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A New Formulation of Finite Element Equations of Motion for Geometrically Nonlinear Dynamics Analysis

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SUMMARY

A geometrically nonlinear dynamic analysis formulation is presented for general space structures which may be subjected to finite rotations in 3-dimensional space. The proposed method has its base on the computer-oriented method reported by one of the authors [1], in which equilibrium equations are represented directly by nodal coordinates instead of conventional nodal displacements. The governning dynamic equations for each member are derived from the static equations by adding the inertia term [2]. To obtain numerical solution, the two-step approximation and iterative correction solution procedure developed for static analysis is adopted and combined with Newmark's β time integration scheme. The proposed formulation utilizes so called floating frame method, in contrast with Simo [3] where fixed frame expression has been preferred.

Equations of Motion

Equations of motion of an element in a global coordinate system at time t can be expressed as the following:

$$d\{[M(t)]\{\dot{u}(t)\}\}/dt + \{R(t)\} = \{F(t)\} \qquad (1)$$

where $[M(t)]$, $\{R(t)\}$, $\{F(t)\}$ and $\{u(t)\}$ are a mass matrix, a restoring force vector, an applied load vector and a displacement vector, respectively. A mark $\cdot$ represents differentiation with respect to time. The first term in Eq.(1) can be rewritten as

$$\begin{aligned} d\{[M(t)]\{\dot{u}(t)\}\}/dt &= [M(t)]\{\ddot{u}(t)\} + [\dot{M}(t)]\{\dot{u}(t)\} \\ &= \{[T(t)]^T[M^*(t)][T(t)]\}\{\ddot{u}(t)\} \\ &= \{d\{[T(t)]^T[M^*(t)][T(t)]\}/dt\}\{\dot{u}(t)\} \end{aligned} \qquad (2)$$

where $[M^*(t)]$, $[T(t)]$ are a mass matrix of the element expressed in the element coordinate system, and a coordinate transformation matrix from the global coordinate system to the element coordinate system, respectively. A superscript T represents transposition of a matrix.The second term of Eq.(1) corresponds to the internal force term of the static equilibrium equations presented in [1], if a dumping term is neglected for simplicity, and can be shown as

$$\{R(t)\} = [T(t)]^T[K^*(t)]\{[T(t)][G](\{u(t)\}+\{Z(t)\}) \\ -[T(0)][G]\{Z(0)\}\} \qquad (3)$$

where, $[K^*(t)]$, $[G]$, are a secant stiffness matrix expressed in the element coordinate system, a constant matrix which shifts the origin of the element coordinate system to the element's first node, respectively. And $\{Z(t)\}^T = \langle\{x(0)\}^T \{-r(t)\}^T\rangle$, where $x(0)$ and $r(t)$ are an initial coordinate vector, and a rigid body rotation vector at time t, respectively, of the element. Presenting equations of motion can be obtained by substitution of Eqs.(2) and (3) into Eq.(1)

Incremental Equations of Motion

Subtracting the equations of motion at time t from those at time $t+\Delta t$ and making some calculations, we obtain next equations

$$\begin{aligned}
&[T(t+\Delta t)]^T[M^*(t+\Delta t)][T(t+\Delta t)]\{\Delta\ddot{u}\} \\
&+ \{[T(t)]^T[M^*(t)][\Delta T] + [\Delta T]^T[M^*(t)][T(t)] \\
&+ [\Delta T]^T[M^*(t)][\Delta T]\}\{\ddot{u}(t)\} \\
&+ [T(t+\Delta t)]^T[\Delta M^*][T(t+\Delta t)]\{\ddot{u}(t)\} \\
&+ \{[\dot{T}(t+\Delta t)]^T[M^*(t+\Delta t)][T(t+\Delta t)] \\
&+ [T(t+\Delta t)]^T[\dot{M}^*(t+\Delta t)][T(t+\Delta t)] \\
&+ [T(t+\Delta t)]^T[M^*(t+\Delta t)][\dot{T}(t+\Delta t)]\}\{\Delta\dot{u}\} \\
&+ \{[\dot{T}(t+\Delta t)]^T[M^*(t+\Delta t)][\Delta T] + [\dot{T}(t+\Delta t)]^T[\Delta M^*][T(t)] \\
&+ [\Delta\dot{T}]^T[M^*(t+\Delta t)][T(t)] + [T(t+\Delta t)]^T[\dot{M}^*(t+\Delta t)][\Delta T] \\
&+ [T(t+\Delta t)]^T[\Delta\dot{M}^*][T(t)] + [\Delta T]^T[\dot{M}^*(t)][T(t)] \\
&+ [T(t+\Delta t)]^T[M^*(t+\Delta t)][\Delta\dot{T}] + [T(t+\Delta t)]^T[\Delta M^*][\dot{T}(t)] \\
&+ [\Delta T]^T[M^*(t)][\dot{T}(t)] \}\{\dot{u}(t)\} \\
&+ [T(t+\Delta t)]^T[K^*(t+\Delta t)][T(t+\Delta t)]\{\Delta u\} \\
&+ [T(t+\Delta t)]^T[K^*(t+\Delta t)]\{[T(t+\Delta t)]\{\Delta Z\} \\
&+ [\Delta T](\{u(t)\}+\{Z(t)\})\} \\
&+ [T(t+\Delta t)]^T[\Delta K^*]\{u^*(t)\} + [\Delta T]^T\{F^*(t)\} \\
&= \{\Delta F\}
\end{aligned} \qquad (4)$$

Eq.(4) is the incremental equations of motion of an element from time t to time $t+\Delta t$. The first four lines represent ordinary inertia terms, next eight lines are generated from the rigid body rotation of the element. The remaining lines correspond to the restoring force.

If strains are small, the stiffness matrix and mass matrix in each element coordinates system can be assumed constant, when sufficiently fine mesh is adopted. Moreover, if time mesh is also as small as the second term in Eq.(2) can be neglected with respect to the first term, Eq.(4) may finally be written as

$$\begin{aligned}
&[T(t+\Delta t)]^T[M^*][T(t+\Delta t)]\{\Delta\ddot{u}\} \\
&+ \{[T(t)]^T[M^*][\Delta T] + [\Delta T]^T[M^*][T(t)] \\
&+ [\Delta T]^T[M^*][\Delta T]\}\{\ddot{u}(t)\} \\
&+ [T(t+\Delta t)]^T[K^*][T(t+\Delta t)]\{\Delta u\} \\
&+ [T(t+\Delta t)]^T[K^*]\{[T(t+\Delta t)]\{\Delta Z\} + [\Delta T](\{u(t)\}+\{Z(t)\})\} \\
&+ [\Delta T]^T\{F^*(t)\} \\
&= [M(t+\Delta t)]\{\Delta\ddot{u}\} + \{g(t+\Delta t)\} \\
&+ [K(t+\Delta t)]\{\Delta u\} + \{h(t+\Delta t)\} \\
&= \{\Delta F\}
\end{aligned}$$

where
$$\begin{aligned}
&[M(t+\Delta t)]=[T(t+\Delta t)]^T[M^*][T(t+\Delta t)] \\
&[K(t+\Delta t)]=[T(t+\Delta t)]^T[K^*][T(t+\Delta t)] \\
&\{g(t+\Delta t)\}=\{[T(t)]^T[M^*][\Delta T]+[\Delta T]^T[M^*][T(t)] \\
&\qquad + [\Delta T]^T[M^*][\Delta T]\}\{\ddot{u}(t)\}
\end{aligned}$$

and
$$\begin{aligned}
\{h(t+\Delta t)\}=&[T(t+\Delta t)]^T[K^*]\{[T(t+\Delta t)]\{\Delta Z\} \\
&+ [\Delta T](\{u(t)\}+\{Z(t)\})\}+[\Delta T]^T\{F^*(t)\}
\end{aligned} \qquad (5)$$

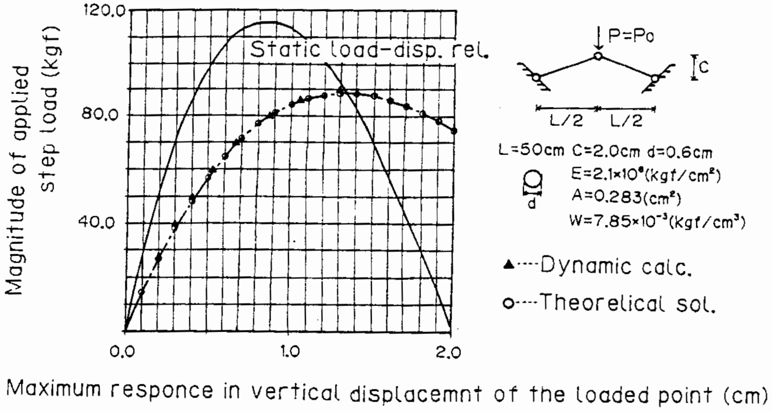
Solution Procedure for the Incremental Equations of Motion

The time-integration scheme to be used here is one of the simplest one, namely, Newmark's β-method with β=1/4. Thus, the velocity and displacement within a time step are assumed to vary according to the next relations.

$$\{\Delta\dot{u}\}=\{\dot{u}(t+\Delta t)\}-\{\dot{u}(t)\}=\{\ddot{u}(t)\}\Delta t+\{\Delta\ddot{u}\}\Delta t/2$$

$$\{\Delta u\}=\{u(t+\Delta t)\}-\{u(t)\}=\{\dot{u}(t)\}\Delta t+\{\ddot{u}\}\Delta t^2/2+\{\Delta\ddot{u}\}\beta\Delta t^2 \qquad (6)$$

Firstly, Eq.(5) is linearized with respect to displacement and acceleration increments $\{\Delta u\}$ and $\{\Delta\ddot{u}\}$, then Eq.(6) is introduced to eliminate the displacement increment to give an equation including only the acceleration increment as unknowns. After superposition of the obtained element equations on whole the structure, the first approximation of the acceleration increment, and then, of the displacement increment can be calculated. Similarly, processes combined with the method presented in [1] gives the second approximating solution and the following corrected solutions, until pre-set convergence criteria are satisfied.



Dynamic responce of a two-bar truss applied a step-load at its top

Numerical Example

Dynamic response analyses are conducted on shallow two-bar-truss loaded with a vertical step load at its top. The result is shown in the figure above. For this simple problem, theoretical solution can be easily obtained, if the system stops entirely at its maximum response and the lost potential of external force is assumed to be converted into strain energy without any loss. Present results and the above theoretical results show good accordance. The solid line in the figure indicates the static load-displacement relation.

Conclusions

The geometrically nonlinear finite element static analysis method developed by the authors, based on the coordinate representation, has been expanded to dynamic analyses. Though the numerical example treated here is very simple one, the method presented here is also valid for plate and shells as well as for frames.

References

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Geometrically Nonlinear Vibration of Cable Structures Considering Stress-Unilateral Behaviours

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Summary

The geometrically nonlinear dynamic behaviour of cable structures is discussed from the view point of stress-unilateral discrete system. In the first part of the paper, the geometrically nonlinear equations of motion considering the slackness of cables are derived by the variational inequality formulation, and in the latter part, the free vibrations and the forced vibrations to the harmonic applied force are numerically analyzed for a cable structure of hyperbolic paraboloidal shape.

Geometrically Nonlinear Equations of Motion

Cable structures as well as membrane structures reveal the structural characteristics such that materials used for these structures cannot transmit the compression stress, and then these structures belong to the stress-unilateral structure system. Also, cable structures are naturally unstable structures so that the initial prestressing is usually introduced to add the initial stiffness, and in order to estimate the amount of the initial prestressing to be introduced, the geometrically nonlinear analyses are required considering the slackness of cables.

Let us consider a cable member 'a' whose nodal points are i and j in a Cartesian coordinate system(x,y,z). Let ${}^t\mathbf{x}_i=(x_i,y_i,z_i)$ and ${}^t\mathbf{x}_j=(x_j,y_j,z_j)$ be the coordinates of i and j (Fig.1). Then, direction cosines are obtained in the vector form as $\boldsymbol{\lambda}_a=(\mathbf{x}_j-\mathbf{x}_i)/|\mathbf{x}_j-\mathbf{x}_i|$. Let $\mathbf{u}_i,\mathbf{u}_j$ and $\mathbf{f}_i,\mathbf{f}_j$ be the displacement vectors and the force vectors for i and j nodal points, respectively. The elongation of a-member is derived considering the first and the second terms of displacement as

$$(u_n)_a = {}^t\lambda_a\,(u_j-u_i) + \frac{1}{2}\,{}^t(u_j-u_i)\,N\,(u_j-u_i) \qquad (1)$$

where $\mathbf{N}=[\mathbf{I}-{}^t\lambda_a\lambda_a]/L_o$ in which L_o denotes the initial member length.

As shown in Fig.2, the axial force-elongation law for a member has the following relation: