

PREDICTION OF CYCLIC PLASTICITY BEHAVIOURS OF STEEL BEAMS

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SUMMARY

A cyclic plasticity model proposed by the authors was introduced to an analysis system for predicting cross sectional behaviours of beams and beam-columns. The tangent stiffness method was used with some modifications to calculate moment-curvature relations. The calculated moment-curvature relations were compared with those obtained from experiments and it was verified that hysteretic moment-curvature relations under consideration can be predicted with high accuracy by means of the proposed model.

INTRODUCTION

In order to perform response analyses of frame structures subjected to complex variable loads, deformation characteristics of the frame members and their joints must be known, which, in turn, lay their bases on the stress-strain relation of the material. The authors have proposed a cyclic plasticity model (hereafter called the Proposed Model) [1,2] based on the multi-surface plasticity model (hereafter called Petersson-Popov Model) proposed by Petersson and Popov [3,4]. The authors have also shown by comparisons of measured and calculated results that it is possible for hysteretic tension-compression stress-strain relations of steel to be accurately estimated if the Proposed Model is used [1,2,5,6].

In this study, with the objective to establish the application method of the Proposed Model to elasto-plastic response analyses of frame structures, the Proposed Model is introduced in an estimation method of the elasto-plastic hysteretic moment-curvature relations of H-shaped steel beams as the first step. And the effectiveness of the cyclic plasticity model proposed by the authors for estimating hysteretic behaviours of beam cross sections is shown through comparisons of the calculation results and the results of

measurements.

OUTLINE OF PROPOSED MODEL[1,2]

The hysteretic stress-strain behaviour is expressed by enlargement, reduction and movement of surfaces defined in the principal stress space as shown in Fig.1 [3,4]. Each surface is defined by a function κ expressing its size and vector $\{\alpha\}$ expressing the central coordinate. κ and $\{\alpha\}$ are functions of state variables $\bar{\epsilon}_p$ and $\bar{\epsilon}_{pi}$, and the effects of the hysteresis on κ and $\{\alpha\}$ are introduced by them. $\bar{\epsilon}_p$ and $\bar{\epsilon}_{pi}$ are respectively called cumulative

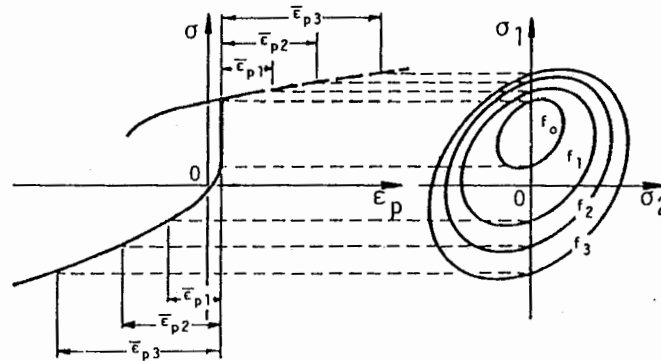


Fig.1. Cyclic plasticity model[3,4]

equivalent plastic strain and equivalent plastic strain increment. $\bar{\epsilon}_p$ is the cumulative amount of equivalent plastic strain from time (t_0) when loading was commenced until the time (t_c) when unloading most recently occurred. Further, $\bar{\epsilon}_{pi}$ is the increment of equivalent plastic strain during monotonous loading toward the outer side of the multi-surface from time (t_c) to a certain time (t_i). In the Proposed Model, $\bar{\epsilon}_p$ is evaluated as follows.

"Of equivalent plastic strains occurring in a monotonous stress-strain path, only the equivalent quantity exceeding the maximum value of the equivalent plastic strain range $\Delta\bar{\epsilon}_p$ occurring up to that time is accumulated to define $\bar{\epsilon}_p$."

Here, by equivalent plastic strain range $\Delta\bar{\epsilon}_p$ is meant the range of equivalent plastic strains calculated from plastic strains occurring from the time an unloading begins until the next time unloading begins.

κ expressing enlargement and reduction of surfaces is defined by the following equation:

$$\kappa = W_i \kappa_i + (1-W_i) \kappa_{i+1} \quad ; i=1, N_b \quad (1)$$

where, κ_i and κ_{i+1} are fundamental surface size functions and W_i is weighting function expressing the variation in surface size according to hysteresis. The equation means that the variation in surface size function in a certain limited range of $\bar{\epsilon}_p$ can be expressed using the fundamental surface size functions κ_i and κ_{i+1} defined as surface size functions at the boundaries determining the range and the weighting function W_i for that range. When the number of boundaries is taken to be N_b , an N_b number of weighting functions and an N_b+1 number of fundamental surface size functions will be required.

NUMERICAL CALCULATION METHOD FOR MOMENT-CURVATURE RELATION

In case of a frame member having a biaxially symmetrical section, calculations of sectional behaviours can be performed by means of the tangent stiffness method by Chen and Atsuta [7] with some modifications. The hypotheses introduced in numerical calculations are given below.

- 1) Stress components other than normal stress in the direction perpendicular to the member cross section are ignored.
- 2) The configuration of the cross section is invariable.

- 3) Unstable behaviour such as local buckling does not occur.
- 4) Stress, strain, and tangent modulus vary linearly inside each element.
- 5) Residual stress exists.

In order to evaluate the tangent stiffness, it is necessary for integration to be done all over the cross section using the tangent modulus at various parts of the cross section. Therefore, the cross section was divided into finite triangular elements and using the above mentioned hypothesis 4), an integrated value concerning an element was obtained by the values at the three nodal points composing the element. According to this method, it is possible to reduce the number of elements, since the distributions of strains, stresses, and tangent moduli can be evaluated more accurately compared with the case that strains, etc. are assumed constant in the elements.

MEASUREMENTS OF MOMENT-CURVATURE RELATIONS AND MATERIAL PROPERTIES

A specimen consists of a test portion cut out from H-shaped steel of SS41 quality and of loading arms with cylindrical cross section. According to mill-sheet, the mechanical properties and chemical compositions of the H-shaped steel in the test portion are given in Table 1. The configuration and dimensions of the specimen are shown in Fig.2(a), while the method of loading is shown in Fig.2(b). Loading speed was controlled by mini-computer so that strain rate at the upper and lower flanges would be approximately 0.0001 mm/mm/sec. The curvature of the test portion was calculated with the assumption that cross sections of the beam remain plane.

Table 1 Mechanical properties and chemical compositions.

	Tension test			Chemical composition %					
	Y.P. (Mpa)	T.S. (Mpa)	EL. (%)	C	Si X100	Mn	P X1000	S X100	Seq X100
SS41	312	436	24.5	10	20	60	40	40	21

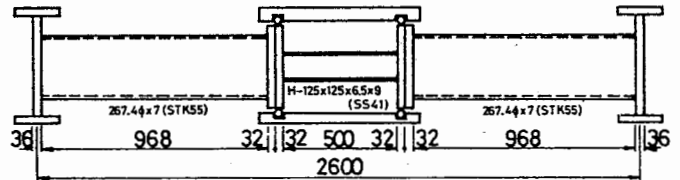
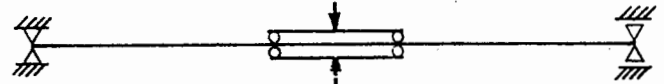


Table 2 Mechanical properties measured by the authors.

unit : MPa

	Upper Yield Point σ_{yu}	Lower Yield Point σ_{yl}	Tensile Strength σ_B	Breaking Stress σ_b	Young's Modulus $x10^5$ E
flange	344	301	440	347	2.10
web	---	402	498	402	2.10

(a) Configuration



(b) Method of loading

Fig. 2. Test specimens

The mechanical properties measured by tension tests are given in Table 2. To obtain the values for the material properties introduced in the Proposed Model, a tension-compression tests each including a single unloading were performed besides tension tests to measure the stress-strain relations[1,2]. Fig.3 shows the results of the tests in terms of the relation between stress non-dimensionalized by lower yield stress and plastic strain.

The fundamental surface size functions determined in accordance with Ref. [1,2] are shown in Fig.4. Since κ_{conv} defined as the fundamental surface size function when the effects of hysteresis had converged was not measured, the fundamental surface size function at the time of start of strain hardening enlarged 30 percent in the direction of stress axis was taken to be κ_{conv} , using the results of measurements on mild steel made previously by the authors

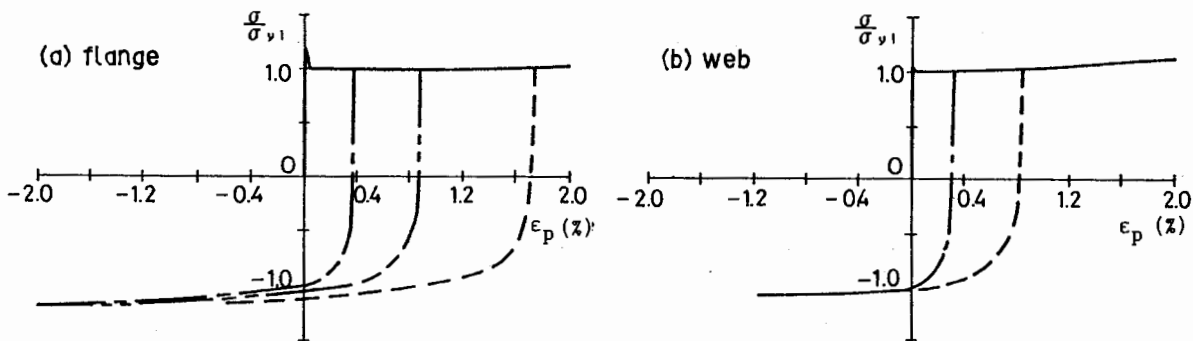


Fig. 3. Experimental results for evaluating material properties.

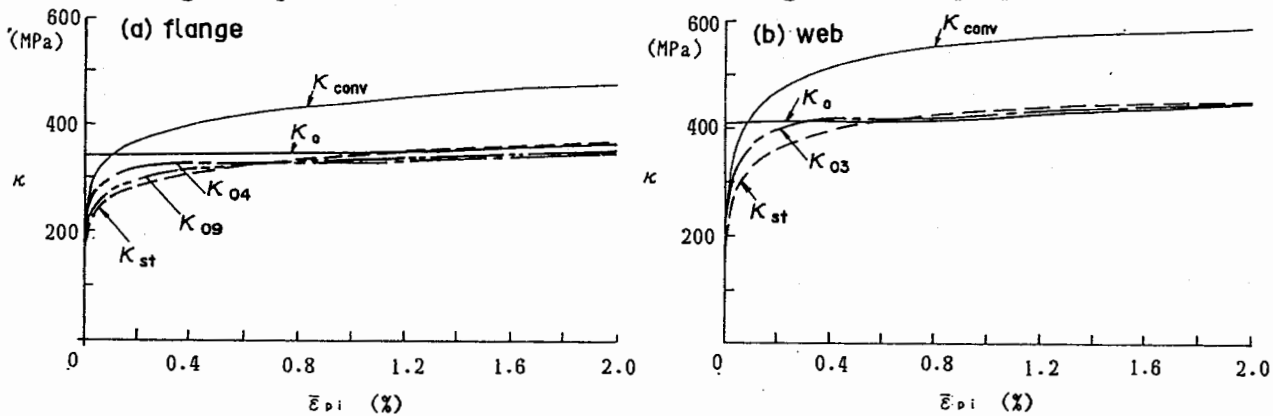


Fig. 4. Fundamental surface size functions measured.

[1,2]. All surface size functions measured were adopted as fundamental surface size functions, and weighting functions were all made to be straight lines. The residual stresses occurring in the specimens were measured by the hole drilling method [8] and a simple distribution type was assumed for calculations.

COMPARISONS OF ANALYTICAL AND MEASURED RESULTS

Petersson-Popov Model

Before estimating the hysteretic moment-curvature relations using the

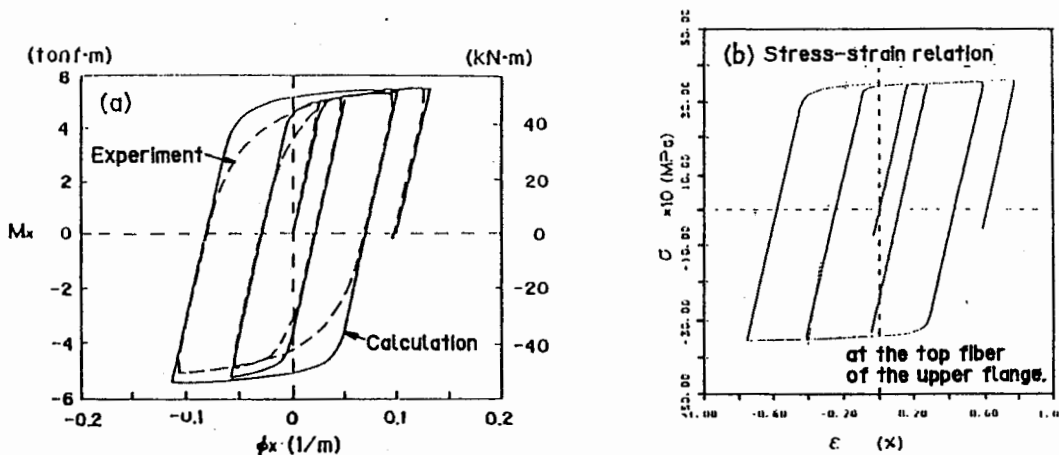


Fig. 5. Comparison of moment-curvature relation; Experiment and Petersson-Popov Model.

stress-strain relation calculated by the Proposed Model, the relations were calculated using stress-strain relations according to Petersson-Popov Model. The moment-curvature relation calculated using this stress-strain relation is shown in Fig.5 compared with that obtained from measurement. The solid line in (a) indicates the calculated moment-curvature relation, and the broken line is the relation obtained from the corresponding loading test result, respectively. The stress-strain relation at the top fibers of the upper flange obtained in the numerical calculation indicated is shown in (b).

The elastic region was large in the hysteretic moment-curvature relation, and the gradual lowering of stiffness as seen in measured result was not expressed. Since only κ_0 and κ_{conv} were used as fundamental surface size functions in the Petersson-Popov Model and the plastic strain produced was in a range not exceeding 0.5 percent, the configuration of the function of κ_0 became dominant. Moreover, since the strain produced in a single path was about 1.5 percent at most the stress-strain relation at the part of κ_0 at the yield plateau appears to become a stress-strain relation close to a bilinear type. As a result, although a matter of course, the moment-curvature relation also became similar to results obtained by means of the bilinear type stress-strain relations.

Proposed Model

Hysteretic moment-curvature relations were calculated using the stress-strain relation proposed by the authors, and the results were compared with the relations obtained by corresponding loading tests. The results are shown

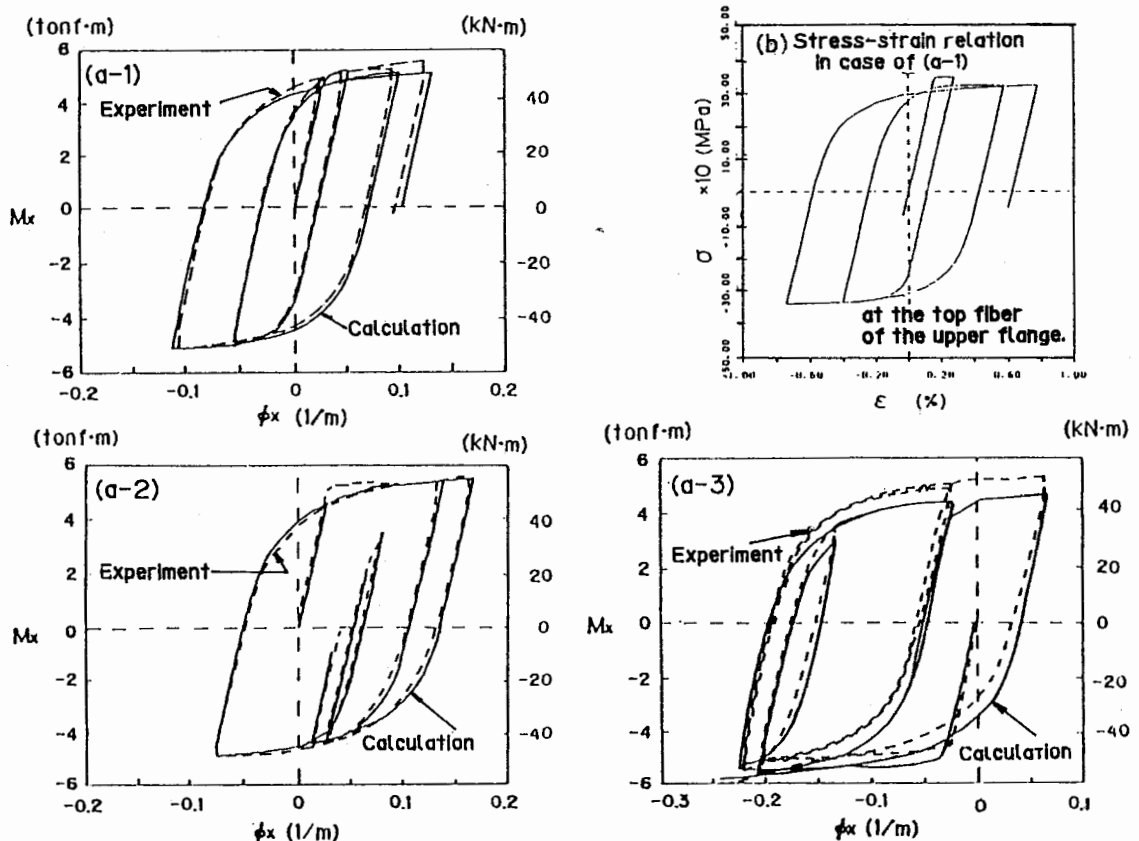


Fig.6. Comparisons of moment-curvature relations; Experiment and the Proposed Model.

in Figs.6(a-1) to (a-3). The solid lines of these figures show the calculated moment-curvature relations and the broken lines the relations obtained from the results of corresponding loading tests, respectively. The stress-strain relation at the top fibers of the upper flange obtained in the numerical calculation indicated in (a-1) is shown in (b).

Since the strain to which the material was subjected was about 2 percent at maximum in terms of value of $\bar{\epsilon}_p$, it can be considered that the effect of having determined κ_{conv} from assumptions based on past measurement results instead of depending on actual measurements has not appeared. As for weighting functions, they were all set linear, and it was found that as a result of having used four or five fundamental surface size functions, serious errors were not brought about in the calculated results. Regarding hysteretic moment-curvature relations, it is thought the errors in the first loading paths were slightly large mainly because of the scatter in upper yield point values of materials, but it was found concerning subsequent cyclic loading processes that moment-curvature relations obtained from measurements could be predicted with high accuracy by using the proposed cyclic plasticity model.

CONCLUSIONS

As discussed in the foregoing, when the Petersson-Popov Model is used, the hysteretic moment-curvature relation cannot be estimated with high accuracy, and it is desirable to rely on the Proposed Model in order to get the stress-strain relation of high precision.

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