

# COMPUTER SIMULATION OF CYCLIC PLASTICITY BEHAVIOURS OF STRUCTURAL MEMBERS

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## 1. OBJECTIVE

A CYCLIC PLASTICITY MODEL IS PROPOSED AND ELASTO-PLASTIC HYSTERETIC BEHAVIOURS OF STEEL AND STEEL BEAMS ARE SIMULATED BY THE PROPOSED MODEL.

## 2. CYCLIC PLASTICITY MODEL

THE PROPOSED MODEL IS DERIVED BY THE REFINEMENTS OF A MULTI SURFACE PLASTICITY MODEL INTRODUCED BY POPOV AND PETERSSON SHOWN IN FIG. 1. WE MODIFIED THIS MODEL TO EXPRESS ACTUAL BEHAVIOURS OF STRUCTURAL STEEL.

## 3. STRESS-STRAIN RELATIONS OF STRUCTURAL STEEL

### [APPLICATION NO.1]

THIS MODEL IS APPLIED TO SIMULATION OF TENSION-COMPRESSION STRESS-STRAIN RELATIONS OF MILD STEEL AND HIGH STRENGTH STEEL.

FIG. 2 SHOWS STRESS-STRAIN RELATIONS PREDICTED BY THE PROPOSED MODEL AND THOSE GAINED BY THE CORRESPONDING EXPERIMENTS.

$\sigma$  : STRESS  
 $\sigma_1$  : PRINCIPAL STRESS  
 $\sigma_2$  : PRINCIPAL STRESS  
 $\epsilon_p$  : PLASTIC STRAIN  
 $\epsilon_e$  : EQUIVALENT PLASTIC STRAIN

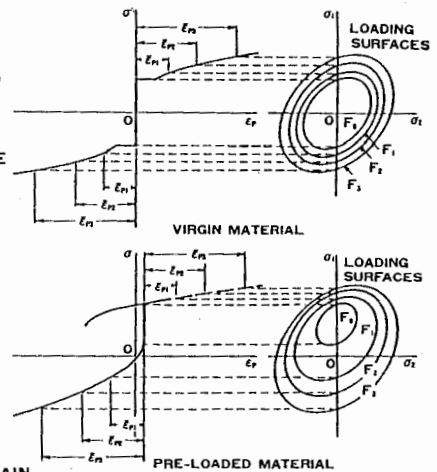


FIG. 1 CYCLIC PLASTICITY MODEL

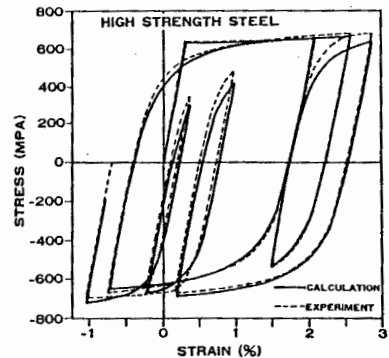
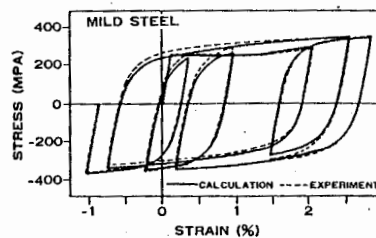
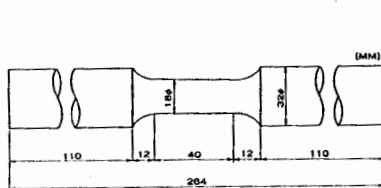


FIG. 2 STRESS-STRAIN RELATIONS OF ROUND BAR SPECIMENS

## 4. MOMENT-CURVATURE RELATIONS OF STEEL BEAMS

### [APPLICATION NO.2]

THE NEXT APPLICATION OF THE CYCLIC PLASTICITY MODEL IS THE EVALUATION OF BENDING MOMENT-CURVATURE RELATIONS (M- $\phi$  RELATIONS) OF STEEL BEAMS. THE TANGENT STIFFNESS METHOD WAS USED WITH SOME MODIFICATIONS TO CALCULATE M- $\phi$  RELATIONS BY THE PROPOSED MODEL. MATERIAL PROPERTY FUNCTIONS WERE DETERMINED BY MATERIAL TESTS AND RESIDUAL STRESS WAS ANALYZED BY HOLE DRILLING METHOD. THE CALCULATED M- $\phi$  RELATIONS AND THOSE OBTAINED BY THE CORRESPONDING EXPERIMENTS ARE SHOWN IN FIG. 3.

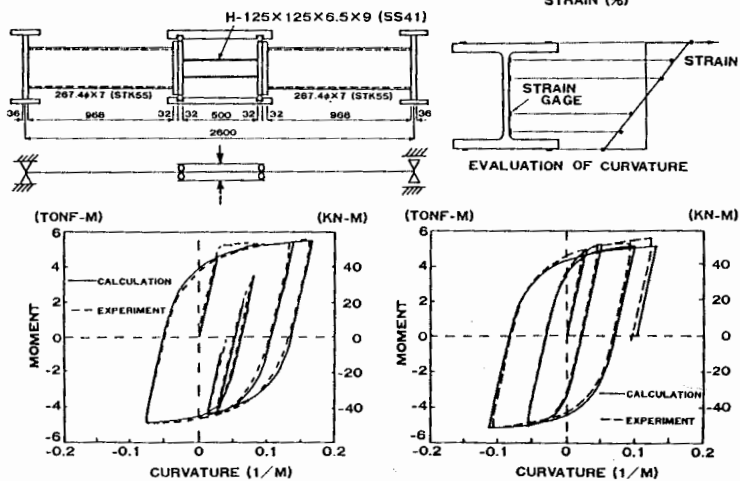


FIG. 3 MOMENT-CURVATURE RELATIONS OF SIMPLE BEAM SPECIMENS.



### Simulation of Cyclic Plasticity Behaviour of Structural Members

Simulation du comportement de plasticité cyclique d'éléments de structures

Rechnerische Simulation des zyklisch-plastischen Verhaltens von Tragelementen

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#### 1. INTRODUCTION

Computer simulation of the cyclic plasticity behaviours of structures or structural members is very important to evaluate structural safety and make them safe against severe cyclic loading due to earthquakes, wind storms and so on. Here a cyclic plasticity theory for elasto-plastic hysteretic behaviours of structural steel is proposed and it is applied to the analyses of structural members subjected to cyclic loads.

#### 2. CYCLIC PLASTICITY MODEL

The proposed model is derived by the refinements of a multi surface plasticity theory introduced by Petersson and Popov[1]. Petersson-Popov model used cumulative equivalent plastic strain as a state variable, two fundamental surface size functions and a weighting function as material property functions.

There are three important differences between Petersson-Popov model and the proposed model. Firstly, effective value of cumulative equivalent plastic strain is defined as a state variable to represent "Return Phenomena". Secondly, additional material property functions are introduced. These functions express strain hardening characteristics of materials after loading histories corresponding to certain values of cumulative equivalent plastic strain. Owing to this modification, the theory can express strain hardening characteristics of materials with both notable strain hardening and non-hardening strain region. Thirdly, all of the material property functions can easily and unambiguously be obtained by a combination of a simple tension test and several simple tension-compression tests.

#### 3. APPLICATION OF THE MODEL

This method is applied to simulation of tension-compression stress-strain relationships of mild steel and high strength steel. Using material property functions measured, the authors carried out elasto-plastic finite element analyses for round-bar specimens subjected to repetitive tension-compression loading under controlled strain. Figure 1 shows stress-strain relationships predicted by the proposed model and those gained by the corresponding experiments. By comparing these results, it was confirmed that the stress-strain relationships calculated by the proposed theory was accurate.

A next application of the cyclic plasticity model is the evaluation of bending moment-curvature relationships ( $M-\phi$  relationships) of steel beams. In the calculation of the  $M-\phi$  relationships of beams and beam-columns by the proposed

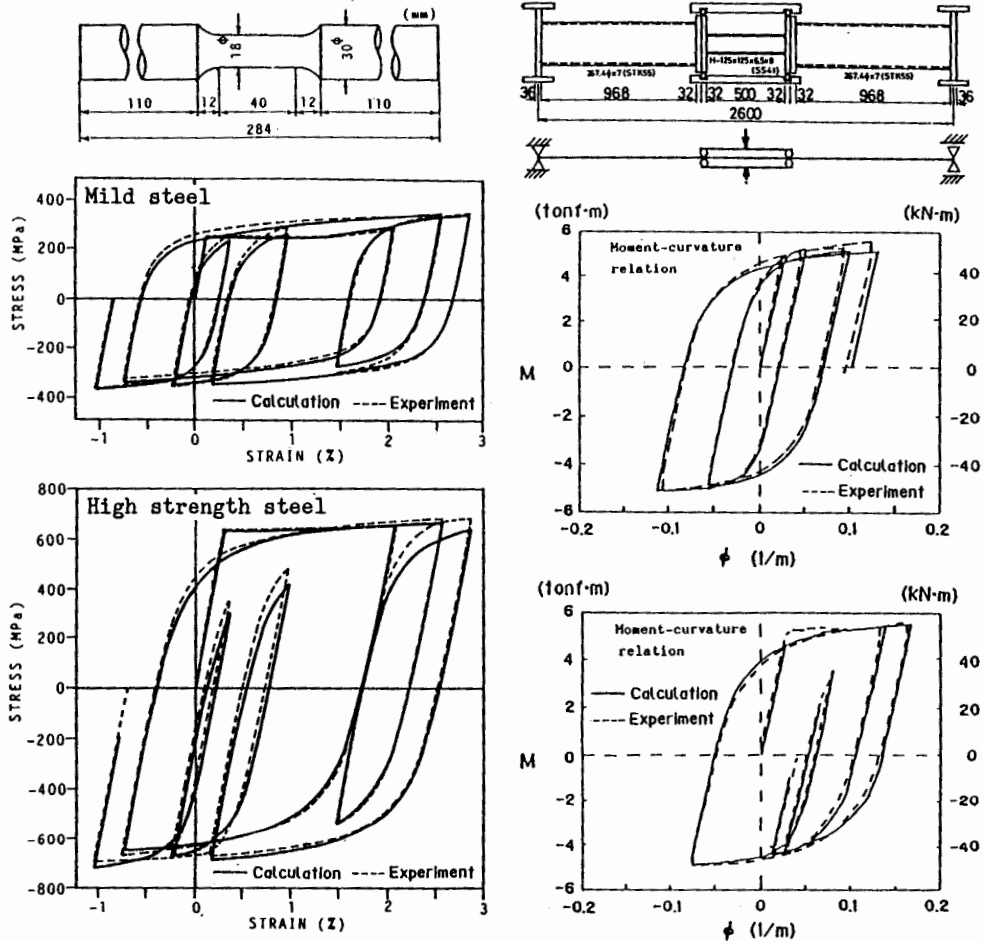


Figure 1 Stress-strain relationships of round bar specimens. Figure 2 Moment-curvature relationships of simple beam specimens.

stress-strain model, the tangent stiffness method introduced by Chen and Atsuta[2] was used with some modifications.

JIS(Japanese Industrial Standards)-5 type specimens were shaped from a H-shaped beam specimen to evaluate material property functions. Material property functions were determined by a combination of tension tests and tension-compression tests. Residual stress characteristics were analyzed by a drilling method and simple type distribution was assumed according to these results.

The calculated  $M-\phi$  relationships are compared with those obtained by experiments which the authors had carried out. Figure 2 shows hysteresis  $M-\phi$  relationships predicted and those gained by the corresponding experiments. The accurate  $M-\phi$  relationships can be evaluated by the authors' theory.

#### REFERENCES

1. PETERSSON H. and POPOV E.P., Constitutive relation for generalized loadings, Proc. of ASCE, Vol.103, No.EM4, pp.611-627, 1977.
2. CHEN W.F. and ATSUTA T., Theory of Beam-Column, McGraw-Hill Inc., 1977.

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ABSTRACTS

Volume I

FIRST

WORLD CONGRESS ON COMPUTATIONAL MECHANICS  
INTERNATIONAL ASSOCIATION FOR  
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The University of Texas at Austin

and

George Washington University

through

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## MODELING OF CYCLIC PLASTICITY OF STRUCTURAL STEELS

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When structures are subjected to complicated fluctuating loads due to earthquakes, wind storms, waves and so on, stresses beyond the elastic-limits of steels break out in members or parts of them repetitively. In this case, even if the structures do not collapse, it is expected that the hysteretic effect reduces the load capacity or deformability of such a structure from the level predicted in design. Because such problems are very important and fundamental for structural design, the authors have investigated the effect of loading histories on mechanical properties of steels which are important structural materials, and have accumulated experimental data.

Petersson & Popov Model (P.P. Model)[1] has the grounding in the multi-surface plasticity theory and has an advantage that only a few tests are required to make up fundamental functions representing material properties.

The final purpose of this investigation is to complete an accurate hysteretic model to predict elasto-plastic behaviors of steel structures or members subjected to external forces in excess of those amount. In this paper, a stress-strain model based on P.P. Model is studied with the emphasis on the evaluation method of hysteretic effects.

In P.P. Model, hysteretic stress-strain behaviors are represented by the concept of expansion, reduction and movement of state surfaces in the stress space. Each surface is defined by a surface size function  $K$ , by which the size of each surfaces is expressed, and a vector  $\{\alpha\}$  indicating its central coordinates. In order to introduce the hysteretic effects to stress-strain relations,  $K$  and  $\{\alpha\}$  are changed with the progress of loading histories. The degree of hysteretic effect is described by accumulative equivalent plastic strain  $\bar{\epsilon}_p$  and increment of equivalent plastic strain  $\bar{\epsilon}_{p_i}$ . The surface size function is defined according to the following equation, introducing  $K_a$ ,  $K_b$  and  $W$  which are the functions of  $\bar{\epsilon}_p$  and  $\bar{\epsilon}_{p_i}$ .

$$K = W \cdot K_a + (1 - W) K_b \quad (1)$$

where  $K_a$  is the surface size in the case where no hysteretic effect is and  $K_b$  is that in the case where the hysteretic effect is stationary. The weighting function  $W$  represents the change in the surface size

function from  $K_a$  to  $K_b$  due to loading histories and is evaluated by means of numerical calculation.  $K_a$  and  $K_b$  which are referred to as Fundamental Surface Size Functions (F.S.S. Functions) in this paper, are key functions for evaluating the surface size function in any phase of loading.

In this paper, basing on experimental results, accumulation of equivalent plastic strain in the process of repetitive loading is accomplished in the following way consistent with results of measurements ; the accumulative equivalent plastic strain is evaluated under the assumption that the plastic strain beyond the preceding plastic strain amplitude is effective.

Characteristic features of stress-strain relations for steels under repetitive loading conditions are as follows ;

- (a) the disappearance of yield plateau in the successive hysteric loading processes when unloading is applied on yield plateau,
- (b) the change in degree of Bauschinger effect when unloading is taken place in strain hardening region.

It seems to be a necessary condition for a stress-strain model compatible with experimental results that the model may represent these features. In N.M.M. Model, to get better compatibility with experimental results, the surface size function  $K_{ab}$ , which is a surface size function at the start point of strain hardening for virgin material, has been introduced as a F.S.S. Function in addition to  $K_a$  and  $K_b$ . Following the introduction of  $K_{ab}$ ,  $W_1$  and  $W_2$  is defined as weighting functions. The function  $W_1$  expresses the phenomenon that stress-strain curve changes continuously from the virgin stress-strain curve, which is characterized by yield plateau and strain hardening, to the smooth curve, on which Bauschinger effect is characteristic, and  $W_2$  stands for the cyclic softening or hardening following the progress of loading histories. These two functions are peculiar to each materials.

Since the model is baed on the assumption that the stress-strain relations on the certain loading path or unloading path is determined by means of accumulative equivalent plastic strain  $\bar{\epsilon}_p$  at the start point of the loading or unloading, material properties can be estimated by a combination of a monotonous tension test and several tension-compression tests each including only one reversed point.

Elasto-plastic FEM analyses are performed. Fig.1 shows an example of stress-strain curves predicted by N.M.M. Model and that gained by an experiment. Close agreement between calculated and experimental stress-strain relation over all strain paths is obtained. Thus N.M.M. stress-strain model proposed by the authors is capable of predicting the actual hysteretic behaviors of steel with high accuracy.

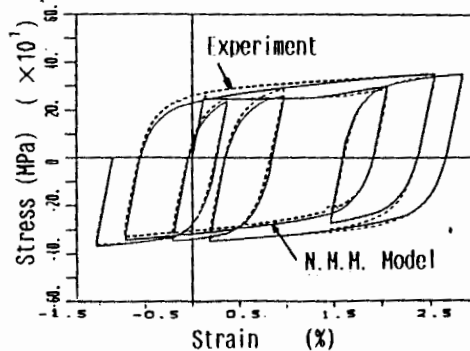


Fig. 1 Comparison; Experiment and calculation.

(Reference) 1. Petersson H. and Popov E.P.: Constitutive Relation for Generalized Loadings, Proc. of ASCE, Vol.103, No.EM4, pp.611-627, 1977.

## NSM.1 - NONLINEAR SOLID MECHANICS I

### **Bass Lecture Hall**

"Solution Algorithms for Fractional Derivatives," Joe Padovan; University of Akron, U.S.A.

"Finite Element Analysis of Shear Localization in Rate and Temperature Dependent Solids," Jeffrey LeMonds; Brown University; and A. Needleman; SRI International, U.S.A.

"Modeling of Cyclic Plasticity of Structural Steels," Masaru Minagawa, Takeo Nishiwaki and Nobutoshi Masuda; Musashi Institute of Technology, Japan

"Crash Simulation of a Convolute Box Beam with a Nonlinear Finite Element Code," Hai Wu, Han Wang and Kin Yeung; Ford Motor Company, U.S.A.

"Nonlinear Analysis of Trusses by Energy Minimization," Sadaji Ohkubo and Yasuo Watada; Ehime University, Japan

"An Algorithm of Contact Problems for Forming Process of Thin Plates - A Numerical Analysis for Seaming Process of a Can," Masao Ishinabe; Toyo Seikan Group, Japan

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# NONLINEAR DYNAMIC ANALYSIS OF FRAME STRUCTURES

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A geometrically nonlinear dynamic analysis method is presented for frames which may be subjected to finite rotations in 3-dimensional space. The proposed method has its base on the method reported by Yoshida et al. on Computer Methods in Applied Mechanics and Engineering, vol.32(1982). This method is called here the YMM method named after the author's names, Yoshida, Masuda and Matsuda.

In the YMM method, governing incremental static equilibrium equation is represented by the coordinates itself rather than conventional displacements. With this equation and the corrective-iterative solution process also proposed, many kinds of geometrically nonlinear structural problems in the field of statics were analyzed satisfactorily.

Here, in this paper, the YMM method is enlarged to be able to deal with dynamic problems. The governing dynamic equilibrium equation for each member is obtained from the static equation given in the paper aforementioned by adding the inertia term, thus;

$$\begin{aligned} \Delta \mathbf{f} = & \mathbf{T}_{(t+\Delta t)}^T \mathbf{m}^* \mathbf{T}_{(t+\Delta t)} \Delta \ddot{\mathbf{u}} + [ \mathbf{T}_{(t+\Delta t)}^T \mathbf{m}^* \mathbf{T}_{(t+\Delta t)} - \mathbf{T}_{(t)}^T \mathbf{m}^* \mathbf{T}_{(t)} ] \ddot{\mathbf{u}}_{(t)} \\ & + d(\mathbf{T}_{(t+\Delta t)}^T \mathbf{m}^* \mathbf{T}_{(t+\Delta t)}) / dt \dot{\mathbf{u}}_{(t+\Delta t)} - d(\mathbf{T}_{(t)}^T \mathbf{m}^* \mathbf{T}_{(t)}) / dt \dot{\mathbf{u}}_{(t)} \\ & + \mathbf{T}_{(t+\Delta t)}^T \mathbf{k}^* \mathbf{T}_{(t+\Delta t)} \Delta \mathbf{u} + \mathbf{T}_{(t+\Delta t)}^T \mathbf{k}^* [ \mathbf{T}_{(t+\Delta t)} \Delta \mathbf{r} + \Delta \mathbf{T} \mathbf{G} (\mathbf{u}_{(t)} + \mathbf{r}_{(t)}) ] \\ & + \Delta \mathbf{T}^T \mathbf{f}_{(t)}^* \end{aligned}$$

where,  $\mathbf{T}$  is the coordinates transformation matrix from global to member coordinates system,  $\mathbf{f}^*$  is the member nodal forces vector,  $\mathbf{k}^*$  is the conventional linear stiffness matrix of a member,  $\mathbf{m}^*$  is the conventional mass matrix of a member in its member coordinates system,  $\mathbf{r}$  is the rigid body rotation vector of a member,  $(t)$  indicates time  $t$ , and  $\Delta$  is a prefix to represent increment. For the sake of simplicity, damping term is neglected.

The contributions from each members are summed up to make the total equilibrium. In practice, the two-step approximation & iterative correction solution procedure developed for static analysis is adopted and combined with a modified Steffensen's iterative process as well as the incremental arc length control method to give a more effective solution procedure.

The modified Steffensen's iterative process is schematically demonstrated as the following:

In solving a fixed point problem expressed as  $X=g(X)$  the following process is iterated

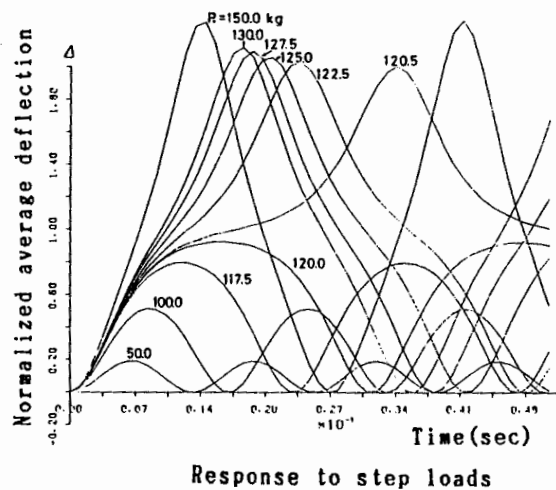
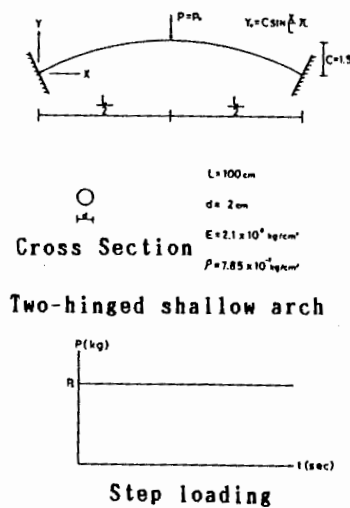
$$X_{i+1} = g(X_i) - \frac{[g(g(X_i)) - g(X_i)]^2}{[g(g(g(X_i))) - 2g(g(X_i)) + g(X_i)]}$$

In other words, starting from  $i$ -th approximation  $X_i$  usual iterations are performed three times giving  $g(X_i)$ ,  $g(g(X_i))$  and  $g(g(g(X_i)))$  respectively. Then the next approximation  $X_{i+1}$  is obtained from the above equation. It is proved that this process has second-order convergence property same as the original Steffensen's iteration process.

A numerical example of a curved cantilever beam under lateral loads illustrates the effectiveness of the proposed method in the field of finite rotations in 3-dimensional space. Among compared are Runge-Kutta type solution processes and a process using original Steffensen's iteration. The former gave divergent solution after certain amount of displacements are obtained. And the effectiveness of the latter was about half of the proposed method in the sense of computational time required to obtain convergent solution.

In order to examine the applicability of the proposed nonlinear dynamic analysis method, it is applied to shallow arches to investigate the dynamic stability. Forced vibration analyses of two-hinged shallow arches under central step loading are conducted with several loading amplitudes. Dynamic instability breaks out at the point where the maximum displacement response drastically increases with respect to the increase of the loading amplitude. The results are compared with those given by Gregory and Plaut in 1982, who used Galerkin method, and show good correspondence. The effect of the number of members on the accuracy is also examined.

Other problems including asymmetric and three-dimensional instability modes are also calculated. All the numerical calculations performed so far confirm the effectiveness of the proposed method.



## SD.2 - STRUCTURAL DYNAMICS II

### **LBJ Auditorium**

"Semi-Adaptive Implicit Method for Dynamics Analysis," Jerzi Kujawski, University of Technology, Poland and Richard H. Gallagher, Worcester Polytechnic Institute, U.S.A.

"Vibrations of a Structure Coupled with an Internal Fluid," R. Ohayon and H. Berger; Office National d'Etudes et de Recherches Aerospatiales, France

"Evaluation of Modal Methods for Dynamic Response," Charles Camarda; NASA Langley Research Center, U.S.A.

"Dynamic Response of Flexible Foundations Using the Boundary Element Method," Juan Jose Benito and Enrique Alarcon; Unversidad Nacional de Educacion a Distancia, Spain

"A Consistent Tension Stiffening Formulation," Johann Kollegger and Gerhard Mehlhorn; Gesamthochschule Kassel, F.R.G.

"Nonlinear Dynamic Analysis of Frame Structures," Nobutoshi Masuda, Takeo Nishiwaki and Masaru Minagawa; Musashi Institute of Technology, Japan

"Impact Analysis of Reinforced Concrete Column," Shigikatsu Ichihashi; Kozo Keikau Engineering, Inc. and Akira Wada; Tokyo Institute of Technology, Japan