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**ON PROPERTIES OF STEEL MEMBERS SUBJECTED TO  
REPETITIVE LOADING HISTORIES**

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**SUMMARY**

In this paper we propose an stress-strain model with high accuracy and clearness to estimate the elasto-plastic hysteretic behaviors of structural steels subjected to complicated repetitive loads. Material's characteristic functions introduced in this model are fundamental surface size functions in the stress space and weighting functions by which the sizes of multi-surfaces corresponding to arbitrary stress-strain conditions are estimated. These functions can be easily and unambiguously estimated by a combination of a tension test and several tension-compression tests each including only one unloading.

**INTRODUCTION**

When structures are subjected to complicated fluctuating loads due to earthquakes, wind storms, waves and so on, stresses beyond the elastic-limits of steels may reliably break out in members or parts of them repetitively. In this case, even if the structures do not collapse, it may be expected that the hysteretic effect reduces the load capacity or deformability of such a structure from the level predicted in design. Because such problems are very important and fundamental for structural design, the authors have investigated the effect of loading histories on mechanical properties of steels, which are the most important materials in civil engineering, and have accumulated experimental data [1].

Although some studies on hysteretic stress-strain relations simulation of steel members have been performed up to this time (for examples [2-8]), most of them may not satisfy the following conditions;

- (a) applicable to arbitrary loading paths and structural steels,
- (b) clear and simple for estimating material's properties.

Petersson & Popov Model (P.P. Model) has the grounding in the multi-surface plasticity theory [7,8] and has an advantage that only a few tests are required to make up fundamental functions representing material properties.

The final purpose of this investigation is to complete an accurate hysteretic model to predict elasto-plastic behaviors of steel structures or members subjected to external forces in excess of those amount. In this paper, a stress-strain model based on P.P. Model is studied with the emphasis on the evaluation method of hysteretic effects. Accumulative equivalent plastic strain and fundamental surface size functions have important roles in the proposed model. To inquire the effect of the difference of the estimating methods of these quantities, elasto-plastic FEM analyses are performed using several methods. And comparisons are made on stress-strain relations of structural steels between actual test results and the calculated ones.

This paper shows that the hysteretic model, by which quasi-static fluctuating stress-strain relations under nearly uniaxial stress fields can be predicted, is sufficiently accurate from the engineering point of view.

#### PROPOSED STRESS-STRAIN MODEL --N.M.M. MODEL--

N.M.M. Model proposed here is based on the multi-surface plasticity theory, adopted in P.P. Model, in which the surface sizes are affected by hysteretic loading. Therefore we refer briefly to P.P. Model and then mention some important features of N.M.M. Model.

In P.P. Model hysteretic stress-strain behaviors are represented by the concept of expansion, reduction and movement of state surfaces in the stress space. Each surface is defined by a surface size function  $K$ , by which the size of each surfaces is expressed, and a vector  $(\alpha)$  indicating its central coordinates. In order to introduce the hysteretic effects to stress-strain relations,  $K$  and  $(\alpha)$  are changed with the progress of loading histories. The degree of hysteretic effect is described by the following state variables;

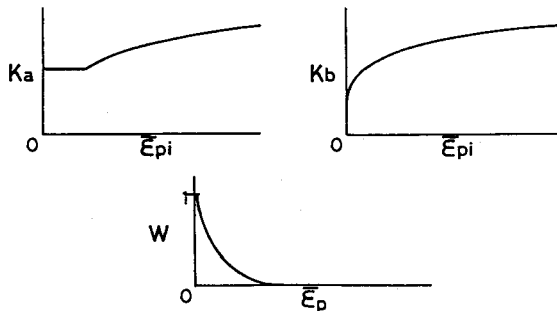


Fig.1. Petersson-Popov Model [7].

$$\bar{\epsilon}_p = \int_{t_0}^{t_c} d\epsilon_p, \quad \bar{\epsilon}_{pi} = \int_{t_c}^{t_i} d\epsilon_p \quad (1)$$

where  $\bar{\epsilon}_p$  is accumulative equivalent plastic strain from the start time ( $t_0$ ) of loading to the time ( $t_c$ ) of the last reversal on the stress-strain paths, and  $\bar{\epsilon}_{pi}$  is equivalent plastic strain increment from the time ( $t_c$ ) to the time ( $t_i$ ) when stress-strain relation is to be predicted. In P.P. Model the surface size function  $K$  is defined according to the following equation, introducing  $K_a$ ,  $K_b$  and  $W$  shown in Fig.1 which are the functions of  $\bar{\epsilon}_p$  and  $\bar{\epsilon}_{pi}$ .

$$K = W K_a + (1 - W) K_b \quad (2)$$

Where  $K_a$  is the surface size in the case where no hysteretic effect is and  $K_b$  is that in the case where the hysteretic effect is stationary. The weighting function  $W$  represents the change in the surface size function from  $K_a$  to  $K_b$  due to loading histories and is evaluated by means of numerical calculation. In this paper,  $K_a$  and  $K_b$  is defined as Fundamental Surface Size Functions [F.S.S. Functions] and these are key functions for evaluating the surface size function in any phase of loading.

A vector  $\{\alpha\}$  indicating the central coordinate of each surface is updated in turn according to the progress of loading by the following equation

$$\{\alpha_i\} = \{\alpha_j\} + \frac{(\dot{\sigma}) - \{\alpha_o\}}{\dot{\sigma}_o} (\dot{K}_j - K_i) \quad (3)$$

where  $\{\sigma\}$  is a stress vector, subscript  $o$  means the initial condition and dot represents the state of before the updating. The size difference of two surfaces ( $i$ -th and  $j$ -th surface) before and after the set of differential increment of equivalent plastic strain and the current loading direction determine the vector  $\{\alpha\}$ .

#### Evaluation of Accumulative Equivalent Plastic Strain

Accumulation of equivalent plastic strain in the process of repetitive loading is accomplished in the following way consistent with the results of measurements. In Fig.2 a solid line shows an example of experimental results

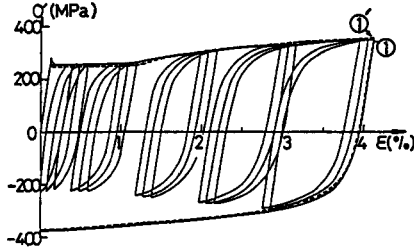


Fig.2. Stress-strain curves in repetitive loading tests.

of repetitive loading tests and a broken line shows one of monotonous loading tests, and in each test unloading is applied at the point ① and the point ①' respectively. If plastic strain is accumulated over all paths, accumulative plastic strain at the point ① are fairly greater than that at the point ①'. In spite of this inference, the stress-strain curve on the path after the point ① is much the same as that on the path after the point ①'. This experimental result has to be understood that the plastic strain produced in repetitive loading processes has to be separated in two components: one has an effect on following stress-strain relations and the other does not.

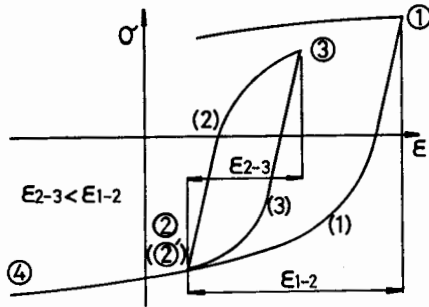


Fig.3. Return phenomenon [2].

Yokoo and Nakamura and et al. [2] confirmed the following phenomenon shown in Fig.3 by experiments and referred to this as "Return Phenomenon";

If a strain amplitude  $E_{2-3}$  corresponding to a path (2) from the point ② to the point ③ is fairly small compared with its preceding strain amplitude  $E_{1-2}$ , the stress-strain curve from the point ③ to the point ④, after passing through the point ②' near the point ②, traces on such path that is the stress-strain curve in the case where a load reversal does not occur at the point ②.

According to such a phenomenon, the stress-strain paths (2) and (3) do not affect the stress-strain curve following to these paths. Basing on the above mentioned experimental data, the accumulative equivalent plastic strain is

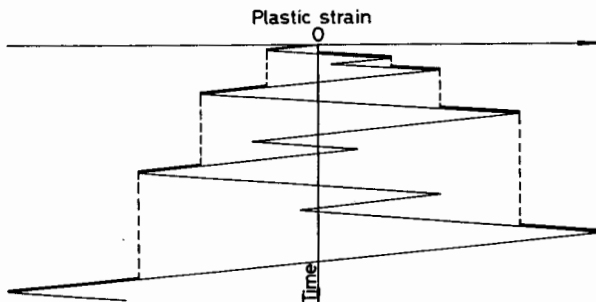


Fig.4. Paths accumulating equivalent plastic strain.

evaluated under the assumption that the plastic strain beyond the preceding plastic strain amplitude is effective. As an example, thick lines in Fig.4 show the paths on which the equivalent plastic strains are accumulated.

Choice of F.S.S. Functions

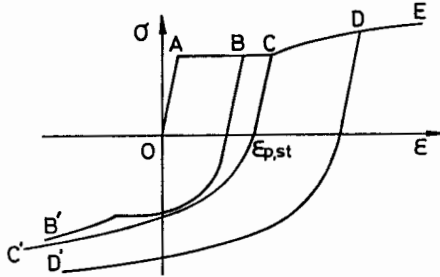


Fig.5. Various types of unloadings.

Fig.5 shows some instances of stress-strain curves for tension-compression tests. In general, when a steel specimen is stretched, the relation between stress and strain is composed of three paths; elastic part, yield plateau and strain hardening (O-A, A-C, C-E). When unloading occurs on the yield plateau, Bauschinger effect takes place which is followed by an yield plateau in compression region for the second time (O-A-B-B'). When it occurs at the start point of strain hardening in tension region, yield plateau in compression region does not appear any longer (O-A-C-C'). Although the curve is similar when unloading occurs in strain hardening region (O-A-D-D'), there appear the different degree of Bauschinger effect. Accordingly, the characteristic features of stress-strain relations for steels under repetitive loading conditions are as follows;

- (a) the disappearance of yield plateau in the successive hysteretic loading processes when unloading is applied on yield plateau,
- (b) the change in degree of Bauschinger effect when unloading is taken place in strain hardening region.

It seems to be one of the necessary conditions for the stress-strain model compatible with experimental results that the model may represent these features.

In N.M.M. Model, to get better compatibility to experimental results, the surface size function  $K_{ab}$  has been introduced as one of F.S.S. Functions in addition to  $K_a$  and  $K_b$ . Following the introduction of  $K_{ab}$ ,  $W_1$  and  $W_2$  is defined as weighting functions so that a surface size function at a certain hysteretic phase is defined as follows by means of these weighting functions;

$$K = W_1 K_j + (1 - W_1) K_k \quad (4)$$

$$\begin{matrix} 0 \leq \epsilon_p < \epsilon_{p,st} & ; i=1, j=a, k=ab \\ \epsilon_{p,st} \leq \epsilon_p & ; i=2, j=ab, k=b \end{matrix}$$

where  $\epsilon_{p,st}$  is  $\bar{\epsilon}_p$  at the start point of strain hardening on virgin stress-strain curve. And  $\epsilon_{p,b}$  is  $\bar{\epsilon}_p$  at the point in which the hysteretic effect is stationary. From the definitions of these functions,  $W_1=1$  at  $\bar{\epsilon}_p=0$ ,  $W_1=0$  and  $W_2=1$  at  $\bar{\epsilon}_p= \epsilon_{p,st}$  and  $W_2=0$  at  $\bar{\epsilon}_p= \epsilon_{p,b}$ . The function  $W_1$  expresses the phenomenon that stress-strain curve changes continuously from the virgin stress-strain curve, which is characterized by yield plateau and strain hardening, to the smooth curve, on which Bauschinger effect is characteristic, and  $W_2$  stands for the cyclic softening or hardening following the progress of loading histories. And these two functions are peculiar to each materials.

ESTIMATION OF MATERIAL PROPERTIES

In this chapter, presented is the method of experiments and its interpretation to evaluate material properties  $K_a$ ,  $K_{ab}$ ,  $K_b$ ,  $W_1$  and  $W_2$ , which are necessary to predict hysteretic behaviors of steel by N.M.M Model. Since the model is based on the assumption that the stress-strain relations on the certain loading path or unloading path is determined by means of accumulative equivalent plastic strain  $\bar{\epsilon}_p$  at the start point of the loading or unloading, material properties can be estimated by a combination of a monotonous tension test and several tension-compression tests each including only one reversed point.

A procedure evaluating material properties is as follows.

Determination of  $K_a$

Virgin stress-plastic strain curve represents  $K_a$ .

Determination of  $K_{ab}$  and  $K_b$

Since  $K_{ab}$  corresponds to such  $K_i$  that its parameter  $\bar{\epsilon}_p$  equals to  $\epsilon_{p,st}$ , it is the combination of two curves. One of them is the virgin stress-plastic strain curve and the other is the stress-plastic strain curve, measured in a test including unloading at the start point of strain hardening.  $K_i$  corresponding to the state in which the hysteretic effect becomes stationary is  $K_b$ . Accordingly, in order to determine  $K_b$ , surface sizes  $K_r$ 's corresponding to the accumulative plastic strains  $\epsilon_{p,r}$ 's have to be determined. Using the stress-plastic strain curve obtained from unloading following the loading till the accumulative plastic strain reaches  $\epsilon_{p,r}$  and the virgin stress-plastic strain curve,  $K_i$  is evaluated as the function of  $\bar{\epsilon}_{pi}$ . And the stationarity in hysteretic effect means that no difference is found among each  $K_r$ 's. In the case where  $K_i$ 's do not converge within the experiments,  $K_i$  for the measured maximum  $\bar{\epsilon}_p$  is adopted as  $K_b$ . In this case it will be possible to correctly predict hysteretic behaviors within the limits where the accumulative equivalent plastic strain does not exceed that maximum  $\bar{\epsilon}_p$ .

Determination of  $W_1$  and  $W_2$

By means of  $K_a$ ,  $K_{ab}$  and  $K_b$ , weighting values in order to evaluate  $K_i$  corresponding to the values of  $\epsilon_{p,r}$ 's is determined by the next equation.

$$W = ( K_i - K_k ) / ( K_j - K_k )$$

$$\begin{matrix} 0 \leq \bar{\epsilon}_p < \epsilon_{p,st} & : & j=a, k=ab \\ \epsilon_{p,st} \leq \bar{\epsilon}_p & : & j=ab, k=b \end{matrix} \quad (5)$$



Weighting function is determined by the formula which shows the relation of the weighting values and the corresponding  $\bar{\epsilon}_p$ .

#### PREDICTION OF HYSTERETIC STRESS-STRAIN RELATIONS

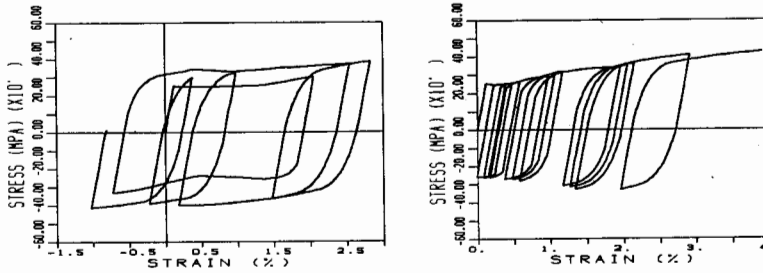
When hysteretic stress-strain relations of steel members are predicted by means of the model, such as N.M.M. Model or P.P. Model, on the ground of multi-surface plasticity theory, the accuracy of prediction seems to be affected by the following two characteristics of these models;

- a) how to evaluate accumulative equivalent plastic strain,
- b) how to choose Fundamental Surface Size Functions.

Noticing above two factors, four stress-strain models are defined as shown in Table 1. And elasto-plastic finite element analyses are carried out for steel members subjected to tension-compression loads repetitively[9-10]. The results are compared with those of corresponding experiments, and investigations are done on the effect of the factors on the accuracy of prediction of stress-strain relations.

Table 1. Definitions of each models.

Models name	accumulation of $\bar{\epsilon}_p$	F.S.S. Functions
Model-1	over all paths	Ka and Kb
Model-2	by the method shown in Fig.4	Ka and Kb
Model-3	over all paths	Ka, Kab and Kb
Model-4 (proposed)	by the method shown in Fig.4	Ka, Kab and Kb

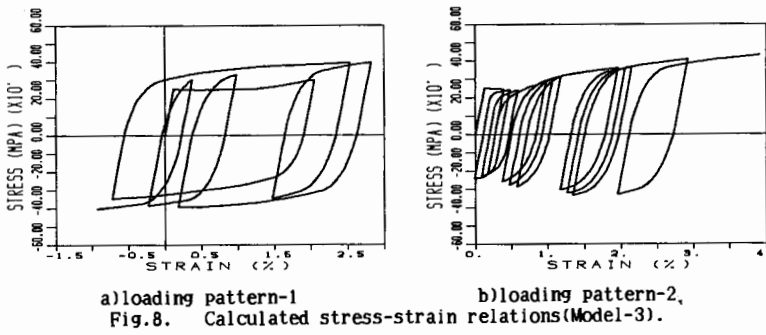
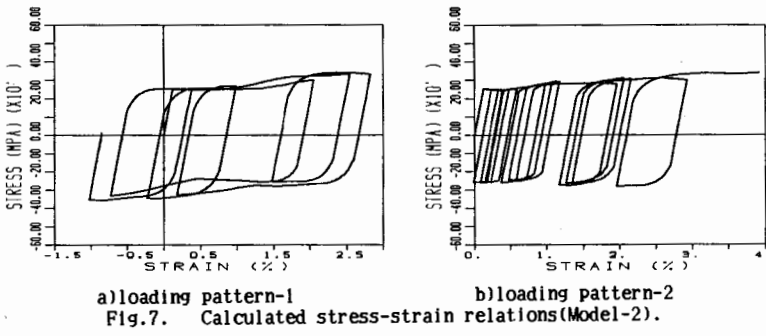


a)loading pattern-1  
b)loading pattern-2  
Fig.6. Calculated stress-strain relations (Model-1).

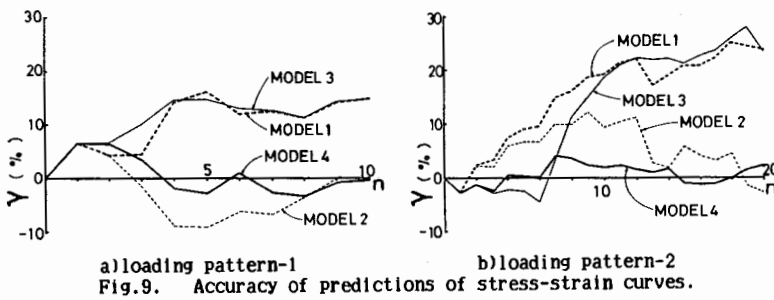
Two examples of stress-strain curves calculated for each models are shown in Fig.6~Fig.8, respectively. Material considered is mild steel SM41A. In this paper, the accuracy of the prediction is defined by the next quantity;

$$Y = [ \sigma_n |_{\text{model}} / \sigma_n |_{\text{experiment}} - 1 ] \times 100 (\%) \quad (6)$$

where  $\sigma_n$  indicates stress range on n-th stress-strain path.



The accuracies of each model is shown in Fig.9- a) and b) with that of proposed model by the relations between the values of  $\gamma$  defined by Eq. 6 and numbers of half cycles  $n$ . These relations correspond to the stress-strain relations shown in Fig.6 ~ Fig.8.



Comparing Fig.6 with Fig.7, prediction using Model-1 and Model-2 is unreasonable, because the effect of yield plateau in virgin stress-strain curve appears in the following processes of load repetitions, and this tendency is remarkable in Model-2. This means that the rate of accumulating equivalent plastic strain for Model-2 is lower than that for Model-1. On the other hand, Fig.9 shows that the value  $\gamma$  for Model-1 is fairly greater than the value for Model-2, and stress ranges in any paths are greater than that of experimental results. These results implies the fact that the effect of  $K_b$  is overestimated on stress-strain relations calculated by Model-1 and the effect of  $K_a$  makes worse the accuracy of prediction by means of Model-2. This results may be brought by the procedure, which makes no distinctions between the phenomenon showing yield plateau and the phenomenon that the strain changes monotonously with stress change.

Model-3 is constructed from Model-1 by introducing  $K_{ab}$  as one of the F.S.S. Function. In Model-1 and Model-3, the accumulation of equivalent plastic strain are carried out over all strain paths. Due to the introduction of the function  $K_{ab}$  in Model-3, the disorder of the hysteretic stress-strain curves caused by the overestimate of  $K_a$  is dissolved in Model-3. The value  $\gamma$  in this case, however, is greater than 10% and notable difference is not found between Model-1 and Model-3 from this point of view. The results obtained by Model-1 being compared with those by Model-2, the above results can be considered to be due to the application of unsuitable method to estimate accumulative equivalent plastic strain.

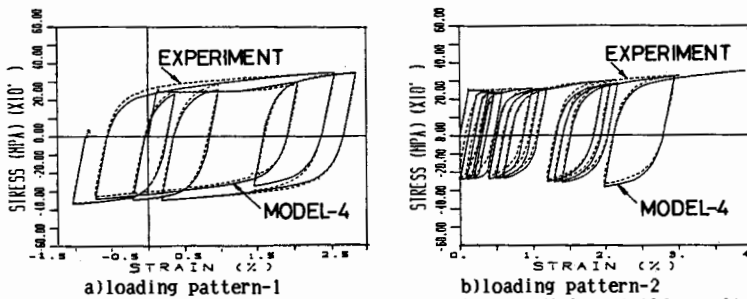
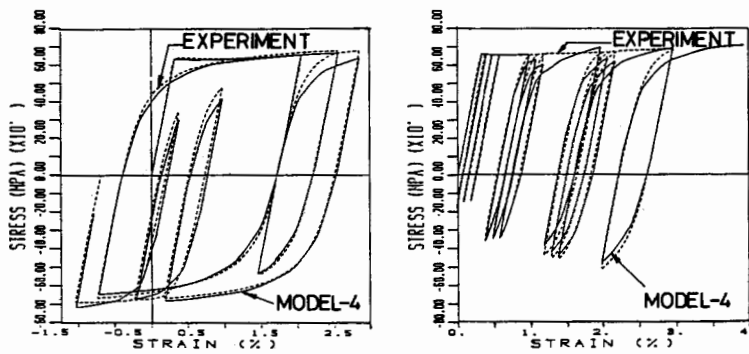


Fig.10. Comparison; experiments and calculations by Model-4(mild steel).

Fig.10- a) and b) shows stress-strain relations predicted by Model-4 (N.M.M. Model) and those gained by the experiments. From these figures, the close agreement between calculated and experimental stress-strain relations over all strain paths is obtained. In Fig.9, the value  $\gamma$  for Model-4 are plotted together with those for the other models. The value  $\gamma$  is 5% at the most. And the accuracy of the prediction by Model-4 is notably improved.

Fig.11- a) and b) shows stress-strain relations of high strength steel HT70 predicted by the application of Model-4 and those measured in the experiments. From these figures and Fig.10, it is observed stress-strain relations predicted by F.E.M. analyses coincide notably well with those obtained by the experiments, in spite of the use of material properties evaluated from experimental results by a few specimens. Moreover, these examples show that the proposed model is adoptable for both mild steels and high strength steels.



a) loading pattern-1                      b) loading pattern-2  
 Fig.11. Comparison; experiments and calculations by Model-4  
 (high strength steel).

#### CONCLUSION

N.M.M. stress-strain model proposed by the authors is capable of predicting the actual hysteretic behaviors of steels with high accuracy so that the error of the predicted stress-strain curves is 5% at the most. If equivalent plastic strain accumulated over all strain paths is taken as the state variable, the expansion of the surface sizes is overestimated and the error is 10% ~ 30% in the examples calculated by the authors. Moreover, introducing the surface size function  $K_{ab}$  at the start point of strain hardening on virgin stress-strain curve as one of the Fundamental Surface Size Functions, the accuracy of the prediction is improved remarkably, in the case of steel with distinct yield plateau.

This paper shows that the proposed model accurately predicts hysteretic behaviors of uniaxial members subjected to tension-compression loads. The method proposed here may be able to be extended to models presenting hysteretic behaviors of structural members such as beams or beam-columns and seems to be of wide applications.

Computations were carried out on HITAC M200H/M280H in the Tokyo University Computer Center.

#### REFERENCES

1. Minagawa, M., Nishiwaki, T. and Masuda, N. : Monotonous stress-strain properties of structural steel subjected to nonstationary repetitive pre-loading, Proc. of the 27th Japan Congress on Materials Research, pp.25-31, 1984.
2. Yokoo, Y., Nakamura, T. and Kawada, Y. : Non-stationary hysteretic uniaxial stress-strain relation of a wide-flange steel (Part I), Proc. of A.I.J., No.259, pp.56-66, 1977. & Yokoo, Y. and Nakamura, T. : Nonstationary

hysteretic uniaxial stress-strain relation of a wide-flange steel (Part II), Proc. of A.I.J., No.260, pp.71-81, 1977.

3. Kato,B., Akiyama,H. and Yamanouchi,Y. : Predictable properties of material under incremental cyclic loading, IABSE Symposium, Theme III, Vol.13, pp.119-124, 1973.
4. Mroz,M. : An attempt to describe the behaviors of metals under cyclic loading using a more general workhardening model, Acta Mechanica, Vol.7, pp.199-212, 1969.
5. Suhara,J., Miyatake,M., Harii,K. and Mitita,R. : Elasto-plastic analysis of steel structures, subject to arbitrarily varying cyclic loads, Proc. of JSNA, Vol.150, pp.398-412, 1981(in Japanese).
6. Fujimoto,M., Nakagome,T. and Yamada,T. : A study based on nonlinear fracture mechanics on fracture of connections in steel structures on cyclic loads, Proc. of the 15th JSSC symposium on matrix methods in engineering, pp.161-166, 1981 (in Japanese).
7. Petersson,H. and Popov,E.P. : Constitutive relation for generalized loadings, Proc. of ASCE, Vol.103, No.EM4, pp.611-627, 1977.
8. Popov,E.P. and Petersson,H. : Cyclic metal plasticity ; experiments and theory, Proc. of ASCE, Vol.104, No.EM6, pp.1371-1388, 1978.
9. Yamada,Y. : Plasticity and Visco-Elasticity, Baihukan, 1970 (in Japanese).
10. Yamada,Y. and Yokouchi,Y. : Programming of Elasto-Plastic Analysis by Finite Element Method - EPIC IV - , Baihukan, 1981 (in Japanese).