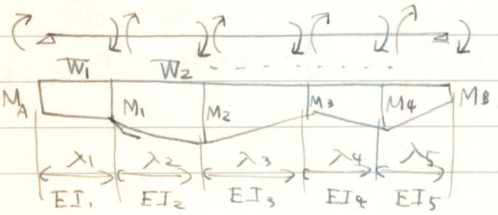
$$\frac{\partial W}{\partial \mathbb{M}} = \begin{bmatrix} \frac{\partial W}{\partial M_1} \\ \frac{\partial W}{\partial M_2} \\ \vdots \\ \frac{\partial W}{\partial M_n} \end{bmatrix} \qquad \frac{\partial W}{\partial \mathbb{M}} = \frac{1}{2}[F]\mathbb{M} + \frac{1}{2}\mathbb{M}^*[F] = \frac{1}{2}\Theta + \frac{1}{2}\Theta^*$$

$$= \frac{1}{2}[F]\mathbb{M} + \frac{1}{2}[F]\mathbb{M} = [F]\mathbb{M}$$

$\Theta = [F]\mathbb{M}$.

$[\ ] + [\ \ ]$

実際 Ed 算はできないが
エレメントが同じなので
おきかえて.



$W_1 = \frac{1}{2}\mathbb{M}_1^*[f_1]\mathbb{M}_1$

$W_2 = \frac{1}{2}\mathbb{M}_2^*[f_2]\mathbb{M}_2$

$$f_1 = \begin{bmatrix} \frac{\lambda_1}{3EI_1} & -\frac{\lambda_1}{6EI_1} \\ -\frac{\lambda_1}{6EI_1} & \frac{\lambda_1}{3EI_1} \end{bmatrix}$$

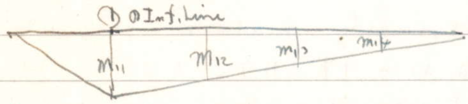
$$\mathbb{M}_1 = \begin{bmatrix} M_A \\ -M_1 \end{bmatrix} \qquad f_2 = \begin{bmatrix} \frac{\lambda_2}{3EI_2} & -\frac{\lambda_2}{6EI_2} \\ -\frac{\lambda_2}{6EI_2} & \frac{\lambda_2}{3EI_2} \end{bmatrix}$$

$$\mathbb{M}_2 = \begin{bmatrix} M_1 \\ -M_2 \end{bmatrix}$$

$$W = W_1 + W_2 + \cdots + W_n$$
$$= \frac{1}{2}\mathbb{M}_1^*[f_1]\mathbb{M}_1 + \frac{1}{2}\mathbb{M}_2^*[f_2]\mathbb{M}_2 + \cdots = \frac{1}{2}\mathbb{M}_1^*\Theta_1 + \frac{1}{2}\mathbb{M}_2^*\Theta_2 + \cdots + \frac{1}{2}\mathbb{M}_n^*\Theta_n$$
$$= \frac{1}{2}\mathbb{M}^*\Theta = \frac{1}{2}\mathbb{M}^*[F]\mathbb{M}$$

$$\mathbb{M} = \begin{bmatrix} \mathbb{M}_1 \\ \mathbb{M}_2 \\ \vdots \\ \mathbb{M}_n \end{bmatrix} = \begin{bmatrix} M_A \\ -M_1 \\ M_1 \\ -M_2 \\ M_2 \\ \vdots \\ M_B \end{bmatrix} \qquad [F] = \begin{bmatrix} [f_1] & & & \\ & [f_2] & & \\ & & \ddots & \\ & & & [f_n] \end{bmatrix}$$

◎ $$\boxed{\frac{\partial W}{\partial \mathbb{M}} = [F]\mathbb{M}}$$



$$\begin{cases} M_1 = m_{11}P_1 + m_{12}P_2 + m_{13}P_3 + \cdots + m_{1,n-1}P_{n-1} \\ M_2 = m_{21}P_1 + m_{22}P_2 + m_{23}P_3 + \cdots + m_{2,n-1}P_{n-1} \\ \vdots \end{cases}$$

$$\overline{\mathbb{M}} = [M]\mathbb{P}$$

列 …… 荷重の数
行 …… モーメントの数

$$\mathbb{M} = [B_0]\mathbb{P} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ -m_{11} & -m_{12} & \cdots & -m_{1,n-1} \\ m_{11} & m_{12} & \cdots & m_{1,n-1} \\ -m_{21} & -m_{22} & \cdots & -m_{2,n-1} \\ m_{21} & m_{22} & \cdots & m_{2,n-1} \\ \vdots & \vdots & & \vdots \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \end{bmatrix}$$

$P_1=1$　$P_2=1$ —— $P_n=1$ のときの各モーメント.

n 列は P_n が 1.0 のときの 各モーメント
m 行は m 番目のモーメント

$\mathbb{M} = [B_0]\mathbb{P}$　　$\mathbb{M}^* = \mathbb{P}^*[B_0]^*$

$$W = \frac{1}{2}\mathbb{M}^*[F]\mathbb{M} = \frac{1}{2}\mathbb{P}^*[B_0]^*[F][B_0]\mathbb{P}$$
$$= \frac{1}{2}\mathbb{P}^*[G_{00}]\mathbb{P}$$
$$\Delta = \frac{\partial W}{\partial \mathbb{P}} = [G_{00}]\mathbb{P}$$

（行がたわみの Inf. Line
列は —— たわみ図

←—— $n\lambda$ ——→

$$[M] = \frac{\lambda}{n}\begin{bmatrix} n-1 & n-2 & \cdots & 1 \\ n-2 & 2(n-2) & \cdots & 2 \\ n-3 & 2(n-3) & 3(n-3) \cdots & 3 \\ \vdots & & & \vdots \\ 1 & \cdots & \cdots & n-1 \end{bmatrix}$$

$n = 10$.

$$[M] = \frac{l}{10} \cdot \frac{1}{10}\begin{bmatrix} 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 8 & 16 & 14 & 12 & 10 & 8 & 6 & 4 & 2 \\ 7 & 14 & 21 & 18 & 15 & 12 & 9 & 6 & 3 \\ 6 & & & & & & & & 4 \\ 5 & & & & & & & & 5 \\ 4 & & & & & & & & 6 \\ 3 & & & & & & & & 7 \\ 2 & & & & & & & & 8 \\ 1 & \cdots & & & & & & & 9 \end{bmatrix}$$

$\Delta = [B]^*[F][B]\mathbb{P}$.
MI.

$$= \begin{bmatrix} 1.307 & 1.188 & 1.188 & 1.069 \end{bmatrix} \frac{2}{3EI} \begin{bmatrix} 2 & -1 & & \\ -1 & 2 & & \\ & & 2 & -1 \\ & & -1 & 2 \end{bmatrix} \begin{bmatrix} -26.14 \\ -23.76 \\ 23.76 \\ 21.38 \end{bmatrix}$$

$$= \frac{2}{3EI} \begin{bmatrix} -1.426 & -1.069 & 1.307 & 0.950 \end{bmatrix} \begin{bmatrix} -26.14 \\ -23.76 \\ 23.76 \\ 21.38 \end{bmatrix} = \frac{2}{3EI} \begin{bmatrix} 114.04 \end{bmatrix}$$ //

8.1 応力法

$$S=([B_0]-[B_1][G_{11}]^{-1}[G_{10}])P=([B_0]+[B_1][X])P$$

$$\eta=([B_0]^*[f][B_0]-[B_0]^*[f][B_1][G_{11}]^{-1}[G_{10}])P$$

$$=([G_{00}]+[G_{10}]^*[X])P$$

$$[G_{11}]=[B_1]^*[f][B_1]$$

$$[G_{00}]=[B_0]^*[f][B_0]$$

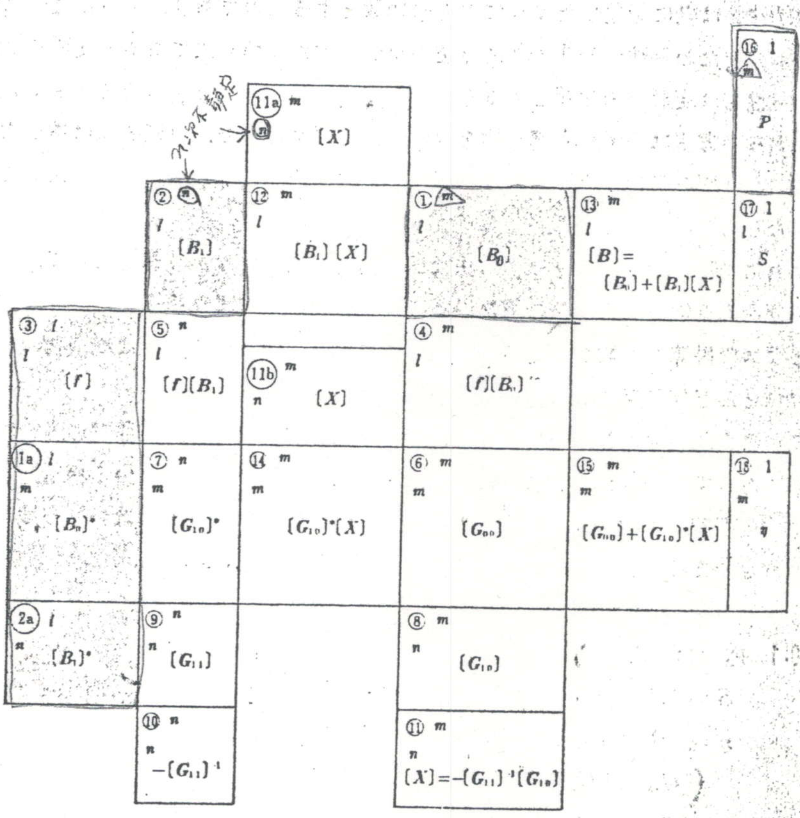
$$[G_{10}]=[B_1]^*[f][B_0]$$

$$[X]=-[G_{11}]^{-1}[G_{10}]$$ (肩つきの * は転置マトリックスを示す)

S：部材応力を要素とするベクトル。要素は全く任意に配置することができるが，構造物の形によってその配置を考えた方が，他のマトリックスの要素の配列が規則正しく作りやすい場合が多い。

$[B_0]$：荷重を基本系の部材応力に変換するマトリックス。$[B_0]$ の i 行 j 列の要素は，S において第 i 番目に配置された部材応力の荷重が $P_j=1$，他は 0 の場合に対する応力値。

$[B_1]$：不静定力を基本系の部材応力に変換するマトリックス。$[B_1]$ の

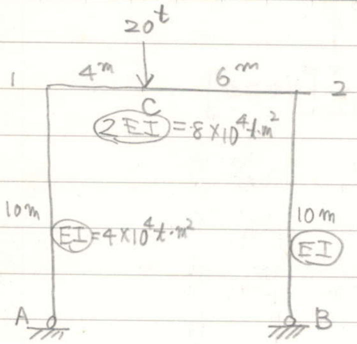


うすずみの部分は，計算の開始時に既知のマトリックス，
l, m, n は各マトリックスの行または列の数を示す。

図 8.1 計算の順序

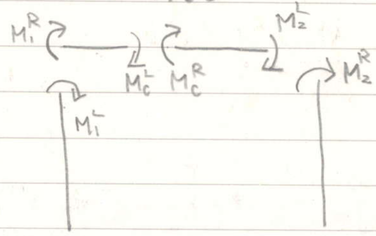
○印の中の数字の順序に計算を行う。ただし部材応力の影響線のみを求める場合は
1, 2, 3, 5, 7, 9, 10, 11, 12, 13

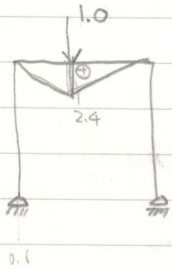
NO.



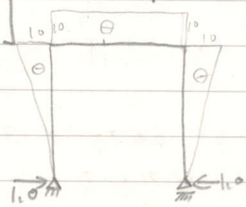
曲げモーメント図とΔcを求めよ。

$$M_1 = \begin{bmatrix} M_{1A}^{L} \\ M_{12}^{R} \\ M_C^{L} \\ M_C^{R} \\ M_2^{L} \\ M_2^{R} \end{bmatrix}$$





$$[B_0] = \begin{bmatrix} 0 \\ 0 \\ -2.4 \\ 2.4 \\ 0 \\ 0 \end{bmatrix}$$



$$[B_1] = \begin{bmatrix} 10 \\ -10 \\ 10 \\ -10 \\ 10 \\ -10 \end{bmatrix}$$

$$[F] = \begin{bmatrix} \frac{10}{3EI} & & & & & \\ & \frac{4}{3(2EI)} & -\frac{4}{6(2EI)} & & & \\ & -\frac{4}{6(2EI)} & \frac{4}{3(2EI)} & & & \\ & & & \frac{6}{3(2EI)} & -\frac{6}{6(2EI)} & \\ & & & -\frac{6}{6(2EI)} & \frac{6}{3(2EI)} & \\ & & & & & \frac{10}{3EI} \end{bmatrix}$$

$$= \frac{1}{6EI} \begin{bmatrix} 20 & & & & & \\ & 4 & -2 & & & \\ & -2 & 4 & & & \\ & & & 6 & -3 & \\ & & & -3 & 6 & \\ & & & & & 20 \end{bmatrix}$$

$$[G_{11}] = [B_1]^{*}[F][B_1]$$

$$= [10 \ -10 \ 10 \ -10 \ 10 \ -10] \frac{1}{6EI} \begin{bmatrix} 20 & & & & & \\ & 4 & -2 & & & \\ & -2 & 4 & & & \\ & & & 6 & -3 & \\ & & & -3 & 6 & \\ & & & & & 20 \end{bmatrix} \begin{bmatrix} 10 \\ -10 \\ 10 \\ -10 \\ 10 \\ -10 \end{bmatrix}$$

$$= \frac{1}{6EI}\begin{bmatrix} 200 & -60 & 60 & -90 & 90 & -200 \end{bmatrix}\begin{bmatrix} 10 \\ -10 \\ 10 \\ -10 \\ 10 \\ -10 \end{bmatrix} = \frac{1}{6EI}\begin{bmatrix} 7000 \end{bmatrix}$$

$$[G_{10}] = [B_1]^{*}[F][B_0]$$

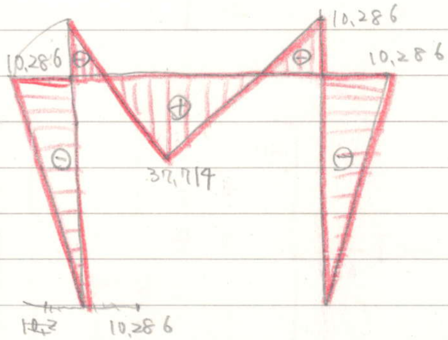
$$= \frac{1}{6EI}\begin{bmatrix} 200 & -60 & 60 & -90 & 90 & -200 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -2{,}4 \\ 2{,}4 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{6EI}\begin{bmatrix} -360 \end{bmatrix}$$

$$X = -[G_{11}]^{-1}[G_{10}][P]$$

$$= -6EI\left[\frac{1}{7000}\right]\frac{1}{6EI}[-360][20] = \left[\frac{360\times 20}{7000}\right] = 1{,}0286$$

$$M = [B_0]P + [B_1]X$$

$$= \begin{bmatrix} 0 \\ 0 \\ -2{,}4 \\ 2{,}4 \\ 0 \\ 0 \end{bmatrix}[20] + \begin{bmatrix} 10 \\ -10 \\ 10 \\ -10 \\ 10 \\ -10 \end{bmatrix}[1{,}0286] = \begin{bmatrix} 10{,}286 \\ -10{,}286 \\ -37{,}714 \\ 37{,}714 \\ 10{,}286 \\ -10{,}286 \end{bmatrix}$$



$$[B] = [B_0] - [B_1][G_{11}]^{-1}[G_{10}] \quad (P^{-1}X)$$

$$= \begin{bmatrix} 0 \\ 0 \\ -2.4 \\ 2.4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 10 \\ -10 \\ 10 \\ -10 \\ 10 \\ -10 \end{bmatrix} [0.0514] = \begin{bmatrix} +0.514 \\ -0.514 \\ \sout{-1.886} \\ \sout{2.914} \\ +0.514 \\ -0.514 \end{bmatrix} \quad \begin{matrix} \\ \\ \sout{-2.914}\ -1.886 \\ +1.886 \\ \\ \\ \end{matrix}$$

$$\sout{\Delta_1} = [B]^*[F][B]P$$

$$= [0.514 \ \ -0.514 \ \ -1.886 \ \ 1.886 \ \ 0.514 \ \ -0.514] \frac{1}{6EI} \begin{bmatrix} 20 & & & & & \\ & 4 & -2 & & & \\ & -2 & 4 & & & \\ & & & 6 & -3 & \\ & & & -3 & 6 & \\ & & & & & 20 \end{bmatrix} \begin{bmatrix} 0.514 \\ -0.514 \\ -1.886 \\ 1.886 \\ 0.514 \\ -0.514 \end{bmatrix}$$

$$= \frac{1}{6EI}[10.28 \ \ 1.716 \ \ -6.516 \ \ 9.794 \ \ -2.574 \ \ -10.28] \begin{bmatrix} 0.514 \\ -0.514 \\ -1.886 \\ 1.886 \\ 0.514 \\ -0.514 \end{bmatrix} P = \frac{1}{6EI}[39.0852]$$

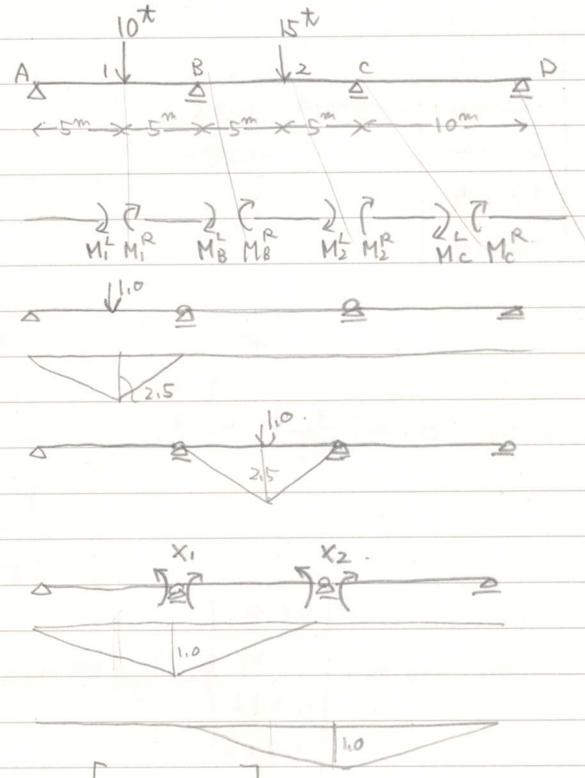
$$\therefore \Delta_1 = [B]^*[F][B]P = \frac{1}{6EI}[781.714]$$

$$\Delta_1 = [B_0]^*[F][B]P$$

$$= \begin{bmatrix} 0 \\ 0 \\ -2.4 \\ 2.4 \\ 0 \\ 0 \end{bmatrix}^* \frac{1}{6EI} \begin{bmatrix} 20 & & & & & \\ & 4 & -2 & & & \\ & -2 & 4 & & & \\ & & & 6 & -3 & \\ & & & -3 & 6 & \\ & & & & & 20 \end{bmatrix} \begin{bmatrix} 0.514 \\ -0.514 \\ -1.886 \\ 1.886 \\ 0.514 \\ -0.514 \end{bmatrix} [20]$$

$$= \frac{1}{6EI}[0 \ \ 4.8 \ \ -9.6 \ \ 14.4 \ \ -7.2 \ \ 0] \begin{bmatrix} 0.514 \\ -0.514 \\ -1.886 \\ 1.886 \\ 0.514 \\ -0.514 \end{bmatrix} [20] = \frac{1}{6EI}[781.92]$$

~~$\Delta = [B]^{*} [F] [B_0] \mathbb{P}$.~~



EI = const.

M図, Δ_1, Δ_2 を求める。

$$\mathbb{M} = \begin{bmatrix} M_1^L \\ M_1^R \\ M_B^L \\ M_B^R \\ M_2^L \\ M_2^R \\ M_C^L \\ M_C^R \end{bmatrix} \qquad \mathbb{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad \mathbb{P} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$$[B_0] = \begin{bmatrix} -2.5 & 0 \\ 2.5 & 0 \\ 0 & \cancel{-2.5} \\ 0 & \cancel{2.5} \\ 0 & \cancel{0}\ -2.5 \\ 0 & \cancel{0}\ 2.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[B_1] = \begin{bmatrix} -0.5 & 0 \\ 0.5 & 0 \\ -1.0 & 0 \\ 1.0 & 0 \\ -0.5 & \cancel{-1.0}\ -0.5 \\ 0.5 & \cancel{1.0}\ 0.5 \\ 0 & \cancel{-0.5}\ -1.0 \\ 0 & \cancel{0.5}\ 1.0 \end{bmatrix}$$

$$[F] = \begin{bmatrix} \frac{5}{3EI} &&&&&&& \\ & \frac{5}{3EI} & -\frac{5}{6EI} &&&&& \\ & -\frac{5}{6EI} & \frac{5}{3EI} &&&&& \\ &&& \frac{5}{3EI} & -\frac{5}{6EI} &&& \\ &&& -\frac{5}{6EI} & \frac{5}{3EI} &&& \\ &&&&& \frac{5}{3EI} & -\frac{5}{6EI} & \\ &&&&& -\frac{5}{6EI} & \frac{5}{3EI} & \\ &&&&&&& \frac{10}{3EI} \end{bmatrix}$$

$$= \frac{5}{6EI} \begin{bmatrix} 2 &&&&&&& \\ & 2 & -1 &&&&& \\ & -1 & 2 &&&&& \\ &&& 2 & -1 &&& \\ &&& -1 & 2 &&& \\ &&&&& 2 & -1 & \\ &&&&& -1 & 2 & \\ &&&&&&& \cancel{20}\ 4 \end{bmatrix}$$

$$[G_{11}] = [B_1]^{*}[F][B_1].$$

$$= \begin{bmatrix} -0.5 & 0.5 & -1.0 & 1.0 & -0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0.5 & -1.0 & 1.0 \end{bmatrix} \frac{5}{6EI} \begin{bmatrix} 2 & & & & & & & \\ & 2 & -1 & & & & & \\ & -1 & 2 & & & & & \\ & & & 2 & -1 & & & \\ & & & -1 & 2 & & & \\ & & & & & 2 & -1 & \\ & & & & & -1 & 2 & \\ & & & & & & & 4 \end{bmatrix} \begin{bmatrix} -0.5 & 0 \\ 0.5 & 0 \\ -1.0 & 0 \\ 1.0 & 0 \\ -0.5 & -0.5 \\ 0.5 & -0.5 \\ 0 & -1.0 \\ 0 & 1.0 \end{bmatrix}$$

$$= \frac{5}{6EI} \begin{bmatrix} -1.0 & 2.0 & -2.5 & 2.5 & -2.0 & 1.0 & -0.5 & 0 \\ 0 & 0 & 0 & 0.5 & -1.0 & 2.0 & -2.5 & 4.0 \end{bmatrix} \begin{bmatrix} -0.5 & 0 \\ 0.5 & 0 \\ -1.0 & 0 \\ 1.0 & 0 \\ -0.5 & -0.5 \\ 0.5 & +0.5 \\ 0 & -1.0 \\ 0 & 1.0 \end{bmatrix} = \frac{5}{6EI} \begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}$$

$$[G_{10}] = [B_1]^{*}[F][B_0]$$

$$= \frac{5}{6EI} \begin{bmatrix} -1 & 2 & -2.5 & 2.5 & -2 & 1 & -0.5 & 0 \\ 0 & 0 & 0 & 0.5 & -1 & 2 & -2.5 & 4 \end{bmatrix} \begin{bmatrix} -2.5 & 0 \\ 2.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2.5 \\ 0 & 2.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \frac{5}{6EI} \begin{bmatrix} 7.5 & 7.5 \\ 0 & 7.5 \end{bmatrix}$$

$$X = -[G_{11}]^{-1}[G_{10}] P$$

$$= -\frac{6EI}{5} \cdot \frac{1}{60} \begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \frac{5}{6EI} \begin{bmatrix} 7.5 & 7.5 \\ 0 & 7.5 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \end{bmatrix} = -\frac{1}{60} \begin{bmatrix} 60 & 45 \\ -15 & 45 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \end{bmatrix} = -\frac{1}{60} \begin{bmatrix} 1275 \\ 525 \end{bmatrix}$$

$$= \begin{bmatrix} 21.25 \\ -8.75 \end{bmatrix}$$

NO.

$$M_1 = [B_0]\,\mathbb{P} + [B_1]\,\mathbb{X}$$

$$= \begin{bmatrix} -2.5 & 0 \\ 2.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2.5 \\ 0 & 2.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \end{bmatrix} + \begin{bmatrix} -0.5 & 0 \\ 0.5 & 0 \\ -1 & 0 \\ 1 & 0 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -21.25 \\ -8.75 \end{bmatrix} = \begin{bmatrix} -25 \\ 25 \\ 0 \\ 0 \\ -37.5 \\ 37.5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 10.625 \\ -10.625 \\ 21.25 \\ -21.25 \\ 15 \\ -15 \\ 8.75 \\ -8.75 \end{bmatrix} = \begin{bmatrix} -14.375 \\ 14.375 \\ 21.25 \\ -21.25 \\ -22.5 \\ 22.5 \\ 8.75 \\ -8.75 \end{bmatrix}$$

$$[B] = [B_0] - [B_1][G_{11}]^{-1}[G_{10}].$$

$$= \begin{bmatrix} -2.5 & 0 \\ 2.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2.5 \\ 0 & 2.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -0.5 & 0 \\ 0.5 & 0 \\ -1 & 0 \\ 1 & 0 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.75 \\ -0.25 & 0.75 \end{bmatrix} = \begin{bmatrix} -2.5 & 0 \\ 2.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2.5 \\ 0 & 2.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -0.5 & -0.375 \\ 0.5 & 0.375 \\ -1 & -0.75 \\ 1 & 0.75 \\ -0.375 & -0.75 \\ 0.375 & 0.75 \\ 0.25 & -0.75 \\ -0.25 & 0.75 \end{bmatrix} = \begin{bmatrix} -2 & +0.375 \\ 2 & -0.375 \\ +1 & +0.75 \\ -1 & -0.75 \\ +0.375 & -1.75 \\ -0.375 & 1.75 \\ -0.25 & +0.75 \\ +0.25 & -0.75 \end{bmatrix}$$

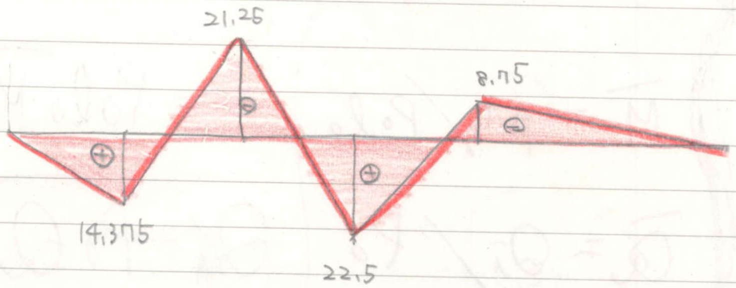
$$\Delta_1 = [B_0]^* [F] [B]\,\mathbb{P}$$

$$= \begin{bmatrix} -2.5 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.5 & 2.5 & 0 & 0 \end{bmatrix} \frac{5}{6EI} \begin{bmatrix} 2 & & & & & & & \\ & 2 & -1 & & & & & \\ & -1 & 2 & & & & & \\ & & & 2 & -1 & & & \\ & & & -1 & 2 & & & \\ & & & & & 2 & -1 & \\ & & & & & -1 & 2 & \\ & & & & & & & 4 \end{bmatrix} \begin{bmatrix} -2 & -0.375 \\ 2 & +0.375 \\ 1 & 0.75 \\ -1 & -0.75 \\ -0.375 & -1.75 \\ -0.375 & 1.75 \\ -0.25 & 0.75 \\ 0.25 & -0.75 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

NO.

$$= \frac{5}{6EI}\begin{bmatrix} -5 & 5 & -2.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.5 & -5 & 5 & -2.5 & 0 \end{bmatrix}\begin{bmatrix} -14.375 \\ 14.375 \\ 21.25 \\ -21.25 \\ -22.5 \\ 22.5 \\ 8.75 \\ -8.75 \end{bmatrix}$$

$$= \frac{5}{6EI}\begin{bmatrix} 90.625 \\ 150 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 75.5 \\ 125 \end{bmatrix} \quad //$$



NO.

遷元法.

$$[F]_i = \begin{bmatrix} 1 & \beta & \frac{\beta^2}{2\alpha} & \frac{\beta^3}{6\alpha} & \bar{y}^* \\ 0 & 1 & \frac{\beta}{\alpha} & \frac{\beta^2}{2\alpha} & \bar{\varphi}^* \\ 0 & 0 & 1 & \beta & \bar{M}^* \\ 0 & 0 & 0 & 1 & \bar{Q}^* \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i$$

$$\beta_i = \frac{l_i}{l_0}$$

$$\alpha_i = \frac{EI_i}{100 P_0 l_0^2}$$

$$[Z] = \begin{bmatrix} \bar{y} \\ \bar{\varphi} \\ \bar{M} \\ \bar{Q} \\ 1 \end{bmatrix}$$

$$\bar{y} = -100 \frac{y_i}{l_0} \qquad y = -\frac{\bar{y} l_0}{100}$$

$$\bar{\varphi} = -100 \varphi \qquad \varphi = -\frac{\bar{\varphi}}{100}$$

$$\bar{M} = M / P_0 l_0 \qquad M = P_0 l_0 \bar{M}$$

$$\bar{Q} = Q_i / P_0 \qquad Q_i = P_0 \bar{Q}$$

P_0

(応力法)

[手順]

荷重 $\mathbb{P}$

[f_i]：区間に分ける ⟶ [F]

[B_0]：$\mathbb{P}=1.0$ ⟶ 曲げモーメント（静定）

[B_1]：不静定力 → 曲げモーメント

$$[G_{11}] = [B_1]^*[F][B_1]$$

$$[G_{10}] = [B_1]^*[F][B_0]$$

$$\mathbb{X} = -[G_{11}]^{-1}[G_{10}]\mathbb{P}$$

$$\mathbb{M} = [B_0]\mathbb{P} + [B_1]\mathbb{X}$$

$$\Delta = [B]^*[F][B]\mathbb{P}$$

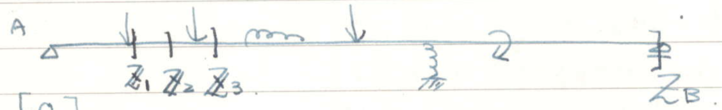
$$[B] = [B_0] - [B_1][G_{11}]^{-1}[G_{10}]$$

$$\mathbb{M} = \{[B_0] - [B_1][G_{11}]^{-1}[G_{10}]\}\mathbb{P}$$

$$= [B]\mathbb{P}$$

還元法

transfer matrix



$$\begin{bmatrix} y \\ \varphi \\ Q \\ M \\ 1 \end{bmatrix} \quad Z_A = \begin{bmatrix} 0 \\ \varphi_A \\ Q_A \\ 0 \\ 1 \end{bmatrix}$$

↑ 状態ベクトル

単純支点...

$$\begin{bmatrix} 0 \\ 0 \\ Q_A \\ M_A \\ 1 \end{bmatrix}$$

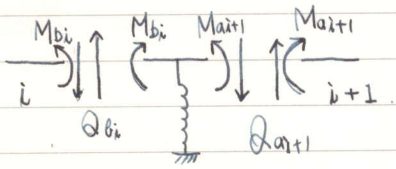
固定端

$$Z_1 = [F_1]\, Z_A$$

$$Z_1 = \begin{bmatrix} y_1 \\ \varphi_1 \\ Q_1 \\ M_1 \\ 1 \end{bmatrix}$$

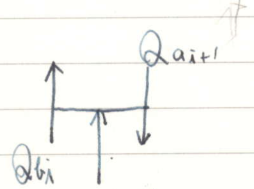
$$Z_2 = [F_2]\, Z_1 = [F_2][F_1]\, Z_A = [F_{21}]\, Z_A$$

$$Z_B = [F_{BA}]\, Z_A = [F_n][F_{n-1}]\cdots[F_2][F_1]\, Z_A$$



$$\left.\begin{array}{l} y_{ai+1} = y_{bi} \\ Q_{ai+1} = Q_{bi} \end{array}\right\}$$ 変位の適合条件

$$\left.\begin{array}{l} M_{ai+1} = M_{bi} \\ Q_{ai+1} = k_{yi} y_{bi} + Q_{bi} \end{array}\right|$$ 釣合の条件.

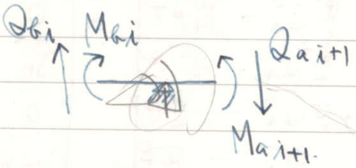


$$R = k_{yi} y_{bi}$$

$$Q_{bi} + k_{yi} y_{bi} - Q_{ai+1} = 0$$

$$\begin{bmatrix} y \\ \theta \\ M \\ Q \\ 1 \end{bmatrix}_{a\,i+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ k_{yi} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ \theta \\ M \\ Q \\ 1 \end{bmatrix}_{b\,i}$$

6/25



$y_{ai+1} = y_{bi}$

$\theta_{ai+1} = \theta_{bi}$

$M_{ai+1} = M_{bi}$

$Q_{ai+1} = R_i + Q_{bi}$

$$\begin{bmatrix} y \\ \theta \\ M \\ Q \\ 1 \end{bmatrix}_{ai+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & R_i \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ \theta \\ M \\ Q \\ 1 \end{bmatrix}_{bi}$$

$$[P_i] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & \Gamma & 1 & 0 & 0 \\ -\mu & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Gamma = k_{ai} / 100 P_0 l_0$

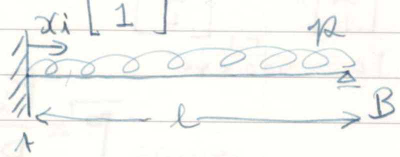
$\mu = k_{yi} l_0 / 100 P_0$

$$Z = \begin{bmatrix} \bar{y} \\ \bar{\varphi} \\ \bar{M} \\ \bar{Q} \\ 1 \end{bmatrix}$$

$\bar{y} = -\dfrac{y_i}{l_0} \times 100$ $\quad \bar{M} = M_i / P_0 l_0$

$\bar{\varphi} = -100 \varphi_i$ $\quad \bar{Q} = Q_i / P_0$

(図18.3)



$$Z_A = \begin{bmatrix} 0 \\ 0 \\ \bar{M} \\ \bar{Q} \\ 1 \end{bmatrix} \quad Z_B = \begin{bmatrix} 0 \\ \bar{\varphi} \\ 0 \\ \bar{Q} \\ 1 \end{bmatrix}$$

無次元化.

$l_0 = l \quad \therefore \beta = 1.$

$\alpha = EI / 100 P_0 l_0^2 = 1$ とP₀を決める $P_0 = \dfrac{EI}{100 l^2}$

$$[F] = \begin{bmatrix} 1 & 1 & 1/2 & 1/6 & -\bar{p}/24 \\ 0 & 1 & 1 & 1/2 & -\bar{p}/6 \\ 0 & 0 & 1 & 1 & -\bar{p}/2 \\ 0 & 0 & 0 & 1 & -\bar{p} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[F]\,Z_A=\begin{bmatrix}1&1\beta&\frac12\beta^2&\frac16\beta^3&-\frac{\bar p}{24}\beta^4\\0&1&1\beta&\frac12\beta^2&-\frac{\bar p}{6}\beta^3\\0&0&1&1\beta&-\frac{\bar p}{2}\beta^2\\0&0&0&1&-\bar p\beta\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\\bar M\\\bar Q\\1\end{bmatrix}_A=\begin{bmatrix}\frac{\beta^2}{2}\bar M_A+\frac{\beta^3}{6}\bar Q_A-\frac{\bar p}{24}\beta^4\\ \beta\bar M_A+\frac{\beta^2}{2}\bar Q_A-\frac{\bar p}{6}\beta^3\\ \bar M_A+\beta\bar Q_A-\frac{\bar p}{2}\beta^2\\ \bar Q_A-\bar p\beta\\ 1\end{bmatrix}=\begin{bmatrix}0\\ \varphi_B\\ 0\\ \bar Q_B\\ 1\end{bmatrix}$$

$\frac12\bar M_A+\frac16\bar Q_A=\frac{\bar p}{24}$

$\bar M_A+\bar Q_A=\frac{\bar p}{2}$

$\bar M_A=-\bar p/8$

$\bar Q_A=5/8\,\bar p$

$\bar M_A+\frac12\bar Q_A-\frac{\bar p}{6}=\varphi_B$

$=(-\frac18+\frac{5}{16}-\frac16)\bar p=-\frac{1}{48}\bar p$

$\bar Q_B=\bar Q_A-\bar p=-\frac{\bar p}{8}$

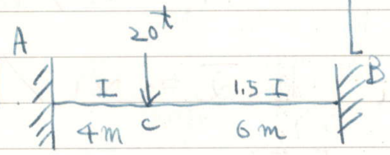
$\bar p=\frac{p l_0}{p_0}=\frac{p l_0}{EI}=\frac{p l\times 100 l^2}{EI}=\frac{100 p l^3}{EI}$

$\varphi_B=-\frac{1}{48}\times\frac{100 p l^3}{100 EI}=-\frac{p l^3}{48EI}$

$\boxed{\beta=x_i/l_0}$

$$Z_x=\begin{bmatrix}\frac{\beta^2}{2}(-\frac{\bar p}{8})+\frac{\beta^3}{6}(\frac58\bar p)-\frac{\bar p}{24}\beta^4\\ \beta(-\frac{\bar p}{8})+\frac{\beta^2}{2}(\frac58\bar p)-\frac{\bar p}{6}\beta^3\\ -\frac{\bar p}{8}+\beta\frac58\bar p-\frac{\bar p}{2}\beta^2\\ \frac58\bar p-\bar p\beta\\ 1\end{bmatrix}$$

[134]



$Z_A=\begin{bmatrix}0\\0\\\bar M_A\\\bar Q_A\\1\end{bmatrix}$ $Z_B=\begin{bmatrix}0\\0\\\bar M_B\\\bar Q_B\\1\end{bmatrix}$

$l_0=4m.\ \beta=1\ \alpha=\frac{EI}{100P_0 l^2}=1,\quad P_0=\frac{EI}{100\times4^2}$ $\times100\times4^2$

$\bar P=\frac{P_i}{P_0}=\frac{20}{EI}$

$\bar p=\frac{20\times4^2\times100}{EI}$

$$[F_{AC}]=\begin{bmatrix}1&1&\frac12&\frac16&0\\0&1&1&\frac12&0\\0&0&1&1&0\\0&0&0&0&\bar p\\0&0&0&0&1\end{bmatrix}$$

$\beta=\frac64=1.5\quad\alpha=\frac{1.5EI}{100P_0 l^2}=\frac{1.5EI}{100\times4^2}\times\frac{4^2\times100}{EI}$

$\alpha=1.5$

$$[F_{CB}]=\begin{bmatrix}1&1.5&\frac{1.5}{2}&\frac{1.5^2}{6}&0\\0&1&1&\frac{1.5}{2}&0\\0&0&1&1.5&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$$

8.2 変形法（変位法）

$S=[K_0][A][K]^{-1}P$

$d=[K]^{-1}P$

$[K]=[A]^*[K_0][A]$

（肩つきの *は転置マトリックスを示す）

S：部材応力を要素とするベクトル。

$[A]$：部材端（格点）の変位を部材変形に変換するマトリックス。$[A]$ の i 行 j 列の要素は第 j 格点の P_j 方向の変位を 1 とし他の変位を 0 としたときの第 i 番目に配置された部材変形。一つの部材について考慮すべき部材変形は必ずしも 1 個でないから，第 i 番目に配置されている部材変形は，必ずしも第 i 番目の部材に関するものではない。可能な格点における部材変形を v とすると，

$v=[A]d$

$[K_0]$：部材の剛性マトリックス。これの要素は表 8.1 参照

P：部材端（格点）において考えられるすべての変位の方向に作用する荷重を要素とするベクトル。要素の配列の順序は任意であるが，格点の順序に従うのが便利である。

d：部材端（格点）において考えられるすべての方向の変位を要素とするベクトル。要素の配列の順序は P と同じ順序に配列する。

$[\bar{K}]=[C_{00}]-[C_{10}]^*[C_{11}]^{-1}[C_{10}]=[C_{00}]+[C_{10}]^*[Y]$

$[C_{11}]=[A_1]^*[K_0][A_1]$

$[C_{00}]=[A_0]^*[K_0][A_0]$

$[C_{10}]=[A_1]^*[K_0][A_0]$

$[Y]=-[C_{11}]^{-1}[C_{10}]$

$[A_0]$：$[A]$ のうち，0 でない荷重に対応する変位を部材変形に変換するマトリックス

$[A_1]$：$[A]$ のうち，0 である荷重に対応する変位を部材変形に変換するマトリックス

NO.

$$S=[K_0][\overline{A}][\overline{K}]^{-1}P$$

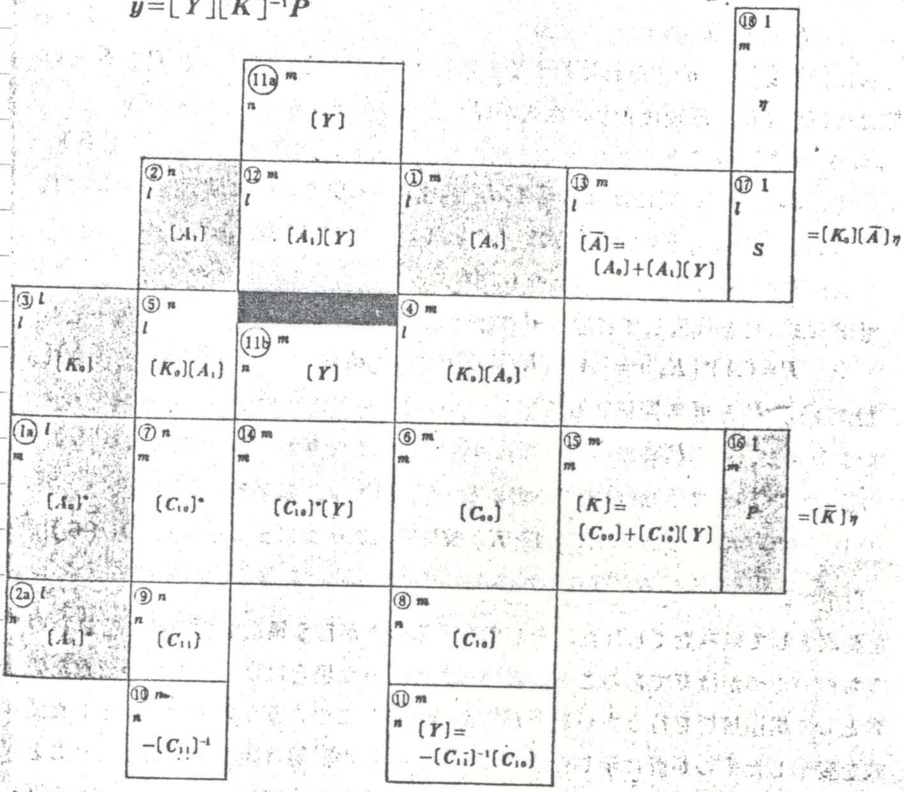
$$[\overline{A}]=[A_0]+[A_1][Y]$$

$$=[A_0]-[A_1][C_{11}]^{-1}[C_{10}]$$

$$\eta=[\overline{K}]^{-1}P$$

荷重が 0 である格点の変位を y とすると

$$y=[Y][\overline{K}]^{-1}P$$

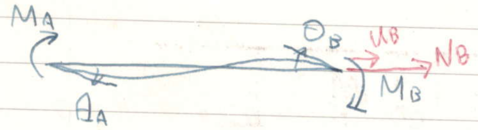


うすずみの部分は計算開始時に既知のマトリックス
l, m, n は各マトリックスの行または列の数を示す。

図 8.20 計算の順序 ○印の中の数字の順に計算を行う

7/2　変位法

$[f]$　　$\mathbb{U} = [f]\mathbb{S}$　　$[f] = \begin{bmatrix} \frac{l}{3EI} & -\frac{l}{6EI} \\ -\frac{l}{6EI} & \frac{l}{3EI} \end{bmatrix}$



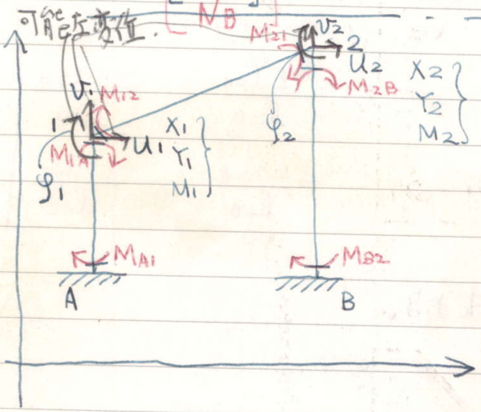
$\mathbb{S}_i = [f]_i^{-1}\mathbb{U}_i$
$= [k]_i \mathbb{U}_i$

軸力を考える場合

$u = \begin{bmatrix} \theta_A \\ \theta_B \\ u_B \end{bmatrix}$

$\mathbb{S} = \begin{bmatrix} M_A \\ M_B \\ N_B \end{bmatrix}$

$[k]_i = \begin{bmatrix} \frac{4EI}{l} & \frac{2EI}{l} & 0 \\ \frac{2EI}{l} & \frac{4EI}{l} & 0 \\ 0 & 0 & \frac{EA}{l} \end{bmatrix}$



$\mathbb{S} = \begin{bmatrix} M_{A1} \\ M_{1A} \\ M_{12} \\ M_{21} \\ M_{2B} \\ M_{B2} \end{bmatrix}$　　$\mathbb{U} = \begin{bmatrix} \theta_{A1} \\ \theta_{1A} \\ \theta_{12} \\ \theta_{21} \\ \theta_{2B} \\ \theta_{B2} \end{bmatrix}$

（対応）

可能な変位 $\mathbb{u}$　（d）　　　　可能な荷重（外力）

$\mathbb{u} = \begin{bmatrix} u_1 \\ v_1 \\ \varphi_1 \\ u_2 \\ v_2 \\ \varphi_2 \end{bmatrix}$ ← 並べ方は任意　　$\mathbb{P} = \begin{bmatrix} X_1 \\ Y_1 \\ M_1 \\ X_2 \\ Y_2 \\ M_2 \end{bmatrix}$

（対応）

$\mathbb{S}_{A1} = [k]_{A1}\mathbb{U}_{A1}$　　$\mathbb{S} = [k_0]\mathbb{U}$
$\mathbb{S}_{12} = [k]_{12}\mathbb{U}_{12}$
$\mathbb{S}_{2B} = [k]_{2B}\mathbb{U}_{2B}$

$\mathbb{S} = \begin{bmatrix} \mathbb{S}_{A1} \\ \mathbb{S}_{12} \\ \mathbb{S}_{2B} \end{bmatrix}$　　$\mathbb{U} = \begin{bmatrix} \mathbb{U}_{A1} \\ \mathbb{U}_{12} \\ \mathbb{U}_{2B} \end{bmatrix}$

NO.

☆ $[k_0] = \begin{bmatrix} [k]_{A1} & & \\ & [k]_{12} & \\ & & [k]_{2B} \end{bmatrix}$

$$= \begin{bmatrix} \frac{4EI_{A1}}{l_{A1}} & \frac{2EI_{A1}}{l_{A1}} & 0 & 0 & 0 & 0 \\ \frac{2EI_{A1}}{l_{A1}} & \frac{4EI_{A1}}{l_{A1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4EI_{12}}{l_{12}} & \frac{2EI_{12}}{l_{12}} & 0 & 0 \\ 0 & 0 & \frac{2EI_{12}}{l_{12}} & \frac{4EI_{12}}{l_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4EI_{2B}}{l_{2B}} & \frac{2EI_{2B}}{l_{2B}} \\ 0 & 0 & 0 & 0 & \frac{2EI_{2B}}{l_{2B}} & \frac{4EI_{2B}}{l_{2B}} \end{bmatrix}$$

$\mathbb{P}:S$, $\mathbb{P}:\mathbb{U}$ の関係を求める。

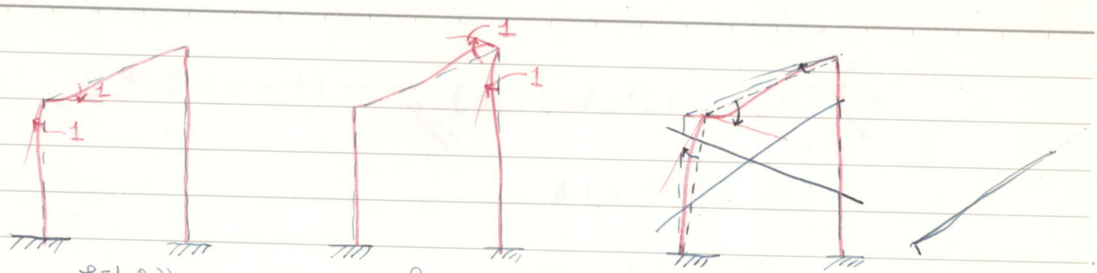
変位の適合条件.

☆ $\mathbb{V} = [A]\mathbb{U}$

←その他の変位が0のときの変位図.

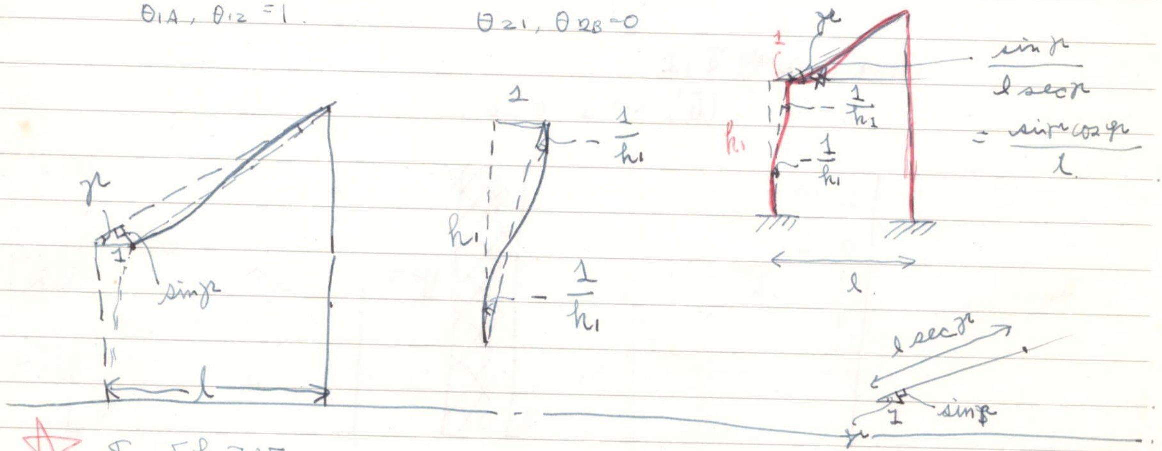
$$\begin{bmatrix} \theta_{A1} \\ \theta_{1A} \\ \theta_{12} \\ \theta_{21} \\ \theta_{2B} \\ \theta_{B2} \end{bmatrix} = \overset{\quad u_1=1 \quad v_1=1 \quad \varphi_1=1 \quad u_2=1 \quad v_2=1 \quad \varphi_2=1}{\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}} \begin{bmatrix} u_1 \\ v_1 \\ \varphi_1 \\ u_2 \\ v_2 \\ \varphi_2 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{h_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{h_1} & 0 & 1 & 0 & 0 & 0 \\ \frac{\sin\gamma\cos\gamma}{l} & -\frac{\cos^2\gamma}{l} & 1 & -\frac{\sin\gamma\cos\gamma}{l} & \frac{\cos^2\gamma}{l} & 0 \\ \frac{\sin\gamma\cos\gamma}{l} & -\frac{\cos^2\gamma}{l} & 0 & -\frac{\sin\gamma\cos\gamma}{l} & \frac{\cos^2\gamma}{l} & 1 \\ 0 & 0 & 0 & -\frac{1}{h_2} & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{h_2} & 0 & 0 \end{bmatrix}$$



$\varphi_1 = 1$ のみ　　　$\varphi_2 = 1$ のみ　　　$u_1 = 1$ のみ

$\theta_{1A},\ \theta_{12} = 1$　　　$\theta_{21},\ \theta_{2B} = 0$



★ $S = [k_0]\, U$

★ $U = [A]\, \mathbb{U}$　　　$U^T = \mathbb{U}^T [A]^T$

仮想仕事の原理　$\delta W_E = \delta W_I$

仮想変位 $\bar{\mathbb{U}}$

$$\delta W_E = \begin{bmatrix} \bar{u}_1 & \bar{v}_1 & \bar{\varphi}_1 & \bar{u}_2 & \bar{v}_2 & \bar{\varphi}_2 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ M_1 \\ X_2 \\ Y_2 \\ M_2 \end{bmatrix}$$

$$= \bar{\mathbb{U}}^* \mathbb{P} = \bar{\mathbb{U}}^T \mathbb{P}$$

仮想変形 $\bar{U}$

$$\delta W_I = \begin{bmatrix} \bar{\theta}_{A1} & \bar{\theta}_{1A} & \bar{\theta}_{12} & \bar{\theta}_{21} & \bar{\theta}_{2B} & \bar{\theta}_{B2} \end{bmatrix} \begin{bmatrix} M_{A1} \\ M_{1A} \\ M_{12} \\ M_{21} \\ M_{2B} \\ M_{B2} \end{bmatrix}$$

$$= \bar{U}^* S = \bar{U}^T S$$

$$\delta W_I = \bar{\mathbb{U}}^T [A]^T S = \bar{\mathbb{U}}^T [A]^T [k_0]\, U$$

$$= \bar{\mathbb{U}}^T \underbrace{[A]^T [k_0] [A]}_{\text{対称マトリックス}} \mathbb{U}$$

☆ $[k] = [A]^T [k_0] [A]$ ・・・・（たわみの影響線）

$\therefore\ SW_I = \bar{\mathbb{U}}^T [k] \mathbb{U}$

$\therefore\ \bar{\mathbb{U}}^T \mathbb{P} = \bar{\mathbb{U}}^T [k] \mathbb{U}$

特殊な仮想変位

$\bar{\mathbb{U}}_1^T = [1, 0, 0, \cdots\cdots]$

$$\begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & & & \\ \vdots & & 1 & & \\ \vdots & & & \ddots & \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix} \bar{\mathbb{U}} \ \mathbb{P} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} [k] \mathbb{U}$$

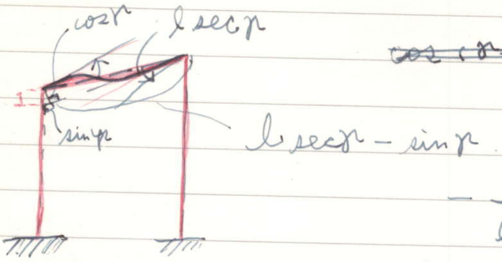
$\therefore\ \mathbb{P} = [k] \mathbb{U}$

☆ $\mathbb{U} = [k]^{-1} \mathbb{P}$

☆ $\mathbb{V} = [A] \mathbb{U} = [A] [k]^{-1} \mathbb{P}$

☆ $\mathbb{S} = [k_0] \mathbb{V} = [k_0] [A] [k]^{-1} \mathbb{P}$

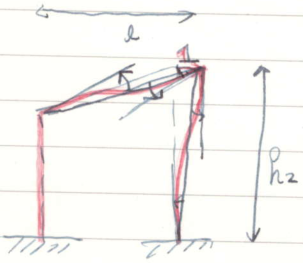
NO.



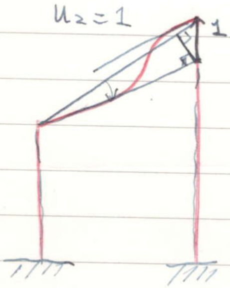
~~cos φ~~

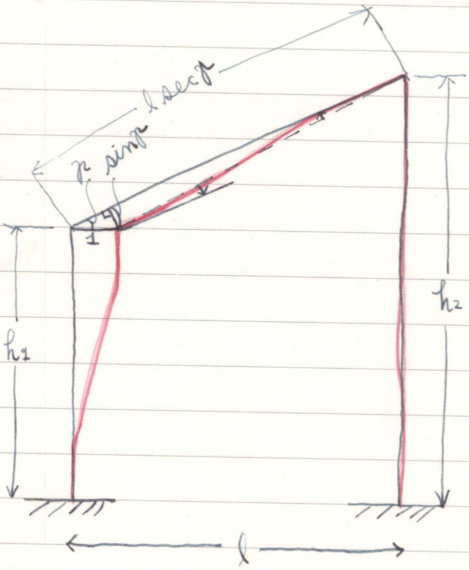
$$-\frac{\cos\varphi}{l\sec\varphi-\sin\varphi}$$

$v_1 = 1$.

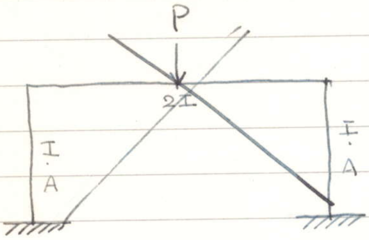


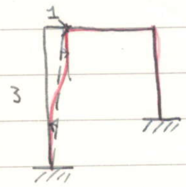
$u_2 = 1$



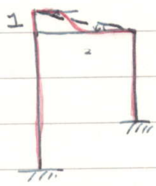


$u_1 = 1$.

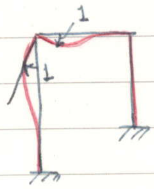




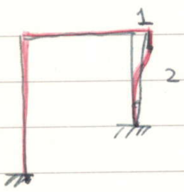
$u_1 = 1$



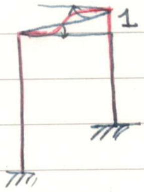
$v_1 = 1$



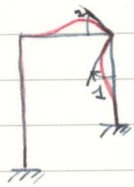
$\varphi_1 = 1$



$u_2 = 1$

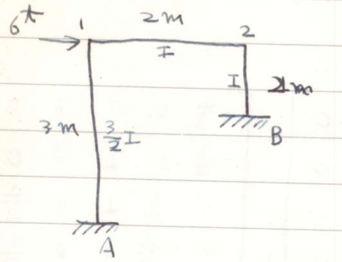


$v_2 = 1$



$\varphi_2 = 1$

$\frac{EI}{l}$



~~EI=一定とする.~~

$$S = \begin{bmatrix} M_{A1} \\ M_{1A} \\ M_{12} \\ M_{21} \\ M_{2B} \\ M_{B2} \end{bmatrix} \qquad V = \begin{bmatrix} \theta_{A1} \\ \theta_{1A} \\ \theta_{12} \\ \theta_{21} \\ \theta_{2B} \\ \theta_{B2} \end{bmatrix}$$

$$U = \begin{bmatrix} u_1 \\ v_1 \\ \varphi_1 \\ u_2 \\ v_2 \\ \varphi_2 \end{bmatrix} \qquad P = \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$4E\cdot\frac{3}{2}I$ $\qquad \frac{2E\cdot\frac{3}{2}I}{3}$

$$[k_0] = \begin{bmatrix} 2EI & EI & & & & \\ EI & 2EI & & & & \\ & & 2EI & EI & & \\ & & EI & 2EI & & \\ & & & & 2EI & EI \\ & & & & EI & 2EI \end{bmatrix}$$

$$= EI \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$[A] = \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \end{bmatrix}$$

$[k] = [A]^{*}[k_0][A]$

$$= \begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} EI \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \end{bmatrix}$$

$$= EI \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.5 & -1.5 & 0 & 0 \\ 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.5 & -1.5 \\ 0 & 0 & 1.5 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \end{bmatrix}$$

$$= EI \begin{bmatrix} \frac{2}{3} & 0 & -1 & 0 & 0 & 0 \\ 0 & 0.75 & -1.5 & 0 & -1.5 & -1.5 \\ -1 & -1.5 & 4 & 0 & 1.5 & 1 \\ 0 & 0 & 0 & 1.5 & 0 & -1.5 \\ 0 & -0.75 & 1.5 & 0 & 1.5 & 1.5 \\ 0 & -1.5 & 1 & -1.5 & 1.5 & 4 \end{bmatrix} = \frac{EI}{6} \begin{bmatrix} 4 & 0 & -6 & 0 & 0 & 0 \\ 0 & 4.5 & -9 & 0 & -9 & -9 \\ -6 & -9 & 24 & 0 & 9 & 6 \\ 0 & 0 & 0 & 9 & 0 & -9 \\ 0 & -4.5 & 9 & 0 & 9 & 9 \\ 0 & -9 & 6 & -9 & 9 & 24 \end{bmatrix}$$

$[A]^{-1} =$

$$\left[\begin{array}{cccccc|cccccc} 4 & 0 & -6 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.5 & -9 & 0 & -9 & -9 & 0 & 1 & 0 & 0 & 0 & 0 \\ -6 & -9 & 24 & 0 & 9 & 6 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & -9 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -4.5 & 9 & 0 & 9 & 9 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -9 & 6 & -9 & 9 & 24 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

NO.

⇒

1	0	-1.5	0	0	0	0.25	0	0	0	0	0
0	1	-2	0	-2	-2	0	0.222	0	0	0	0
-0.25	-0.375	1	0	0.375	0.25	0	0	0.0417	0	0	0
0	0	0	1	0	-1	0	0	0	0.111	0	0
0	-0.5	1	0	1	1	0	0	0	0	0.111	0
0	-0.375	0.25	-0.375	0.375	1	0	0	0	0	0	0.0417

⇒

1	0	-1.5	0	0	0	0.25	0	0	0	0	0
0	1	-2	0	-2	-2	0	0.222	0	0	0	0
0	-0.375	0.625	0	0.375	0.25	0.0625	0.	0.0417	0	0	0
0	0	0	1	0	-1	0	0	0	0.111	0	0
0	-0.5	1	0	1	1	0	0	0	0	0.111	0
0	-0.375	0.25	-0.375	0.375	1	0	0	0	0	0	0.0417

⇒

1	0	-1.5	0	0	0	0.25	0	0	0	0	0
0	1	-2	0	-2	-2	0	0.222	0	0	0	0
0	0	-0.125	0	-0.375	-0.5	0.0625	0.0834	0.0417	0	0	0
0	0	0	1	0	-1	0	0	0	0.111	0	0
0	0	0	0	0	0	0	0.111	0	0	0.111	0
~~0~~	~~0.125~~	~~-0.75~~	~~-0.375~~	~~0~~	~~0~~	~~0~~					
0	0	~~0.0625~~ -0.5	-0.375	-0.375	0.25	0	0.0834	0	0	0.	0.0417

⇒

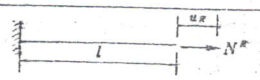
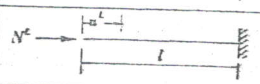
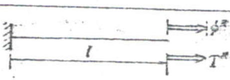
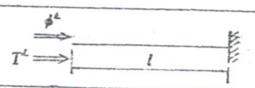
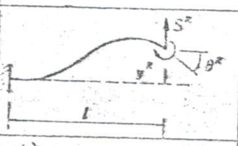
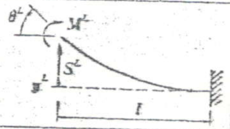
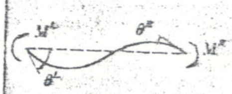
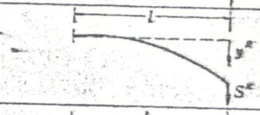
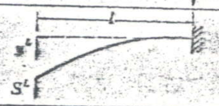
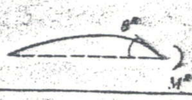
1	0	-1.5	0	0	0	0.25	0	0	0	0	0
0	1	-2	0	-2	-2	0	0.222	0	0	0	0
0	0	1	0	3	4.	0.5.	0.667	0.334	0	0	0
0	0	0	1	0	-1	0	0	0	0.111	0	0
0	0	0	0	0	0	0	0.111	0	0	0.111	0
0	0	0	-0.375	1.275	1.75	0.25	0.417	0.668	0	0	0.0417

⇒

1	0	0	0	4.5	6	1	1.0005	0.501	0	0	0
0	1	0	0	4	6	1	1.556	0.668	0	0	0
0	0	1	0	3	4	0.5	0.667	0.334	0	0	0
0	0	0	1	0	-1	0	0	0	0.111	0	0
0	0	0	0	0	0	0	0.111	0	0	0.111	0
~~0~~	~~0~~	~~0~~	~~0.375~~	~~1.275~~	~~1.75~~	~~0.25~~	~~0.417~~	~~0.668~~	~~0~~	~~0~~	~~0.0417~~
0	0	0	0								

NO.

表 8.1　たわみ性マトリックスと剛性マトリックス

	構造要素とそれに作用する力と対応変形		たわみ性マトリックスの要素	剛性マトリックスの要素
(1)			$\frac{l}{EA}$	$\frac{EA}{l}$
(2)			$\frac{l}{EA}$	$\frac{EA}{l}$
(3)			$\frac{l}{I_T G}$	$\frac{I_T G}{l}$
(4)			$\frac{l}{I_T G}$	$\frac{I_T G}{l}$
(5)		$\begin{bmatrix} M \\ S \end{bmatrix}$ $\begin{bmatrix} \theta \\ y \end{bmatrix}$	$\begin{bmatrix} \frac{l}{EI} & \frac{-l^2}{2EI} \\ \frac{-l^2}{2EI} & \frac{l^3}{3EI} \end{bmatrix}$	$\begin{bmatrix} \frac{4EI}{l} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{12EI}{l^3} \end{bmatrix}$
(6)		$\begin{bmatrix} M \\ S \end{bmatrix}$ $\begin{bmatrix} \theta \\ y \end{bmatrix}$	$\begin{bmatrix} \frac{l}{EI} & \frac{l^2}{2EI} \\ \frac{l^2}{2EI} & \frac{l^3}{3EI} \end{bmatrix}$	$\begin{bmatrix} \frac{4EI}{l} & \frac{-6EI}{l^2} \\ \frac{-6EI}{l^2} & \frac{12EI}{l^3} \end{bmatrix}$
(7)		$\begin{bmatrix} M^L \\ M^R \end{bmatrix}$ $\begin{bmatrix} \theta^L \\ \theta^R \end{bmatrix}$	$\begin{bmatrix} \frac{l}{3EI} & \frac{-l}{6EI} \\ \frac{-l}{6EI} & \frac{l}{3EI} \end{bmatrix}$	$\begin{bmatrix} \frac{4EI}{l} & \frac{2EI}{l} \\ \frac{2EI}{l} & \frac{4EI}{l} \end{bmatrix}$
(8)			$\frac{l^3}{3EI}$	$\frac{3EI}{l^3}$
(9)			$\frac{l^3}{3EI}$	$\frac{3EI}{l^3}$
(10)			$\frac{l}{3EI}$	$\frac{3EI}{l}$
(11)			$\frac{l}{3EI}$	$\frac{3EI}{l}$

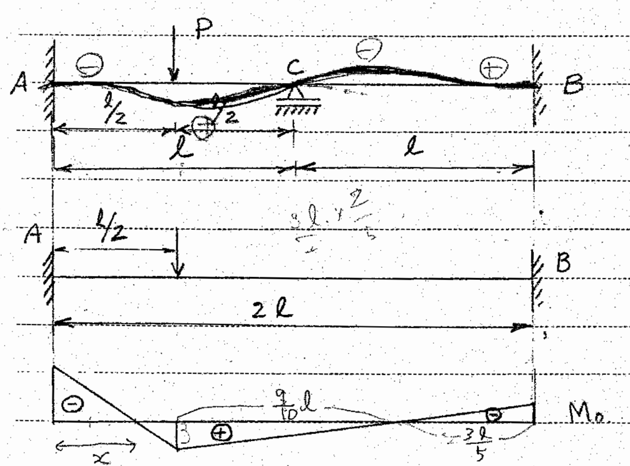
(昭和52年 6月10日 2時限) 試 験 答 案 用 紙

教室番号	試験科目	担当教員	科	学年・組	学籍番号	氏名	採点
633	構造力学第1	西脇	土木	3・2	755089	皆川 勝	100
着席番号							

教科書等
・~~参照一切不可~~
⊙参照可
1. 教科書
2. ノート(他人のノート
3. (それ以外は不
~~計算用紙添付・要・否~~

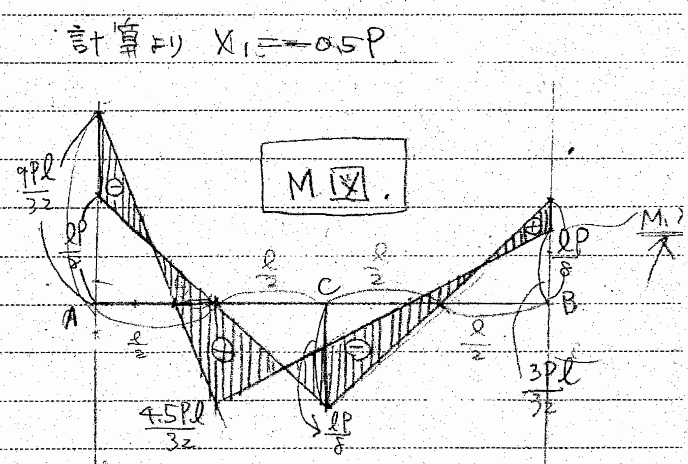
計算用紙添付・要・否 (B4)
解答用紙添付・要・否 (B4)

1. 図に示すはりの曲げモーメント図を求めよ。なお固定ばりABの曲げモーメント図は図に示すものである。但し全長にわたって断面は等しいものとする。



$-\frac{9}{32}Pl$ $\frac{4.5}{32}Pl$ $-\frac{3}{32}Pl$

計算より $X_1 = -0.5P$

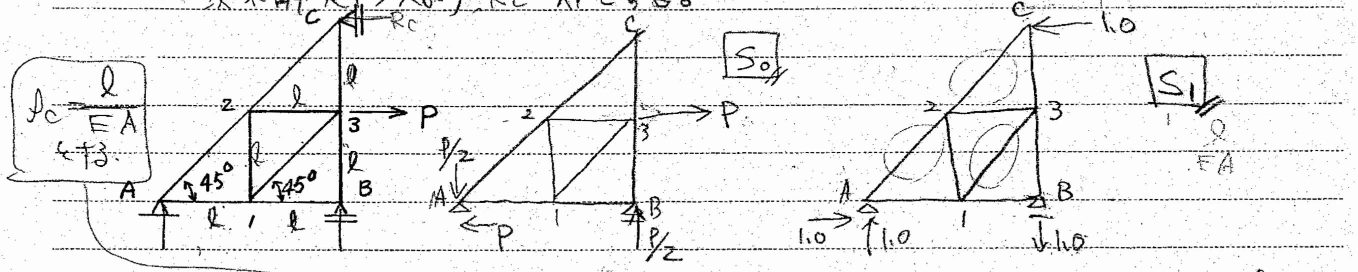


50

2. 図に示すトラスの部材力を求めよ。すべての部材の断面積は等しいものとする。

一次不静定トラスより $R_c = X_1$ とする。

$\rho_c = \frac{l}{EA}$ とする。



部材	S_0	S_1	ρ/ρ_c	$S_0S_1\rho/\rho_c$	$S_1^2\rho/\rho_c$
A1	$\frac{P}{2}$	0	1	0	0
A2	$\frac{P}{2}\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-P\sqrt{2}$	$2\sqrt{2}$
12	$-\frac{P}{2}$	0	1	0	0
2C	0	$-\sqrt{2}$	$\sqrt{2}$	0	$2\sqrt{2}$
23	$\frac{P}{2}$	0	1	0	0
13	$\frac{P}{2}\sqrt{2}$	0	$\sqrt{2}$	0	0
1B	0	0	1	0	0
C3	0	1.0	1	0	1.0
3.B	$-\frac{P}{2}$	1.0	1	$-P/2$	1.0
Total				$-P\sqrt{2}-\frac{P}{2}$	$4\sqrt{2}+2$

$X_1 = -\frac{\Sigma S_0S_1\rho/\rho_c}{\Sigma S_1^2\rho/\rho_c}$ より

$X_1 = \frac{\sqrt{2}+\frac{1}{2}}{4\sqrt{2}+2}P = \frac{1}{4}P$

$S_{A1} = \frac{P}{2}$, $S_{A2} = \frac{\sqrt{2}}{2}P - \frac{\sqrt{2}}{4}P = \frac{\sqrt{2}}{4}P$

$S_{12} = -\frac{P}{2}$, $S_{2C} = -\frac{\sqrt{2}}{4}P$

$S_{23} = \frac{P}{2}$, $S_{13} = \frac{\sqrt{2}}{2}P$

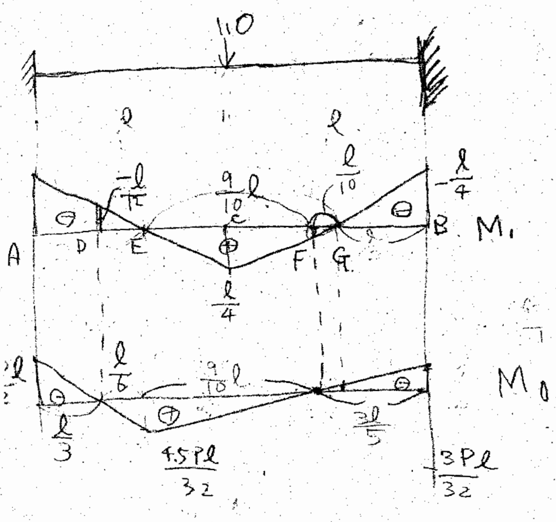
$S_{1B} = 0$, $S_{C3} = \frac{1}{4}P$

$S_{3B} = -\frac{P}{2} + \frac{P}{4} = -\frac{1}{4}P$

($\because S = S_0 + S_1X$)

50

立派な答案です。



$\int M_0 M_1$

AD間 $\frac{1}{6} \times \frac{l}{3} \times \frac{9Pl}{32} \times \left(\frac{l}{2} + \frac{l}{12}\right) = \frac{7Pl^3}{768} = 0.0091Pl^3$

DE間 $-\frac{1}{6} \times \frac{l}{6} \times \frac{l}{12} \times \frac{4.5Pl}{32} = -\frac{4.5Pl^3}{13824} = -0.0003Pl^3$

EF間 $\frac{4.5Pl}{32} \times \frac{l}{4} \times \frac{1}{6} \times \left(\frac{9}{10}l + \frac{9}{20}l\right) = \frac{121.5Pl^3}{15360} = 0.0079Pl^3$

FG間 $-\frac{1}{6} \times \frac{l}{10} \times \left(\frac{l}{4} \times \frac{2}{9}\right) \times \frac{3Pl}{32} \times \frac{1}{6} = \frac{-Pl^3}{69120} = 0.0000144Pl^3 \fallingdotseq 0$

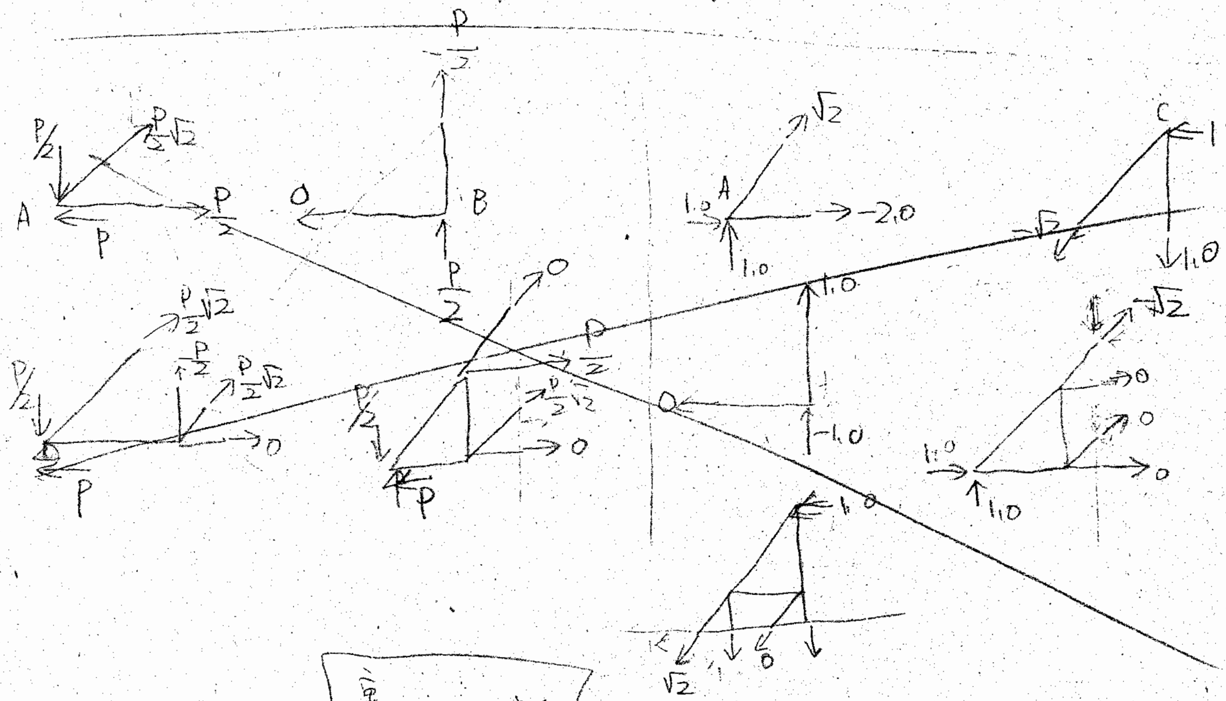
GB間 $\frac{1}{6} \times \frac{l}{2} \times \frac{l}{4} \times \left(\frac{3}{16}Pl + \frac{3Pl}{32} \times \frac{1}{6}\right) = \frac{13Pl^3}{3072} = 0.0042Pl^3$

$\therefore \Sigma \int M_0 M_1 ds = 0.0209Pl^3$

$\Sigma \int M_1^2 ds = 2 \times \frac{1}{3} \cdot \frac{l}{2} \times \frac{l^2}{16} + 2 \times \frac{1}{3} \times \frac{l}{2} \times \left(\frac{l}{4}\right)^2$

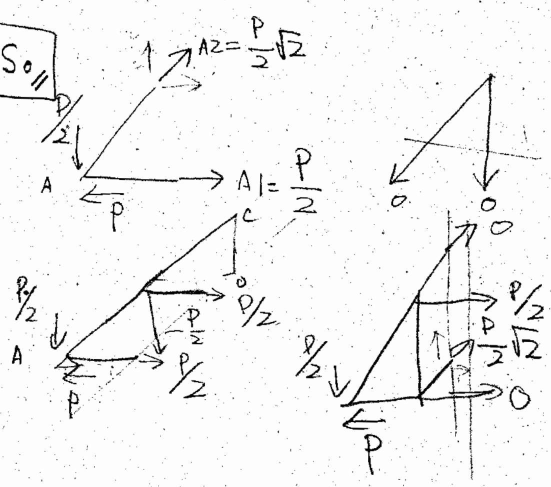
$= \frac{l^3}{48} + \frac{l^3}{48} = \frac{l^3}{24}$

$\therefore X = -0.0209Pl^3 / \frac{l^3}{24} = -0.5016P \fallingdotseq -0.5P$

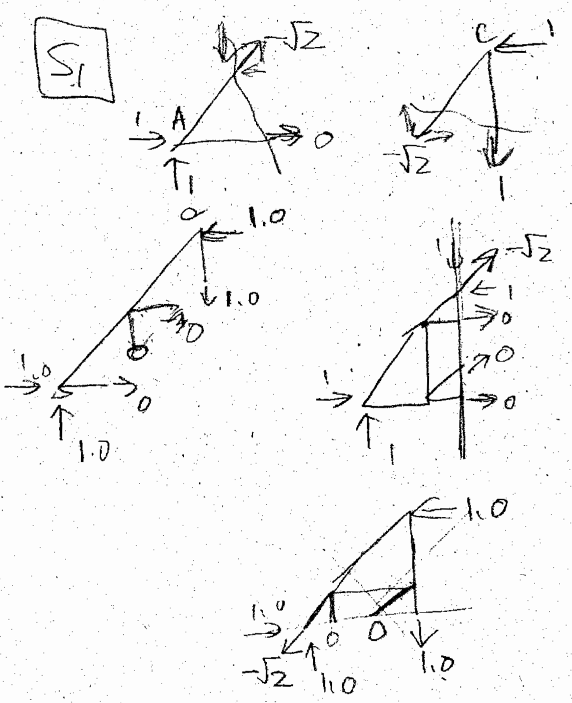


裏へつづく

S_0



S_1



$$\frac{\sqrt{2}+\frac{1}{2}}{4\sqrt{2}+2} \qquad \frac{4-?}{8}$$

$$=\frac{(\sqrt{2}+\frac{1}{2})(4\sqrt{2}-2)}{(4\sqrt{2}+2)(4\sqrt{2}-2)}$$

$$=\frac{8 + 2\sqrt{2}-2\sqrt{2}}{32-4}=\frac{?}{8}P$$

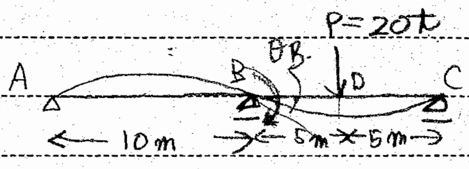
$$-\frac{(\sqrt{2}+\frac{1}{2})}{4\sqrt{2}+2}$$

$$\frac{(\sqrt{2}+\frac{1}{2})(4\sqrt{2}-2)}{(4\sqrt{2}+2)(4\sqrt{2}-2)}=\frac{8-1+2\sqrt{2}-2\sqrt{2}}{32-4}=\frac{?}{28}=\frac{1}{4}$$

$$\therefore x_1=\frac{P}{4}$$

3 . 8　　答　案　用　紙　（52年 6月 4日）

科目	構造力学(1)	科・年	土-3	学番 籍号	755089	氏名	皆川　勝	採点	15

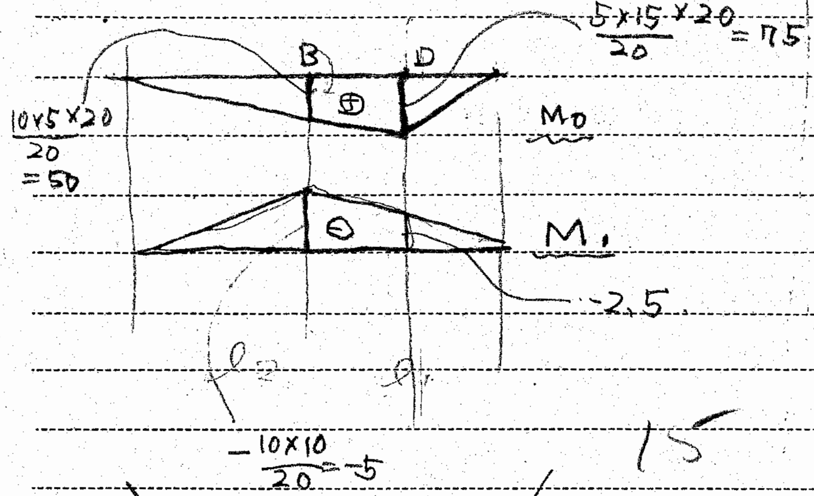


B点のたわみ角を求めよ.

EI = 21350 t·m

$R_B = X_1$とすると.

$$X_1 = -\frac{\delta_{10}}{\delta_{11}}$$



15

$$\int M_1 M_0 dx = \frac{10}{3}\times 50\times(-5)$$

$$+\frac{5}{6}\{50\times(-12.5)+75\times(-10)\}$$

$$+\frac{5}{3}\times 75\times(-2.5) = -2374.5$$

$$\int M_1^2 dx$$

$$= \frac{10}{3}\{5\times 5 + 5\times 5\} = 167.$$

$$\therefore X_1 = \frac{-(-2374.5)}{167} = 14.2$$

$$X_1 = -\frac{\int M_0 M_1 dx}{\int M_1^2 dx} = \left(\frac{x}{l_1}+\frac{x}{2l_2}-\frac{x^3}{2l_1^2 l_2}\right)P$$

$$= \left(\frac{5}{10}+\frac{5}{20}-\frac{5^3}{2\times 10^3}\right)\times 20$$

$$= 1\!\!\!/4.9\ (t)$$

$$M_D = 50 - 5\times 1.19$$

$$= 50 - 5.95$$

$$= 44.05\ t\cdot m$$

$$\therefore \theta_B = \frac{44.05}{EI}$$

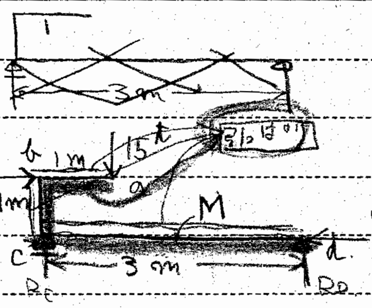
答　案　用　紙

（52年 5 月 20 日）

科目	構造力学(1)	科年	土・3	学籍番号	755089	氏名	皆川　勝	採点	~~40~~ 50

1)

$EI = 3.78 \times 10^{10}\ kg\,cm^2 = 3.78 \times 10^{3}\ t\cdot m^2$

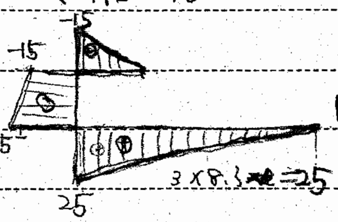


(1) b点の水平変位 ΔH_b

(2) 支点c,dの中点の~~変位~~鉛直変位を求めよ。Δv_M

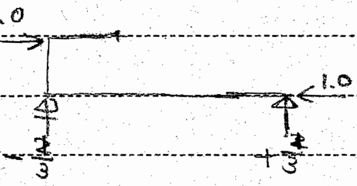
$\left\{ R_C + R_D = 15,\quad \Sigma M_C = 10 + 15 - 3R_D = 0 \quad \therefore R_D = \frac{25}{3} = 8.3^t \quad R_C = 6.7^t \right.$

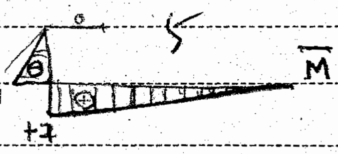
$H_D = 10^t$



10

(1) 1.0





$\int M_i M_k\, ds = \frac{3}{3} \times 25 \times 1 = 25$ （cd間）

$\int M_i M_k\, ds = \frac{l}{6}(2 y_a y_a' + y_b y_a')$

$= \frac{1}{6} \times (2 \times 25 \times 1 + 15 \times 1)$

$= \frac{65}{6} = 10.8$ （bc間）

$\therefore \Delta H_b = \Sigma \int \frac{M_i M_k}{EI} ds = \frac{35.8}{EI}$

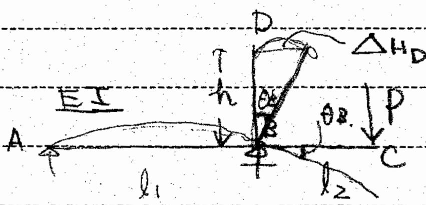
(2)は裏へつづく

$\therefore \Delta H_b = \frac{35.8}{3.78 \times 10^{3}}$

$= 9.5 \times 10^{-3}$ (m)

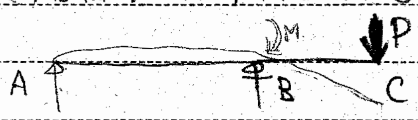
$=$ ~~0.94~~ (cm) 0.95 // 5

2)

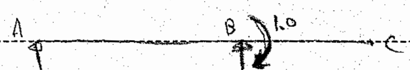


D点の水平変位.

まず,B点のたわみ角をおめる

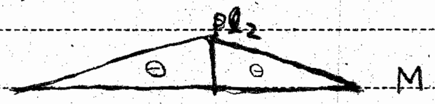


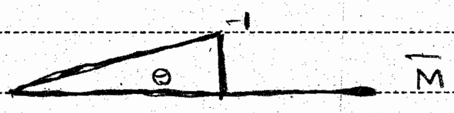
$R_A = -\frac{l_2}{l_1}P,\quad R_B = \frac{l_1 + l_2}{l_1}P$



$R_A + R_B = 0$

$\Sigma M_B = 1.0 + l_1 R_A = 0 \quad R_A = -\frac{1}{l_1}$





	AB	BC
l	l_1	l_2
M_i	0 ～ Pl_2	Pl_2 ～ 0
M_k	0 ～ 1	0 ～ 0

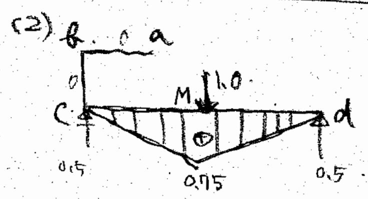
故に $\Delta H_D = \theta_B \cdot h$ 　 $(\because \theta_B \doteqdot \frac{\Delta H_D}{h})$

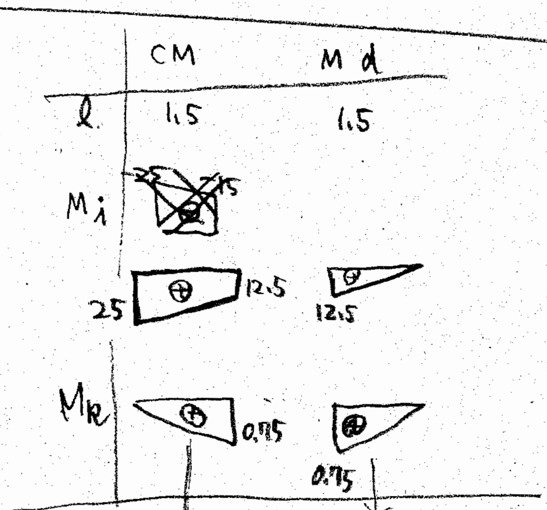
$= \frac{P l_1 l_2 h}{3EI}$ // 20

$\int M_i M_k\, ds = \frac{l_1}{3} \cdot P l_2 \cdot (+1) = +\frac{P l_1 l_2}{3}$

$\therefore \theta_B = \frac{P l_1 l_2}{3EI}$

1) (2)





$\int M_i M_k\, ds$

$\frac{l\, y_a y_a'}{3} = \frac{1.5}{3} \times 12.5 \times 0.75 = 4.7$

$\frac{l}{6}(y_a y_b' + 2 y_b y_b') = \frac{1.5}{6}(25 \times 0.75 + 2 \times 12.5 \times 0.75)$

$= \frac{1}{4} \cdot 0.75 \cdot 50 = \frac{3}{4} \times \frac{1}{4} \times 50 = 9.4$

$\therefore \Delta v_M = \Sigma \int \frac{M_i M_k}{EI} ds = \frac{4.7 + 9.4}{EI} = \frac{14.1}{EI}$

$= \frac{14.1}{3.78 \times 10^3} = 3.7 \times 10^{-3}$ (m)

$= 0.37$ (cm)

5

5

37
378) 1410
1134
2760
2646
1140

459
8) 37.5

$\frac{1}{3} \cdot \frac{3}{4}$ 12.5

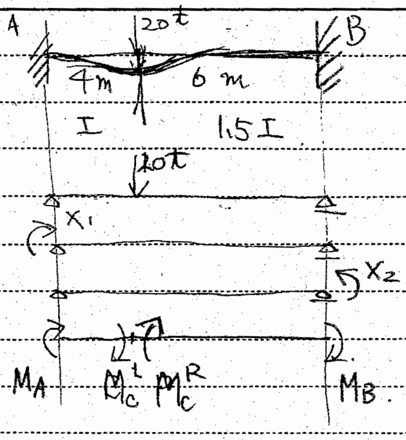
4_9

答　案　用　紙

(昭和　年　月　日　時限)

NO1

教室番号	着席番号	試験科目	担当教員	学科	学年	組	学籍番号	氏名	採点
			西脇	土	3	2	755089	皆川 勝	42

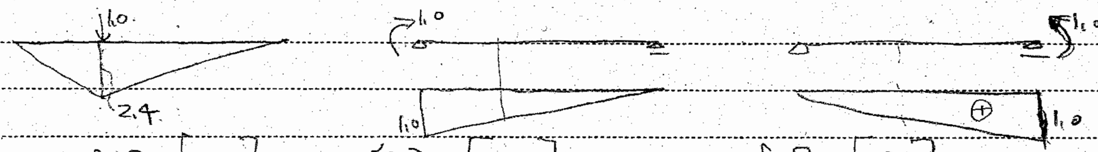


$\mathbb{P} = [20]$

$-\frac{2}{3}$　$-\frac{6}{9}$　$\frac{12}{9}$

$$[F] = \begin{bmatrix} \frac{4}{3EI} & -\frac{4}{6EI} & 0 & 0 \\ -\frac{4}{6EI} & \frac{4}{3EI} & 0 & 0 \\ 0 & 0 & \frac{6}{4.5EI} & -\frac{6}{9EI} \\ 0 & 0 & -\frac{6}{9EI} & \frac{6}{4.5EI} \end{bmatrix} = \frac{2}{9EI}\begin{bmatrix} 6 & -3 & 0 & 0 \\ -3 & 6 & 0 & 0 \\ 0 & 0 & 6 & -3 \\ 0 & 0 & -3 & 6 \end{bmatrix}$$

10



$$[B_0] = \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \\ 0 \end{bmatrix} \quad 10 \qquad [B_1] = \begin{bmatrix} 1.0 \\ -0.6 \\ 0.6 \\ 0 \end{bmatrix} \qquad [B_2] = \begin{bmatrix} 0 \\ -0.4 \\ 0.4 \\ -1.0 \end{bmatrix} \quad 8$$

6 + 1.8

−3 −3.6

$$[G_{11}] = [B_1]^*[F][B_1] = [1.0\ -0.6\ \ 0.6\ \ 0]\ \frac{2}{9EI}\begin{bmatrix} 6 & -3 & 0 & 0 \\ -3 & 6 & 0 & 0 \\ 0 & 0 & 6 & -3 \\ 0 & 0 & -3 & 6 \end{bmatrix}\begin{bmatrix} 1.0 \\ -0.6 \\ 0.6 \\ 0 \end{bmatrix} = \frac{2}{9EI}[7.8\ -6.6\ \ 3.6\ -1.8]\begin{bmatrix} 1.0 \\ -0.6 \\ 0.6 \\ 0 \end{bmatrix}$$

$[G_{11}]X = [G_{10}]P$

$$= \frac{2}{9EI}[13.92]\ ✓$$

$$[G_{10}] = \frac{2}{9EI}[7.8\ -6.6\ \ 3.6\ -1.8]\begin{bmatrix} 0 \\ -2.4 \\ 2.4 \\ 0 \end{bmatrix} = \frac{2}{9EI}[24.48]$$

−1.2 −6

2.4 + 3

$$[G_{22}] = [B_2]^*[F][B_2] = [0\ -0.4\ \ 0.4\ -1.0]\ \frac{2}{9EI}\begin{bmatrix} 6 & -3 & 0 & 0 \\ -3 & 6 & 0 & 0 \\ 0 & 0 & 6 & -3 \\ 0 & 0 & -3 & 6 \end{bmatrix}\begin{bmatrix} 0 \\ -0.4 \\ 0.4 \\ -1.0 \end{bmatrix} = \frac{2}{9EI}[1.2\ -2.4\ \ 5.4\ -7.2]\begin{bmatrix} 0 \\ -0.4 \\ 0.4 \\ -1.0 \end{bmatrix}$$

~~8~~ 5

$$= \frac{2}{9EI}[6.32]$$

~~8~~ 5

$$[G_{20}] = \frac{2}{9EI}[1.2\ -2.4\ \ 5.4\ -7.2]\begin{bmatrix} 0 \\ -2.4 \\ 2.4 \\ 0 \end{bmatrix} = \frac{2}{9EI}[18.72]\ ✓$$

$$[G_{12}] = [G_{21}] = [B_1]^*[F][B_2] = \frac{2}{9EI}[7.8\ -6.6\ \ 3.6\ -1.8]\begin{bmatrix} 0 \\ -0.4 \\ 0.4 \\ -1.0 \end{bmatrix} = \frac{2}{9EI}[5.88]\ ✓$$

NO2へ

414

(昭和　年　月　日　時限)

答　案　用　紙

No2

教室番号	着席番号	試験科目	担当教員	学科	学年	組	学籍番号	氏名	採点
			西脇	土	3	2	155089	皆川 勝	

$$\begin{cases} [G_{11}]X_1 + [G_{12}]X_2 = [G_{10}]P & \text{— ①} \\ [G_{21}]X_1 + [G_{22}]X_2 = [G_{20}]P & \text{— ②} \end{cases}$$

$$\begin{bmatrix} [G_{11}] & [G_{12}] \\ [G_{21}] & [G_{22}] \end{bmatrix} X = \begin{bmatrix} [G_{10}] \\ [G_{20}] \end{bmatrix} P$$

$$\frac{2}{9EI}\begin{bmatrix} 13.92 & 5.88 \\ 5.88 & 10.32 \end{bmatrix} X = \begin{bmatrix} 24.48 \\ 18.72 \end{bmatrix} [20] \frac{2}{9EI}$$

2

$$\left[\begin{array}{cc|cc} 13.92 & 5.88 & 1 & 0 \\ 5.88 & 10.32 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0.42 & 0.07 & 0 \\ 0 & 7.85 & -0.41 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0.42 & 0.07 & 0 \\ 0 & 1 & -0.05 & 0.13 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 0.09 & -0.05 \\ 0 & 1 & -0.05 & 0.13 \end{array}\right] \quad \therefore \begin{bmatrix} 13.92 & 5.88 \\ 5.88 & 10.32 \end{bmatrix}^{-1} = \begin{bmatrix} 0.09 & -0.05 \\ -0.05 & 0.13 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 0.09 & -0.05 \\ -0.05 & 0.13 \end{bmatrix} \begin{bmatrix} 24.48 \\ 18.72 \end{bmatrix} [20] = \begin{bmatrix} 1.2672 \\ -9.\cancel{8064} \end{bmatrix} [20]$$

2

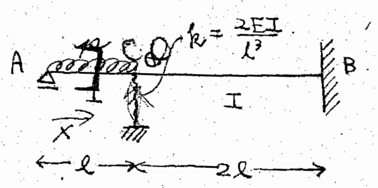
数値の精度に注意すること，数値計算で重要なことは小数値の位置もさることながら、精度は有効数値のけた数による。上の逆マトリックスの数値計算は3〜4けたのマトリックスから、1けたの精度せに落している。従って、そこから先きの計算は高々1けたしか保証されない。すなわち $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ になってしまう。これを逆マトリックスからさき、3けたを確保して計算すると

$$\begin{bmatrix} 13.92 & 5.88 \\ 5.88 & 10.32 \end{bmatrix}^{-1} = \begin{bmatrix} 0.0946 & -0.0539 \\ -0.0539 & 0.128 \end{bmatrix}$$

$$X = -\begin{bmatrix} 0.0946 & -0.0539 \\ -0.0539 & 0.128 \end{bmatrix} \begin{bmatrix} 24.48 \\ 18.72 \end{bmatrix} [20] = -\begin{bmatrix} 1.31 \\ 1.08 \end{bmatrix} [20] = -\begin{bmatrix} -26.2 \\ 21.6 \end{bmatrix}$$

4けたを確保すれば　$X = -\begin{bmatrix} -26.14 \\ 21.38 \end{bmatrix}$

となり、3けために誤差が入っていることになるが $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ では、求まっているもすべに誤差が入っているので、結局公解になっている保証は全くなくなってしまう。技術者にとって数値は重要である。



① 各区間の常数 α, 3
② 〃 の格間 matrix
③ バネ支点の節点 matrix
④ A点の Z_A
⑤ B点の Z_B
⑥ マトリックスの積の計算
⑦ 境界条件 M_B Q_B
⑧ φ_A Q_A
⑨ 任意点に対する力学量

$$[Z]=\begin{bmatrix} y \\ \varphi \\ M \\ Q \\ 1 \end{bmatrix}$$

---

① $l_0 = l$ とする。

AC
$\begin{cases} \beta_1 = 1 \\ \alpha_1 = \dfrac{EI}{100P_0 l^2} = 1 \end{cases}$ と P_0 を定める。

$\left(P_0 = \dfrac{EI}{100 l^2} \right)$

CB
$\begin{cases} \beta_2 = 2 \\ \alpha_2 = 1 \end{cases}$

②
$$[F]_{AC} = \begin{bmatrix} 1 & 1 & 1/2 & 1/6 & -\frac{\bar{P}}{24} \\ 0 & 1 & 1 & 1/2 & -\frac{\bar{P}}{6} \\ 0 & 0 & 1 & 1 & -\frac{\bar{P}}{2} \\ 0 & 0 & 0 & 1 & -\bar{P} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[F]_{CB} = \begin{bmatrix} 1 & 2 & 2 & 4/3 & 0 \\ 0 & 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

③
$$[P]_C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\left(k = \dfrac{k_{y1} l_0}{100 P_0} = \dfrac{\frac{2EI}{l^3} \cdot l}{100 \cdot \frac{EI}{100 l^2}} = 2 \right)$

④
$$Z_A = \begin{bmatrix} 0 \\ \varphi_A \\ 0 \\ Q_A \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix}$$

⑤
$$Z_B = \begin{bmatrix} 0 \\ 0 \\ M_B \\ Q_B \\ 1 \end{bmatrix}$$

⑥
$Z_{C\,left} = [F_{AC}]\, Z_A$

$$= \begin{bmatrix} 1 & 1 & 1/2 & 1/6 & -\bar{P}/24 \\ 0 & 1 & 1 & 1/2 & -\bar{P}/6 \\ 0 & 0 & 1 & 1 & -\bar{P}/2 \\ 0 & 0 & 0 & 1 & -\bar{P} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/6 & -\bar{P}/24 \\ 1 & 1/2 & -\bar{P}/6 \\ 0 & 1 & -\bar{P}/2 \\ 0 & 1 & -\bar{P} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix}$$

$Z_{C\,right} = [P]_C\, Z_{C\,left}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/6 & -\bar{P}/24 \\ 1 & 1/2 & -\bar{P}/6 \\ 0 & 1 & -\bar{P}/2 \\ 0 & 1 & -\bar{P} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/6 & -\bar{P}/24 \\ 1 & 1/2 & -\bar{P}/6 \\ 0 & 1 & -\bar{P}/2 \\ -2 & 2/3 & -\frac{11\bar{P}}{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix}$$

$Z_B = [F]_{CB}\, Z_{C\,right}$

$$= \begin{bmatrix} 1 & 2 & 2 & 4/3 & 0 \\ 0 & 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/6 & -\bar{P}/24 \\ 1 & 1/2 & -\bar{P}/6 \\ 0 & 1 & -\bar{P}/2 \\ -2 & 2/3 & -11\bar{P}/12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.333 & 4.056 & -2.597\bar{P} \\ -3 & 3.833 & -3\bar{P} \\ -4 & 2.333 & -2.333\bar{P} \\ -2 & 0.667 & -0.917\bar{P} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ M_B \\ Q_B \\ 1 \end{bmatrix}$$

$-2 - \frac{22}{12}$

$\frac{6+22}{12}$

$-\frac{1}{6} - 1 - \frac{22}{12} = \frac{2+12+22}{12} = \frac{36}{12}$

$$\begin{cases} 0.333\,\varphi_A + 4.056\,Q_A - 2.597\bar{k} = 0 & \text{—①} \\ -3\,\varphi_A + 3.833\,Q_A - 3\bar{k} = 0 & \text{—②} \end{cases}$$

①より $\varphi_A = 7.799\bar{k} - 12.180\,Q_A$

②に代入

$$-3(7.799\bar{k} - 12.180\,Q_A) + 3.833\,Q_A - 3\bar{k} = 0$$

$$\therefore Q_A = \frac{3\bar{k} + 3\times 7.799\bar{k}}{3\times 12.180 + 3.833} = \frac{26.397}{40.373}\bar{k} = 0.654\bar{k}$$

$$\varphi_A = 7.799\bar{k} - 12.180\times 0.654\bar{k} = -0.167\bar{k}$$

⑤

$$\begin{cases} M_B = -4\times(-0.167\bar{k}) + 2.333\times 0.654\bar{k} - 2.333\bar{k} = -0.139\bar{k} \\ Q_B = -2\times(-0.167\bar{k}) + 0.667\times(0.654\bar{k}) - 0.917\bar{k} = -0.147\bar{k} \end{cases}$$

$$\boxed{\xi = x/l}$$

~~Z c right~~ Z_C = (left)

$$\begin{bmatrix} 1 & 1/6 & -\bar{k}/24 \\ 1 & 1/2 & -\bar{k}/6 \\ 0 & 1 & -\bar{k}/2 \\ 0 & 1 & -\bar{k} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.167\bar{k} \\ 0.654\bar{k} \\ 1 \end{bmatrix} = \begin{bmatrix} y_C \\ \varphi_C \\ M_C \\ Q_C \\ 1 \end{bmatrix}$$

$$y_C = \left(-0.167 + \frac{0.654}{6} - \frac{1}{24}\right)\bar{k} = -0.100\bar{k}$$

$$\varphi_C = \left(-0.167 + 0.327 - \frac{1}{6}\right)\bar{k} = -0.007\bar{k}$$

$$M_C = \left(0.654 - \frac{1}{2}\right)\bar{k} = 0.154\bar{k}$$

$$Q_C = (0.654 - 1)\bar{k} = -0.346\bar{k}$$

$$Z_D = \begin{bmatrix} 1 & 1/6 & -\bar{k}/24 \\ 1 & 1/2 & -\bar{k}/6 \\ 0 & 1 & -\bar{k}/2 \\ -2 & 2/3 & -11\bar{k}/12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.167\bar{k} \\ 0.654\bar{k} \\ 1 \end{bmatrix} = \begin{bmatrix} y_D \\ \varphi_D \\ M_D \\ Q_D \\ 1 \end{bmatrix}$$

$$y_D = \left(-0.167 + \frac{0.654}{6} - \frac{1}{24}\right)\bar{k} = -0.100\bar{k}$$

$$\varphi_D = \left(-0.167 + \frac{0.654}{2} - \frac{1}{6}\right)\bar{k} = -0.007\bar{k}$$

$$M_D = \left(0.654 - \frac{1}{2}\right)\bar{k} = 0.154\bar{k}$$

$$Q_D = \left\{-2\times(-0.167) + \frac{2}{3}\times 0.654 - \frac{11}{12}\right\}\bar{k} = -0.147\bar{k}$$

$$[F_{Ax}] = \begin{bmatrix} 1 & \xi & \xi^2/2 & \xi^3/6 & -\bar{k}/24\,\xi^4 \\ 0 & 1 & \xi & \xi^2/2 & -\bar{k}/6\,\xi^3 \\ 0 & 0 & 1 & \xi & -\bar{k}/2\,\xi^2 \\ 0 & 0 & 0 & 1 & -\bar{k}\,\xi \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Z_x = [F_{Ax}] \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_A \\ Q_A \\ 1 \end{bmatrix} = \begin{bmatrix} y_x \\ \varphi_x \\ M_x \\ Q_x \\ 1 \end{bmatrix}$$

$$y_x = \xi\varphi_A + \frac{\xi^3}{6}Q_A - \frac{\bar{k}}{24}\xi^4$$

$$\varphi_x = \varphi_A + \frac{\xi^2}{2}Q_A - \frac{\bar{k}}{6}\xi^3$$

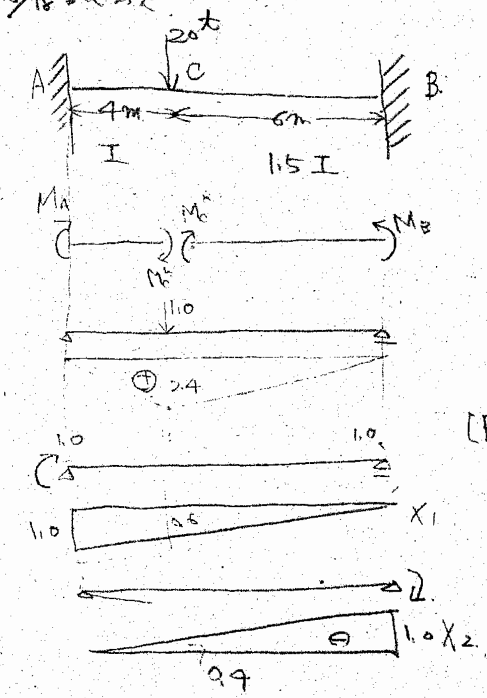
$$M_x = \xi Q_A - \frac{\bar{k}}{2}\xi^2$$

$$Q_x = Q_A - \bar{k}\xi$$

$\xi \to 0.1, 0.2, 0.3, \cdots 1$ とやればすべての ξ の Z が求まる。

9/18 試験

問



M図を求めよ。

$$M = \begin{bmatrix} M_A \\ M_C^L \\ M_C^R \\ M_B \end{bmatrix}$$

$$[F] = \frac{2}{3EI}\begin{bmatrix} 2 & -1 & & \\ -1 & 2 & & \\ & & 2 & -1 \\ & & -1 & 2 \end{bmatrix}$$

$$[B_0] = \begin{bmatrix} 0 \\ 2.4 \\ 2.4 \\ 0 \end{bmatrix} \quad [B_1] = \begin{bmatrix} 1.0 & 0 \\ -0.6 & 0.4 \\ 0.6 & -0.4 \\ 0 & 1.0 \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$[G_{11}] = \frac{2}{3EI}\begin{bmatrix} 4.64 & -1.96 \\ -1.96 & 3.44 \end{bmatrix} \qquad [G_{11}]^{-1} = \frac{3EI}{2 \times 12.12}\begin{bmatrix} 3.44 & 1.96 \\ 1.96 & 4.64 \end{bmatrix}$$

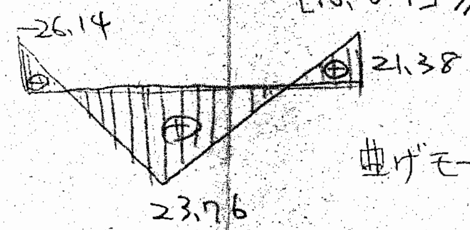
$12.12 \leftarrow 3.44 \times 4.64 - (1.96)^2$

$$[G_{10}] = \frac{2}{3EI}\begin{bmatrix} 8.16 \\ -6.24 \end{bmatrix}$$

$$[G_{11}]^{-1}[G_{10}] = \begin{bmatrix} 1.307 \\ -1.069 \end{bmatrix} \qquad X = -\begin{bmatrix} 1.307 \\ -1.069 \end{bmatrix}[20] = -\begin{bmatrix} 26.07 \\ -10.69 \end{bmatrix}$$

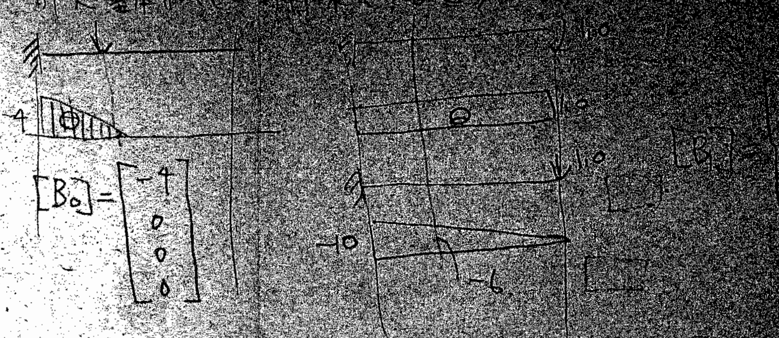
$$= \begin{bmatrix} -26.07 \\ 10.69 \end{bmatrix} \quad //$$

$$M = \begin{bmatrix} -26.14 \\ -23.76 \\ 23.76 \\ 21.38 \end{bmatrix}$$



曲げモーメント図

静定基本形として[illegible]



$$[B_0] = \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

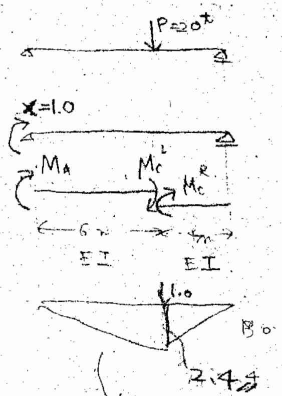
$$[G_{11}] = \frac{2}{3EI}\begin{bmatrix} 12 & 6.6 \\ 6.6 & 4.64 \end{bmatrix} \qquad [G_{11}]^{-1} = \frac{2}{3EI \cdot 12.12}\begin{bmatrix} 4.64 & [illegible] \\ -6.6 & [illegible] \end{bmatrix}$$

$$[G_{10}] = \frac{2}{3EI}\begin{bmatrix} 12 \\ 10.4 \end{bmatrix}$$

$$X = \begin{bmatrix} 1.069 \\ -0.3762 \end{bmatrix}[20] = \begin{bmatrix} 20.69 \\ -7.524 \end{bmatrix}$$

[illegible]

あとは同じ手順。



$[f_{AC}] = \begin{bmatrix} \frac{2}{EI} & -\frac{1}{EI} \\ -\frac{1}{EI} & \frac{2}{EI} \end{bmatrix}$ ⟶ $\begin{bmatrix} \frac{\ell}{3EI} & -\frac{\ell}{6EI} \\ -\frac{\ell}{6EI} & \frac{\ell}{3EI} \end{bmatrix}$

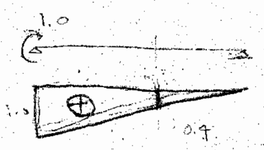
$[f_{CB}] = \left[\frac{4}{3EI} \right]$ ⟶ $\left[\frac{\ell}{3EI} \right]$

$\therefore [F] = \begin{bmatrix} \frac{2}{EI} & -\frac{1}{EI} & 0 \\ -\frac{1}{EI} & \frac{2}{EI} & 0 \\ 0 & 0 & \frac{4}{3EI} \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & \frac{4}{3} \end{bmatrix} = \frac{1}{3EI}\begin{bmatrix} 6 & -3 & 0 \\ -3 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$M_I = \begin{bmatrix} M_A \\ M_c^L \\ M_c^R \end{bmatrix}$

$[B_0] = \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \end{bmatrix}$ 荷重の数

列の数は不静定力の数



$[B_1] = \begin{bmatrix} 1.0 \\ -0.4 \\ 0.4 \end{bmatrix}$ 不静定力の数

$[G_{11}] = [B_1]^* [F] [B_1]$

$= [1.0 \quad -0.4 \quad 0.4] \frac{1}{EI} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 4/3 \end{bmatrix} \begin{bmatrix} 1.0 \\ -0.4 \\ 0.4 \end{bmatrix}$

$= \frac{1}{EI} [2.4 \quad -1.8 \quad 0.53] \begin{bmatrix} 1.0 \\ -0.4 \\ 0.4 \end{bmatrix} = \frac{1}{EI} \left[\frac{10}{3} \right]$ ⟶ $\frac{1}{3EI}[10]$

$[G_{10}] = [B_1]^* [F] [B_0]$

$= \frac{1}{EI} [2.4 \quad -1.8 \quad 0.53] \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \end{bmatrix} = \frac{1}{EI}[5.592]$ ⟶ $\frac{1}{3EI}[16.8]$

$X = -[G_{11}]^{-1} [G_{10}] P$

$= -EI \cdot \left[\frac{3}{10} \right] \frac{1}{EI} [5.592] [20] = -33.6^{tm}$

$M_I = [B_0] P + [B_1] X$

$= \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \end{bmatrix} [20] + \begin{bmatrix} 1.0 \\ -0.4 \\ 0.4 \end{bmatrix} 33.6 = \begin{bmatrix} -33.6 \\ -34.56 \\ 34.56 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \end{bmatrix} - \begin{bmatrix} 1.0 \\ -0.4 \\ 0.4 \end{bmatrix} 3EI \left[\frac{1}{10} \right] \cdot \frac{1}{3EI} [16.8]$

$= \begin{bmatrix} 0 \\ -2.4 \\ 2.4 \end{bmatrix} - \begin{bmatrix} 1.0 \\ -0.4 \\ 0.4 \end{bmatrix} [1.68]$

$= \begin{bmatrix} -1.68 \\ -1.728 \\ 1.728 \end{bmatrix}$

$\Delta = [B]^* [F] [B] P$

$= [-1.68 \quad -1.728 \quad 1.728] \frac{1}{3EI} \begin{bmatrix} 6 & -3 & 0 \\ -3 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} -1.68 \\ -1.728 \\ 1.728 \end{bmatrix} [20]$

$= \frac{1}{3EI} [-4.896 \quad -5.328 \quad 6.912] \begin{bmatrix} -1.68 \\ -1.728 \\ 1.728 \end{bmatrix} [20]$

$= \frac{20}{3EI} \cdot [29.376] = \frac{195.84}{EI}$

$X = -$ [illegible]

$\frac{9EI}{2} \quad \frac{2}{9EI} \quad \frac{2}{9EI} [13.92, 10.32]$ 2.48 [illegible]