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APPLICABILITY OF INFERENCE SYSTEM WITH KNOWLEDGE REFINEMENT FUNCTION FOR SELECTING RETROFITTING METHOD FOR BRIDGES DAMAGED BY FATIGUE

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ABSTRACT

The authors have proposed a knowledge refinement method for a crack diagnostic inference system. The inference engine was constructed for a reciprocal network based on a min-max composition algorithm. The knowledge refinement function was installed in the engine by applying the concept of the back-propagation algorithm. In this paper, the inference and refinement methods are applied to the rule-base system for selecting the retrofitting method for steel bridges damaged by fatigue. Through some inferences for practical cases, it is confirmed that the inference engine can be applied to this domain.

Key words: inference systems, knowledge acquisition, case base, network systems.

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1. INTRODUCTION

Knowledge-base acquisition, which includes the adding and modifying of rules, is still one of the most important issue for constructing an inference system. Research has been conducted on this issue in the field of civil engineering. Miyamoto et al. [1991] proposed a method to refine knowledge for concrete bridge diagnosis. Kushida and Miyamoto [1995] also proposed a knowledge update method by introducing the concepts of possibility and necessity.

Mikami et al. [Mikami 1992a, 1992b, 1994] acquired the knowledge base rules by adding a learning function to an expert system with a causal network using neural networks automatically generating undefined causal relations. Tanaka, Mikami, et al. also constructed a system simultaneously using a rule-baed reasoning and a case-based reasoning [Tanaka 1995, 1996]. A case-based approach was used to design and optimize steel frames by Arciszewski and Ziarko [1991]. By combining multimedia and case-based reasoning technology, Maher and Balachandran [1994] developed a prototype case-based system to assist structural designers. Reich et al. [1993] used machine learning programs to model engineering decision-making procedures. Melhem et al. [1996] found that such a tool was not necessarily effective, when attempting to use a commercial machine learning tool as a means of knowledge acquisition in addition to explicit domain extraction.

These inference systems employed various knowledge expression methods. From the standpoint of sharing knowledge, it is important to establish a methodology theory making it possible to reconstruct a rule base that easily reflects the inference results by using existing systems.

For the purpose, we assumed hypotheses as nodes, and formed rule bases into a network consisting of mutually connected nodes via links representing the relations between the hypotheses. We thus constructed an inference system with a knowledge refinement function and applied the inference system to the problem of inferring the factors of damage to the reinforced-concrete (RC) floor slab of highway bridges [Minagawa 1998 and 1999]. This paper reports on the application of our inference system to the selection of retrofitting methods for steel highway bridges damaged by fatigue, and shows that the rule-base refinement and inference functions of this inference system are effective.

2. OVERVIEW OF INFERENCE SYSTEM WITH RULE-BASE REFINEMENT FUNCTION [MINAGAWA 1999]

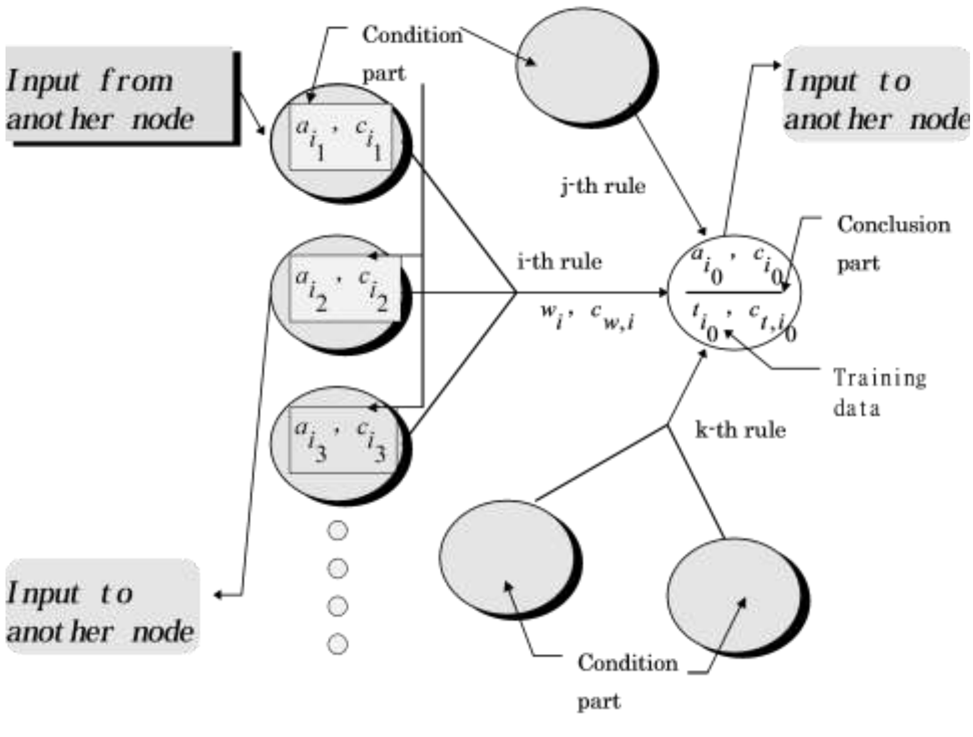

Figure 1 Configuration of inference system with rule-base refinement function

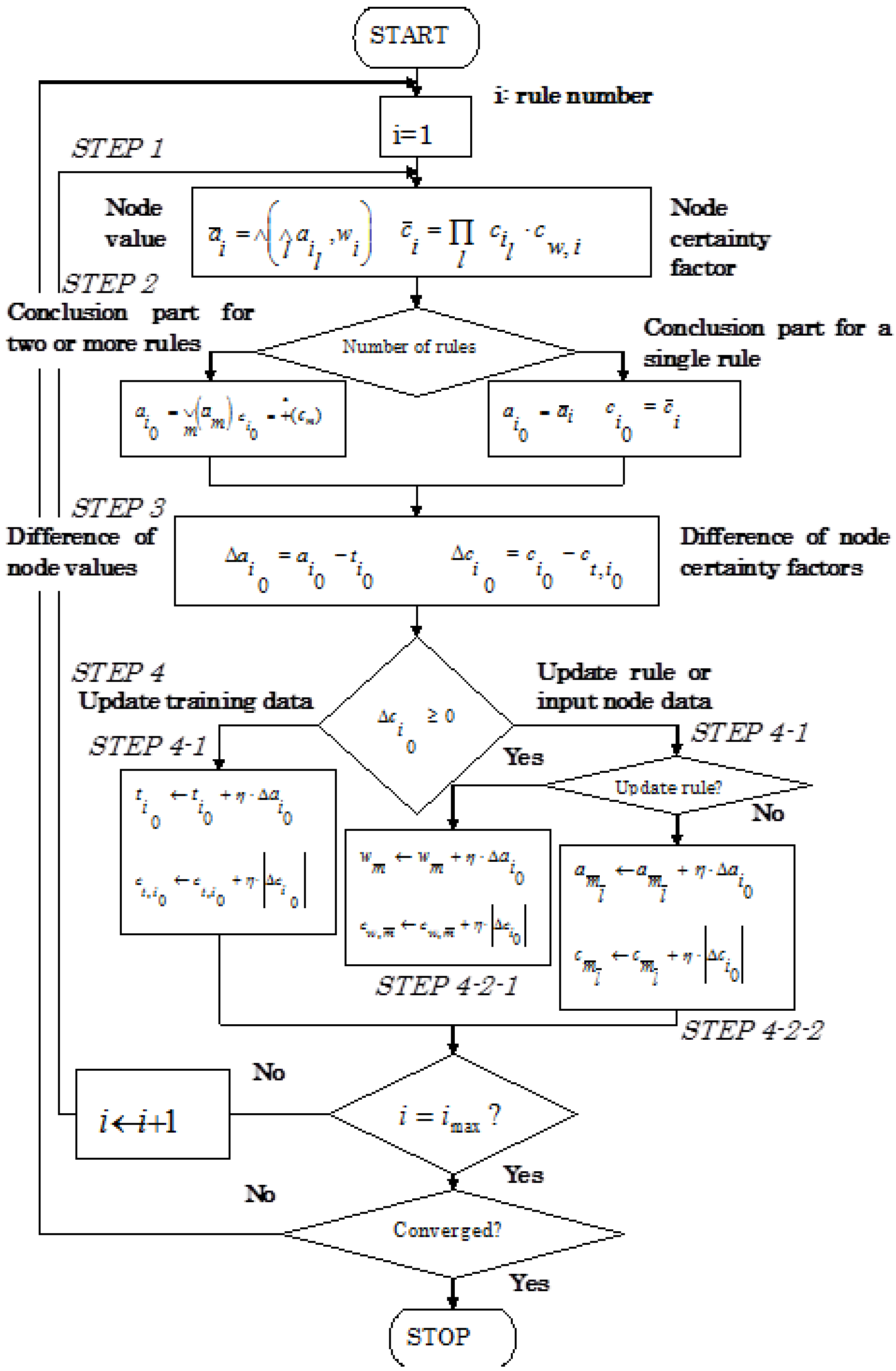

Figure 2 Flowchart of the system

Fig.1 shows a schematic diagram of the inference system, which consists of nodes representing hypotheses and links representing the relations between the nodes in the form of a node network. The inference system has a rule-base refinement function that allows us to refine rule bases and ordinary hypothetical inference by applying min-max operation and a new algorithm similar to the back-propagation algorithm used in neural networks.

The possible application of effectuating each hypothesis and the strength of the relations between hypotheses are respectively assumed as the attributes of each node and relations between nodes, which are represented by a node value and weight given as real numbers in the range of [0, 1]. Certainty factors are also endowed with each attribute in the range of [0, 1]. Each node can retain the results of inferences in past cases as training samples. Accordingly, each node representing a hypothesis has [node value, node certainty factor] and [node training sample value, node training certainty factor] as attributes. The link between nodes representing a relation between hypotheses is given attributes [weight, rule certainty factor]. The certainty factors given to each nodes and links express the uncertainty of information. If the certainty factor of the node sample is lower than the rule certainty factor, an ordinary hypothetical inference is applied. If the certainty factor of the node sample is higher than the rule certainty factor, rule-base refinement is applied. Thus, the inference system also controls the hypothetical inference and rule refinement functions.

Fig.2 shows the flowchart for the hypothetical inference and rule refinement. For an inference, a node value is calculated from a rule by a minimum operation. A node certainty factor is also calculated from the same rule (STEP 1). If a node is the conclusion part for two or more rules, a node value and certainty factor are linked to each set of rules by a maximum operation and ordinary linkage of the certainty factor (STEP 2).

The following shows the refinement method. First, the difference between the node value and node training sample obtained by inference, and the difference between the certainty factor of the node and certainty factor of the node training sample are calculated (STEP 3). If the difference of node certainty factors results in a positive value, the certainty factor of the training sample is assumed to be lower than that of the node, then the node sample and its certainty factor are updated (STEP 4-1). Here, η indicates a learning factor. If the difference results in a negative value and a node value has been used for min-max operation for the inference, the node value and certainty factor given to that node are updated (STEP 4-2-1). In this case, if the weight of a rule has been adopted for min-max operation, the weight and rule certainty factor given to that rule are updated (STEP 4-2-2).

3. RULE BASE FOR SELECTING RETROFITTING METHOD FOR STEEL BRIDGES DAMAGED BY FATIGUE

Tanaka and others [1992b] developed an expert system (here after called TANAKA's system) which intended to select a retrofitting method for steel highway bridges damaged by fatigue as the target problem. In this research, we applied the rule base used for that expert system to our inference system.

3.1. Network configuration

As shown in Fig.3, TANAKA's system is composed of a checklist presentation system, factor/action force inference system and retrofitting method selection system. When damage is found through checking and investigation based on the checklist, the external factors, internal factors, and action force at welding are inferred from the damaged structure, damaged part, type of welding, type of splice and damaged element by using a factor/action force inference system. If retrofitting is judged necessary, a retrofitting method is selected by the retrofitting method selection system based on such information as external factors, internal factors, and action force at welding inferred by the factor/action force inference system and the observable forms of cracks.

The checklist presentation system and factor/action force inference system, however, draw inferences by using frames, and the processes of inference were not made clear. We assumed the external and internal factors of cracks and action force at welding as observed facts, or already known information, like the forms of cracks, and linked the hypotheses together to form a network in the system as shown in Fig.4.

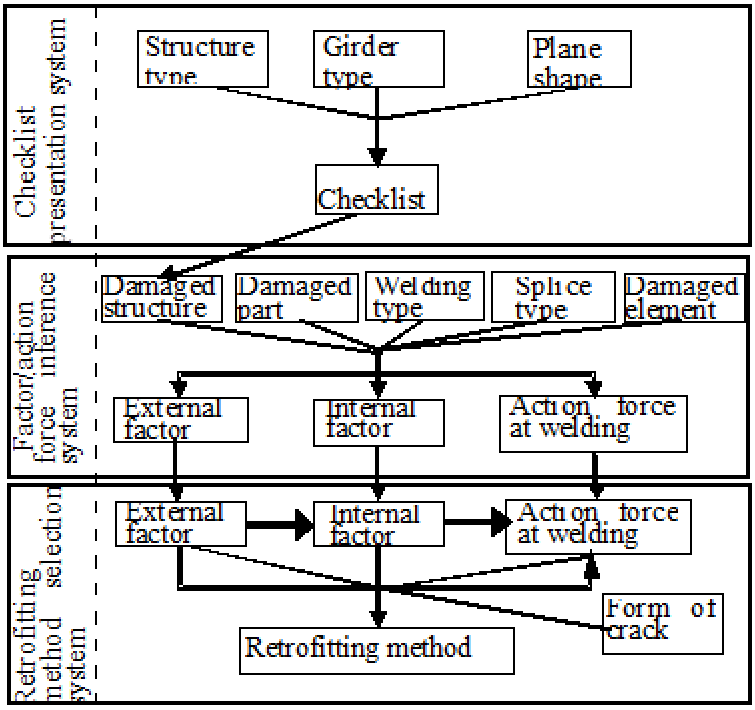

Figure 3 Outline of retrofitting method selection system

3.2. Setting of hypotheses (Nodes)

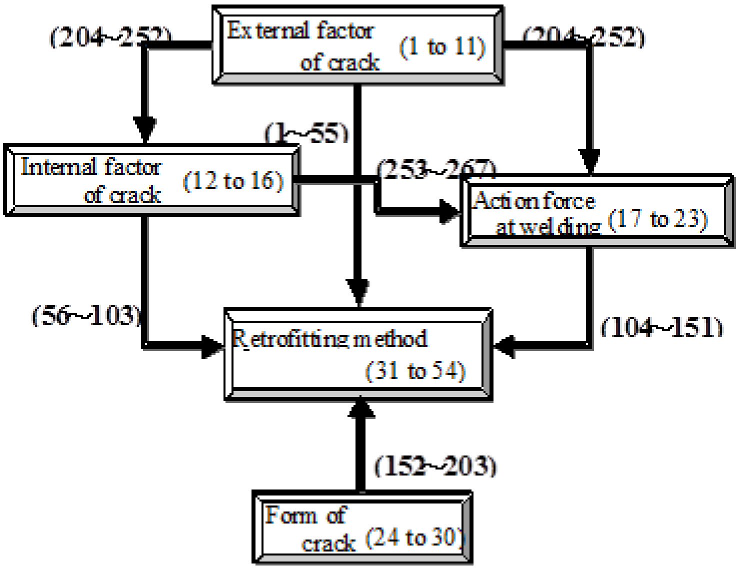

Figure 4 Network configuration

Table 1 Items of input information

No.	External factor
1	Vibration due to wind
2	Action of live load
3	Vibration due toEarthquake
4	Low temperature
5	Lateral distribution effect
6	incompatible of detail
7	Secondary deformation
8	Material inferiority
9	Welding inferiority
10	Construction error
11	Load during transportation or construction
No.	**Internal factor**
12	Stress concentration
13	Stress concentration of secondary stress
14	Secondary stress
15	Secondary stress of buckling due to stress concentration
16	Residual stress
No.	**Action force at welding**
17	T-shape fillet welding(2)
18	T-shape fillet welding(3)
19	T-shape fillet welding(6)
20	T-shape fillet welding(7)
21	Butt-welding(9)
22	Butt-welding(10)
23	Lap welding(12)
No.	**Form of crack**
24	T-shape fillet welding(a)
25	T-shape fillet welding(b)
26	T-shape fillet welding(c)
27	T-shape fillet welding(d)
28	T-shape fillet welding(f)
29	Lap welding(i)
30	Lap welding(j)

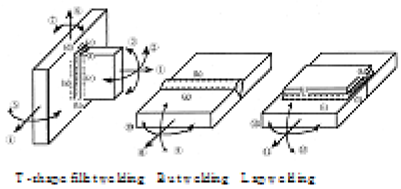

Table 2 Items of retrofitting method

No.	Retrofitting method
31	Stop hole
32	Gouging
33	Grinder
34	Bearing
35	Increase in web plate gap
36	Increase in web plate thickness
37	Cutting out
38	Re-welding
39	Welding of flange and stiffener
40	Re-melting
41	Splice plate with stiffener
42	High-strength bolt
43	Splice plate
44	Insertion plate
45	Mutual binding of hangers by cables
46	Connection of flanges of cross and main beams
47	Connection of flanges of transverse bracing and main bear
48	Connection of flanges of diaphragm and main beam
49	Connection of flanges of cross beam and arch rib
50	Replacement of stringer bearing plate
51	Replacement of main beam
52	Replacement of connecting plate
53	Installation of new stiffener
54	Installation of vibration-mitigating equipment

Table 3 Presented cases

Bridge name	Case 1
Country	U.S.A
Year constructed	1958
Year damage was found	12
Structure type	Simple bridge
Girder type	Grillage composite plate girder
Plane shape	Straight bridge
Damaged structure	Cover plate mounting section on main beam
Damaged part	Lower flange of main beam
	Cover plate
Welding type	Fillet welding
Splice type	Lap splice
Damaged element	Fillet welding
External factor	Live-load stress
Internal factor	Stress concentration
Action force at welding	Action force at lap splice 12
Form of crack	Form of crack on lap splice (j)
Direction of cracks	Cracks parallel with welding
Retrofitting method	Bearing
	Re-welding
	High-strength bolt
	Splice plate
Damage condition	

Bridge name	Case 2
Country	U.S.A
Year constructed	1973
Year damage was found	0
Structure type	Continuous bridge
Girder type	Composite plate girder
Plane shape	Straight bridge
Damaged structure	Vertical stiffener mounting section on main beam
Damaged part	Web plate of main beam
	Vertical stiffener
Welding type	Fillet welding
Splice type	T-splice
Damaged element	Web plate of main beam
External factor	Transport and erection load
Internal factor	Secondary stress
Action force at welding	Action force at T-splice 3
Form of crack	Form of crack on T-splice (d)
Direction of cracks	Cracks parallel with welding
Retrofitting method	Stop hole
	Grinder
Damage condition	

A total of 54 nodes used in TANAKA's system were set for this inference system. Among these, 30 nodes (Nos. 1 to 30 listed in Table 1) were prepared as input items, or input information based on observed facts. A total of 24 nodes (Nos. 31 to 54 listed in Table 2) were prepared as output items, or inference objects.

3.3. Presentation of rules

For the rule base used in TANAKA's system, the strength of causal relation is classified into four levels: "necessity," "high possible application," "possible application," and "low possible application." We applied the four weights (0.8, 0.6, 0.4, and 0.2) corresponding to these four levels and constructed the initial states of the rule base. The rule base applied in our inference system contained a total of 267 rules.

4. INFERENCE PERFORMANCE IN INDIVIDUAL CASES

4.1. Inference assuming all input information as known information

When the above rule base was applied to the inference system, node value "1.0" (true) or "0.0" (false) was given to each of the external and internal factors of cracks and action force at welding which were assumed to be known information, and node value "0.5" and certainty factor "0.1" were given to the retrofitting method which were assumed to be unknown information.

Table 3 shows the cases of actual bridges to which the inference system was applied in this research. In this research, the certainty factor and the rule certainty factor resulting from inference by TANAKA's system were respectively assumed to be "1.0" and "0.5," and rule base refinement was done first. Next, with the node certainty factors lowered (due to unknown retrofitting methods in the cases) and rule certainty factors increased, hypothetical inferences were conducted to select retrofitting methods. Fig.5 show the inference results. In each figure, the horizontal axis indicates the node numbers representing the retrofitting method items listed in Table 2; the vertical axis indicates the node values representing the possible application of using each retrofitting method. Each figure also shows training sample data for comparison purposes.

Fig.6 shows variations in the weight by each refinement count according to changes in the learning factor. The variations were calculated by the formula shown in each figure. The horizontal axis in each figure indicates the rule-base refinement count (logarithm); the vertical axis indicates weight variation.

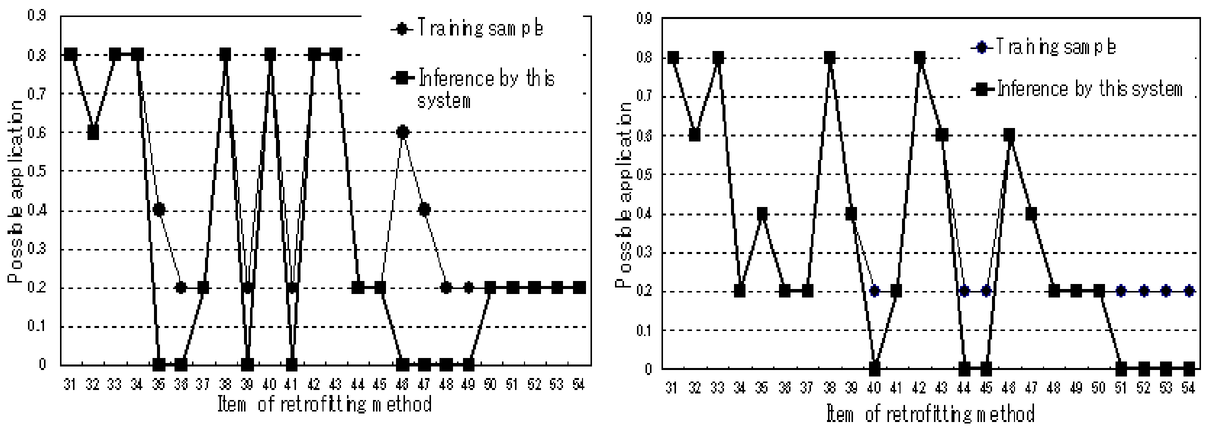

Figure 5 Results of inference of retrofitting method

As a result, the inference system inferred that stop holes and a grinder (i.e., retrofitting methods actually implemented) would have the highest possible application, and the overall inference showed a tendency similar to that of training sample data. Thus, the inference system was considered to have an effective function for selecting retrofitting methods.

Regarding rule base refinement, the tendency of variation depended on differences in the learning factor. However, the variations converged as the refinement count increased, and the inference result suggested a valid solution.

Because this inference system was originally based on learning with training samples, the inference result may have to match training sample data when a case is presented as the training sample and the inference system is applied to the case. The inference result, however, showed a slight difference from the training sample data. One reason for this difference is that the external and internal factors of cracks and action force at welding were assumed to be known information for inference by this inference system. Conversely, TANAKA’s system organized a network as shown in Fig.4 for selecting retrofitting methods. The following section discusses how unknown information (as part of input information) affects the inference result.

4.2. Inference assuming some input information as unknown information

The action force at welding is an item of input information used to select a retrofitting method as shown in Fig.4, “Network configuration.” The action force at welding also has a causal relation to each of the external and internal factors of cracks, and is an output destination inferred from information on external and internal factors. Therefore, the relation between the action force at welding and retrofitting method must be inferred based on all these items of information. When the action force at welding was assumed to be unknown information, the knowledge of its causal relations to the external and internal factors of cracks in the network configuration would not be reflected in the inference result. This can also be true regarding the relation between the external and internal factors of cracks.

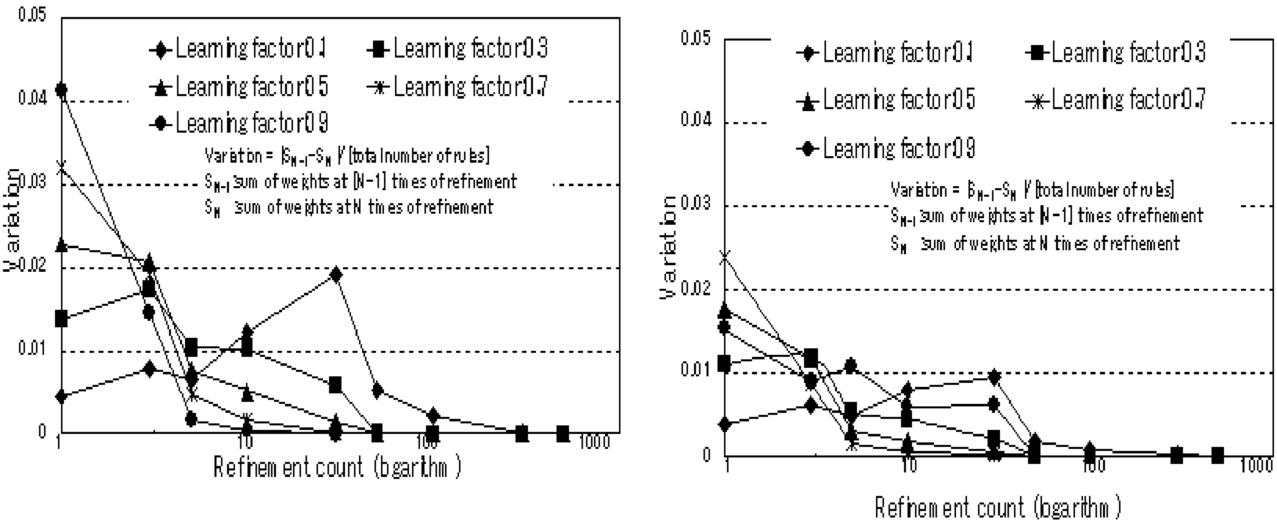

Figure 6 Variation of weights

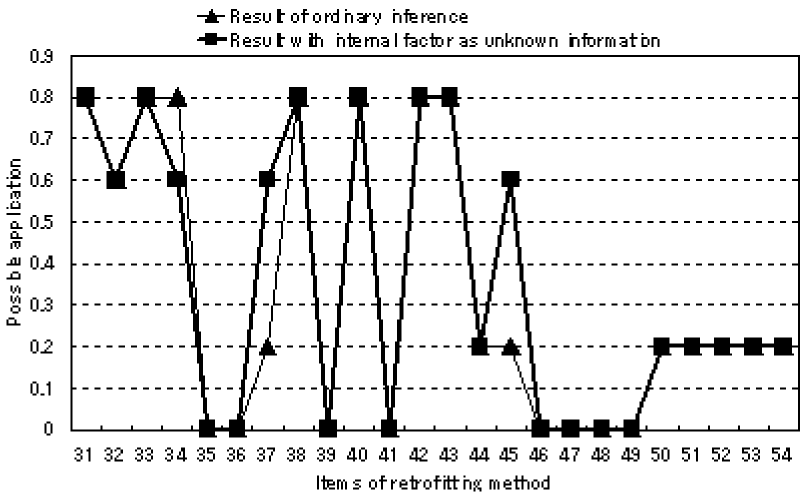

Figure 7 Result of inference with internal factors assumed asunknown information (case 1)

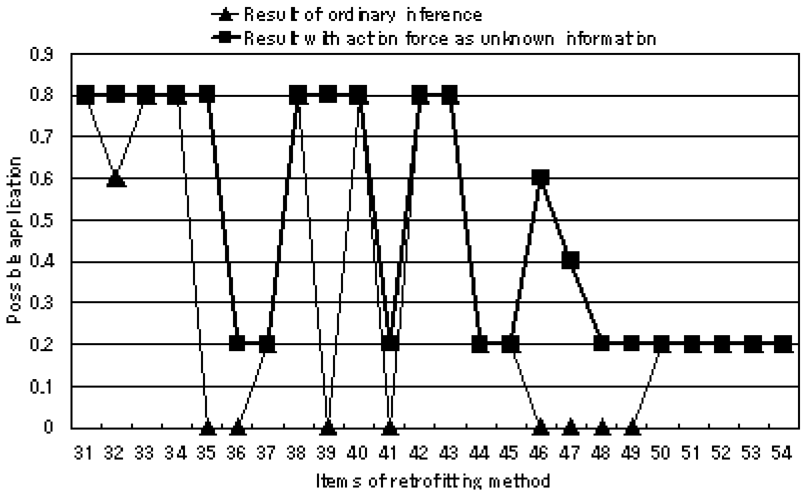

Figure 8 Result of inference with action force at welding assumed as unknown information (case 1)

For the above reason, we applied the inference system under the following two conditions to check each item of information assumed as unknown information before the inference result was affected:

(I) Inference assuming internal factors of cracks as unknown information

(II) Inference assuming action force at splicing as unknown information

Here, “0.5” and “0.1” were respectively given as “node value” and “node certainty factor” to the node assumed to be unknown information.

Fig.7 and 8 show the results of inferences under conditions (I) and (II) above in case 1. As in Fig.5, the horizontal axis indicates node numbers representing the retrofitting method items listed in Table 2; the vertical axis indicates node values representing the possible application of retrofitting methods. Each figure also shows training sample data for comparison purposes.

As shown in Fig.4, each item of information assumed as unknown information here is an object of inference from another item of information. Fig.9 and 10 show the node values of each unknown information inferred when it becomes an object of inference under conditions (I) and (II) together with the node value given as known information for ordinary inference.

The horizontal axis indicates the item numbers of input information; the vertical axis indicates node values.

4.2.1. Inference assuming internal factors of cracks as unknown information

As shown in Fig.7, the inference result differed slightly from the respective results of inferences assuming the relevant items of information as known information. For this reason, the internal factors of cracks could hardly affect the selection of a retrofitting method. This was also suggested by the inference results in Fig.9. The figure shows that the node value for the same item of information as the one assumed beforehand as known information is “0.6” and the node values of most other items are “0” (except node value “0.2” for node No. 14). This result suggests that the internal factors of cracks only have a marginal effect on retrofitting method selection.

4.2.2. Inference assuming action force at welding as unknown information

As shown in Fig.8, the inference results largely differed from the result of inference assuming the action force at welding as known information. The inference result was hardly identical to the training sample data. This meant that information on the action force at welding could largely affect retrofitting method selection in the network configuration. This large affect was also suggested by the fact that the action force at welding is the object of inference from many other nodes (input information) in the network as shown in Fig.4.

The inference result gave an extremely high possibility of “0.8” to some items given low possibility as training sample data and by inference assuming the action force at welding as known information. When the action force at welding was assumed to be unknown information, only the node value of node No. 23 was “1.0.” Conversely, when the action force at welding was assumed to be unknown information, nodes Nos. 17, 18, and 20 were given “0.8” (i.e., extremely high value), but node No. 23 was given a very low node value of “0.2” as shown in Fig.10. For this reason, information on the action force at welding could largely affect the causal relations to retrofitting methods and, accordingly, may have a large effect on retrofitting method selection.

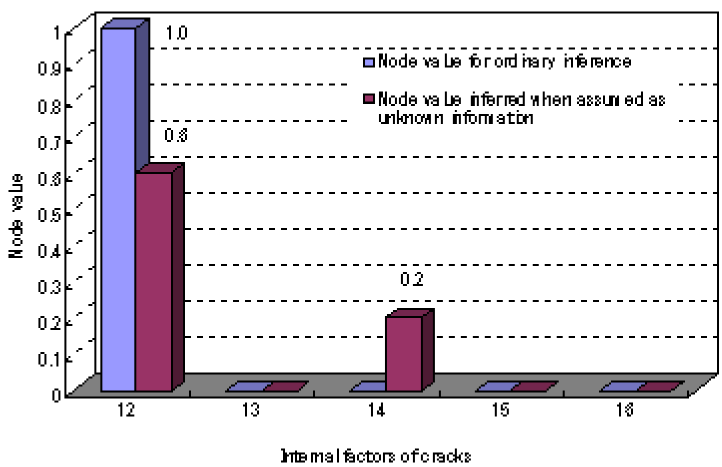

Figure 9 Node value representing internal factors inferred when assumed as unknown information (case 1)

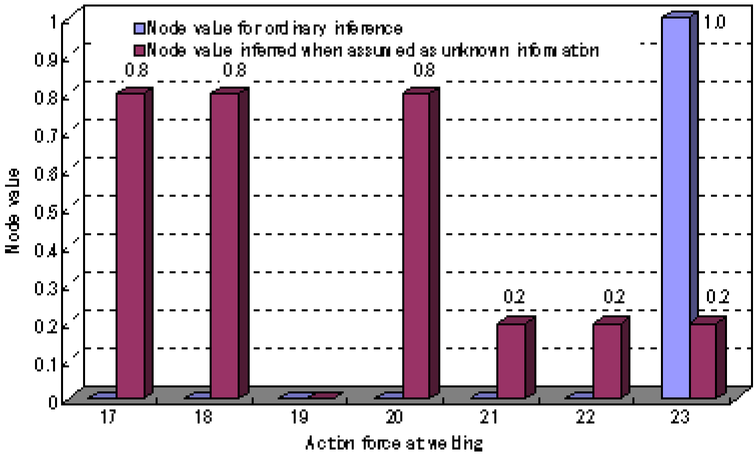

Figure 10 Node value representing action force at welding inferred when assumed as unknown information (case 1)

5. APPLICATION OF INFERENCE SYSTEM TO SIMILAR CASES

Table 4 Similar cases

Case No.	Bridge name	Form of crack	External factor	Internal factor	Action force	Retrofitting method
1.1	Tomei Expressway	Unknown	Transversely distributed stres Live-load stress	Stress concentration	⑫	Gas-cutting of transverse bracing
1.2	King's Bridge	i	Welding defect Material defect	Stress concentration	⑫	Unknown
1.3	King's Bridge	i	Welding defect Material defect	Stress concentration	⑫	Unknown
1.4	U.S. Bridge 51	h	Welding defect	Stress concentration	⑨	Splice plate with high-strength bolts and grinde
1.5	Aquasabon River Bridge	g	Welding defect Low temperature	Stress concentration	⑩	Mounting of cover plate by welding

Case No.	Bridge name	Form of crack	External factor	Internal factor	Action force	Retrofitting method
2.1	Prairio Du Chein Bridge	d	Secondary deformation	Secondary stress	③	Unknown
2.2	Prairio Du Chein Bridge	b	Secondary deformation	Secondary stress	③	Connection of flange of cross beam and diaphragm by high-strength bolts and stop hole
2.3	Poplar Street Bridge	d	Inappropriate details	Secondary-stress concentration	③	Connection of flange of main beam and cross beam, gouging, stop hole, and re-welding
2.4	Poplar Street Bridge	a	Inappropriate details	Secondary-stress concentration	③	Stop hole
2.5	Poplar Street Bridge	a	Inappropriate details	Stress concentration	③	Splice plate with high-strength bolts
2.6	Chamberlain Bridge	b	Inappropriate details	Secondary-stress concentration	③	Welding of flange of main beam and vertical stiffener at top and bottom edges
2.7	Unknown	b	Live-load stress	Secondary stress	③	Unknown

Table 5 Retrofitting methods inferred for similar cases

Case No.	Inferred retrofitting method
1.1	31 32 33 34 38 42 43 47
1.2	31 33 34 38 40 42 43
1.3	31 32 33 34 38 42 43 47
1.4	31 33 34 38 42 43
1.5	31 32 33 34 38 42 43
2.1	31 33 38 42
2.2	31 33 38 42
2.3	31 32 33 35 38 39 42 43 47 46
2.4	31 32 33 35 38 39 42 43 47
2.5	31 32 33 34 35 38 39 42 43 47
2.6	31 32 33 35 38 39 42 43 47

For the retrofitting methods indicated by numbers, see Tables 3 a

We applied the inference system to the selection of retrofitting methods for similar cases by using the rule base refined for cases 1 and 2. The similar cases adopted here were those searched for through the case-based reasoning by Tanaka and others. Tables 4 show similar cases relevant to cases 1 and 2. These similar cases were used as verification data. In other words, we assumed the retrofitting methods for the similar cases as unknown information and other information as known information, and inferred the retrofitting methods by using the rule base refined for cases 1 and 2.

Table 5 lists the results of inference. The numbers in the table indicate the retrofitting methods selected by inference. All retrofitting methods given the highest node value of “0.8” are listed. The numbers printed in bold face indicate the retrofitting methods inferred by Tanaka and others. The underlined number printed in bold face (46 for case 2.3) indicates the retrofitting method not selected by our inference, but selected by Tanaka and others. The numbers with a shaded background indicate retrofitting methods that were actually implemented in the past. The contents of retrofitting methods were unknown in cases for which no bold face numbers are printed, and are not shown. Except for retrofitting method No. 46 (connection of flanges of cross and main beams) for case 2.3, all retrofitting methods selected by Tanaka and others were selected by our inference. Our inference could also select some retrofitting methods not selected by Tanaka and others, but which were actually implemented (as indicated by the shaded numbers in plain typeface.) This result suggests that the inference system could be improved through rule base refinement for flexible application to various cases.

6. CONCLUSION

This paper discussed whether an inference system with a rule-base refinement function mainly designed for sharing and reusing knowledge is effective for selecting retrofitting methods for steel bridges damaged by fatigue.

We first explained that inference results consistent with individual cases could be obtained by refining the rule base using individual cases as training samples. Then we clarified how much input information affects the inference result under complex causal relations among hypotheses through trial inferences assuming that some input information as unknown.

We also applied the inference system to similar cases different from the training samples. For this application, the inference result was affected depending on the degree of case similarities. The information on the action force at welding, which largely affected the inference of retrofitting methods, allowed the authors to select the same retrofitting methods for similar cases. This result suggests that appropriate training samples improve proper rule base refinement.

We must, therefore, increase the accuracy of rule base refinement for application to cases having complex relations among the rules. It is also necessary to discuss how to present knowledge with a set of rules by using such technology as knowledge discovery in database (KDD) and data mining (DM).

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NOTATION

The following symbols are used in this paper:

a_{i_l} = node value of l-th node for i-th rule

w_i = weight for i-th rule

c_{i_l} = certainty factor of node value of l-th node for i-th rule

$c_{w,i}$ = certainty factor of weight for i-th rule

a_{i_o} = node value of conclusion part for i-th rule

c_{i_o} = node certainty factor of conclusion part for i-th rule

Δa_{i_o} = modification value of a_{i_o}

t_{i_o} = training data of conclusion part for i-th rule

Δc_{i_o} = modification value of c_{i_o}

c_{t,i_o} = certainty factor of training data of conclusion part for i-th rule