Localized vs. Overall Competition in Monopolistic Competition

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Introduction

Monopolistic competition attracts renewed attention, not only as a model for market competition but also as a micro-economic base for new macroeconomics. It would also be a more realistic base, than perfect competition, for normative analyses in the field of environment, services industry etc. where location factors play an important role. However, its general properties do not seem to have been sufficiently clarified. One important reason for this is that most analyses start from a specific micro-economic model and we can not decompose their results into two parts: those with general applicability, and those dependent on specific micro-economic model adopted in respective analyses.

In this paper, I examine properties of symmetric monopolistic competition with respect to three controversial respects.

First is whether or not it approaches to perfect competition as the number of the firm increases. There, it is shown that the answer depends on whether competition is "localized" or "overall". In localized competition, the equilibrium converges to perfect competition, whereas, in overall competition, firms may remain monopolistic even in the limit. This limit property is rather the opposite to the difference when the number of firms is finite. In localized competition, firms are oligopolistic even when there are a large number firms in the whole market. On the other hand, in overall competition, oligopolistic factor quickly diminishes as the number of firms increases. This twisted situation may be one of the reasons of the confused arguments on monopolistic competition.

Second issue is whether or not monopolistic competition results in too much variety, and it is shown that variety elasticity or potential heterogeneity of the market is the key parameter.

The third issue is the size of the welfare loss under the economies of scale and efficiency of possible remedies. Here, it is shown that welfare loss can be very substantial if the demand is price-inelastic and variety-elastic. The prescription depends on these two elasticities. In localized competition, a measure of inequality turns out to be a measure of inefficiency.

Although we can theoretically think of monopolistic competition on homogeneous market such as Cournot competition, the subject of this paper is mainly heterogeneous or differentiated market. There are two reasons for it. One is the fact that products supplied by different firms are more or less differentiated. The other is that the properties of heterogeneous market is quite different from homogeneous market, even if potential or observed heterogeneity is small, as is discussed in this

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1 Price Elasticities

In a heterogeneous market, demand for market products can be expressed in the following way:

\[ x_i = x_i \left( p_1, p_2, \ldots, p_i, \ldots, p_n, I \right) \]

where \( x_i \): demand for product \( i \), \( p_i \): price of product \( i \)

\( I \): income level

Let us introduce an assumption of symmetry, that all the products have the same quantity and price at equilibrium. This is a restrictive assumption, but enables us to concentrate on some essential properties of heterogeneous markets. Our another assumption is that each firm produces one product only. This assumption will be relaxed to a certain extent later.

Let us now examine mutual relationship between different notions of elasticity. Hicks, (1956) made a distinction between p-elasticity and q-elasticity as the following: p-elasticity is a price elasticity when all the other prices are kept unchanged, and q-elasticity is a price elasticity, when quantity of particular product is changed while quantity of all the others are kept constant but all the prices are allowed to adjust so that an equilibrium would be restored. Here we concentrate on the two p-elasticities as the following:

\[ \xi_o = -\frac{\partial \log x_i}{\partial \log p_i} = -\frac{p}{x} \frac{\partial x_i}{\partial p_i} \quad : \text{own p-elasticity} \]

\[ \xi_c = \frac{\partial \log x_i}{\partial \log p_j} = \frac{p}{x} \frac{\partial x_i}{\partial p_j} \quad : \text{cross p-elasticity} \]

Note that the sign of the own elasticity is reversed so that the two elasticities are both positive.

There are two kinds of symmetric monopolistic competition. One is a symmetry in the sense that each firm is in an identical competitive situation, but that competition is localized in such a way that there is a distinction between near neighbors and remote firms, with the former being direct rivals. Symmetry assumptions in most of the spatial competition models are of this type. Salop (1979) and Capozza and Van Oder (1978) are typical examples. The other is an overall symmetry such that each firm has all the rest as equal competitors with no distinction between near neighbors and others. This is the case typically discussed by Dixit and Stiglitz (1977), and by Spence (1976b). This difference will matter substantially as will be shown later. In order to make the distinction clear, let us use the word localized for the former and overall for the latter.

To be more precise, in localized competition,

\[ \xi_c^{ij} = \frac{\partial \log x_i}{\partial \log p_j} = \begin{cases} \xi_c & \text{for some } j \neq i \ (\text{for m products}) \\ 0 & \text{for other } j \neq i \ (\text{for n} - 1 - m \text{ products}) \end{cases} \]

..............(2)
where \( n \) is the total number of products in the market and \( m \) is the number of direct rivals for each product.

On the other hand, in overall competition,

\[
\frac{\partial \log x_i}{\partial \log p_j} = \xi_c \quad \text{for any pair of } j \neq i
\]

(3)

It goes without saying that the relevant price elasticity in monopolistic competition, is not an objective one, but a subjective one which depends on the conjecture concerning the response of rivals. Conjectures can change the perceived marginal revenue and thereby importantly affect the characteristics of market equilibrium. In price competition, the \textit{perceived} price elasticity of demand is

\[
\eta = \xi_o - m \lambda \xi_c
\]

(4)

\( 0 < \lambda < 1 \)

Here \( \lambda \) is a conjectural coefficient in the sense that firms believe that the reaction by rivals are \( \lambda \) times their own action, and \( m \) is the number of rivals. \( \lambda \) is zero in a Bertland-Nash competition.

Suppose there is a \textit{uniform price increase} in the whole market. Under our symmetric assumption, the rate of demand decrease will be the same for all the products, and therefore the total demand will decrease by the same ratio. The former can be decomposed into two factors, the effect of own price increase, and the effects of price increase by rivals. Thus,

\[
\xi = \xi_o - m \xi_c
\]

(5)

where \( \xi \): the price elasticity of the total demand for the market

In localized competition, \( m \) is fixed in (2) and (4), and,

\[
m < n - 1
\]

(6)

combining (2), (4) and (5),

\[
\eta = \xi + m (1 - \lambda) \xi_c
\]

(7)

In contrast, in overall competition,

\[
m = n - 1
\]

(8)

from (3), (4) and (5),

\[
\eta = \xi + (n-1)(1-\lambda)\xi_c
\]

(9)

2 \textbf{Localized Competition}

Competition can not be localized in terms of quantity at the same time as in terms of price. To see this, suppose that competition is localized in terms of price. In the following, let us suppose that the number of neighbors for each firm is two, as is typical for one dimensional market. Then, the demand system can be expressed by a quasi-diagonal matrix as the following:
To complete the symmetry and to avoid the end-firm problem we assumed 1 and n to be neighbors as in a circular market. But the inverse of the matrix in (10) is not quasi-diagonal as the original matrix. The intuitive interpretation of this is that, if quantity of product i is increased with the quantity of its two neighbors unchanged, a change in prices of the two neighbors are necessary, and this, in turn necessitate changes in prices of the next neighbors if their quantities are to be kept unchanged. This spill-over process in terms of price continues, although weakening, until the effect reaches the opposite end of the market.

In what follows, we assume that, in localized competition, what is localized is price competition. This is the case in spatial competition when buyers decide which product to buy based on comparison of prices, not of quantities.

Now, let us see how the cross elasticity changes with an increase in the number of firms. If this approaches infinity, from (7), the perceived price elasticity also converges to infinity, and the price mark-up will tend to zero. Indeed Capozza and Oder (1978) showed that this is the case with spatial models with linear transport cost function and linear demand function. Lancaster (1979, p171-2) also argued that the elasticity of demand approaches infinity as the spacing between adjacent products approaches zero. On the other hand, Chamberlin (1962, Appendix G) argued that the proposition that “the larger the number of sellers in a market, the greater the elasticity of demand for each seller” does not hold in general.

In the following section, the behavior of the cross elasticity is examined from a more general perspective than in the works mentioned above. First we will examine a spatial model with demand and transportation cost functions of a general form. Then we will go on to what we call quasi-spatial model. There, a "compensation" function plays an equivalent role of transportation cost function in spatial models.

For both spatial and quasi-spatial model examined below, let us make the following assumptions. First, we assume that the whole market area is fixed when the number of firms increases. By this we do not mean that the total demand is fixed, but the catchment area of each firm becomes proportionally smaller. Of course, this is a strong assumption and there may be a case where new entrants add a new portion to the market. But in order to capture the pure effects of congestion, let us take this assumption. Second, we ignore the fact that if the market is a plane (two dimensional),
the shape of each symmetric catchment area has to be a hexagon, not a disk, if the whole market is to be filled up by catchment areas. Similar problem exist for higher dimensional market. But, in what follows, this is neglected as a second order problem. Third, we assume that population is uniformly distributed in the market. Fourth, we assume the mill pricing. In other words, that transportation costs are assumed to be paid by buyers.

3 Spatial Competition

Assume that the demand function by individual consumer is identical and given by

\[ x = f(p_d) \] ..................................................(11)

where \( p_d \) is delivery price, as defined by

\[ p_d = p + \phi(u) \] .......................................(12)

where \( p \): mill price
\( u \): distance between consumer's location and the mill
\( \phi(u) \): function of \( u \) such that
\[ \phi(0) = 0, \ \phi'(u) > 0 \] .......................................(13)

Then, demand by a consumer at distance \( u \) is

\[ x_u = f(p + \phi(u)) \] ..............................................(14)

Integrating this over the catchment area, we have total demand for individual product.

\[ x = D \int_0^R f(p + \phi(u)) H(u) du \] .............................................(15)

where \( D \): density of consumers
\( R \): radius of the catchment area
\( H(u) \): function dependent on the dimension of the market,

such that \( D \int_0^R H(u) du \) is the total area of the catchment area

\[ e.g. H(u) = \begin{cases} 
2 & \text{if } \tau = 1 \\
2\pi u & \text{if } \tau = 2 \\
4\pi u^2 & \text{if } \tau = 3 
\end{cases} \]

\( \tau \): number of dimension of the market

Now, by differentiating (15) with respect to \( R \), we have,

\[ \frac{\partial \log x}{\partial \log R} = \frac{DRH(R)}{x} \frac{f(p + \phi(R))}{x} \] ...............................................(16)

But, since \( DRH(R) \) is the number of consumers in the catchment area times the number of
dimension, the average consumption in the catchment area is,

\[ x_A = \frac{\tau x}{DRH(R)} \] .................................................................(17)

on the other hand, the consumption by a consumer located at the border of the catchment area is,

\[ x_M = f(\ p + \phi(R)) \] .................................................................(18)

Thus, (16) can be simplified to,

\[ \frac{\partial \log x}{\partial \log R} = \tau \frac{x_A}{x_B} \] ..................................................(19)

where \( \tau \): number of dimension of the market ( integer )

\( x_M \): demand by consumer at the border of the catchment area

\( x_A \): average demand ( over the catchment area )

Now, let us see how the radius is affected by a change in competitors price. If we denote the distance between the neighboring mills by \( u^* \), condition for the border is given as

\[ p + \phi(R) = p^* + \phi(\ u^* - R) \] .................................................(20)

where \( p^* \): price by neighbor

By total differentiation,

\[ dp + \phi'(R)dR = dp^* + \phi'(u^* - R) \] .................................................(21)

Since in symmetric equilibrium,

\[ u^* = 2R, \ and \ p^* = p \]

effect of a price change by ( all the m ) rivals is,

\[ \frac{\partial \log R}{\partial \log p^*} = \frac{p^* \partial R}{R \partial p^*} = \frac{p}{2\phi'(R)R} \] ..................................................(22)

Combining this with (19), we have a formula for cross p-elasticity as the following.

\[ \xi_c = \frac{1}{m} \frac{\partial \log x}{\partial \log p^*} = \frac{1}{m} \frac{\partial \log R}{\partial \log p^*} \frac{\partial \log x}{\partial \log R} = \frac{\tau p}{2m\phi'(R)R} \frac{x_M}{x_A} \] .................................................(23)

As we have assumed that total market area is fixed, \( R \) and \( n \) are inversely related. Thus, from (7),

\[ \lim_{n \to \infty} \eta = \xi + m(1 - \lambda) \lim_{n \to \infty} \xi_c = \xi + \frac{\tau}{2} \lim_{n \to \infty} \frac{p}{\phi'(R)R} \] .................................................(24)

It is easy to see that, if \( \phi(R) \) is a linear function, this approaches to infinity. This is the case that Capozza and Van Oder (1978) analyzed with a linear demand function in two dimensional market. Let us now examine the general property of this.

Up to certain small value of \( R \), the cross price elasticity can either be an increasing or decreasing
function of $R$. In other words, (23) may well decrease. As to whether or not it converges to infinity as $R$ approaches to zero, it depends on the behavior of $R \phi'(R)$, since $p$ is finite. $\phi'(R)R$ can increase or decrease with $R$ as shown by Figure 1. However, if $\phi'(R)R$ is a well behaved function of $R$, this approaches to zero as $R$ becomes small. A sufficient condition is that $\phi'(R)R$ is a monotonous function of $R$ when $R$ is sufficiently small. Appendix 1 gives a mathematical proof.

Thus from (7), unless $\lambda$ is one,

$$\lim_{n \to \infty} \eta = \infty$$ ...........................................(25)

and the spatial competition converges to perfect competition in the limit.

4 Quasi-spatial Competition

Assume that all the consumers have the following identical utility function.

$$U = x_0 + f \left[ \frac{x}{\phi(u)} \right]$$ ...........................................(26)

where $x_0$: numeraire, $x$: market product

$f$: function such that $f' > 0$, and $f'' < 0$ ...........................................(27)

$u$: “distance” in specification between the most preferred product and the best available product.

$\psi(u)$: function of $u$ such that $\psi(0) = 1$,

$$\psi' \geq 0$$, but $\psi' > 0$ at the border ...........................................(28)

$\phi(u)$ can be taken as Lancaster's compensation function. But our specification is more general than his, since we require (28) only, whereas he (1979, p42) required that

$$\psi(0) = 1, \psi'(0) = 0, \psi'(u) > 0 \text{ for } u \neq 0, \psi'' > 0$$ ...........................................(29)
The first order condition for the utility maximization is given by
\[ p = \frac{\partial U}{\partial x} = f' \left[ \frac{x}{\phi(u)} \right] \phi(u)^{-1} \] .................................................................(30)

or equivalently,
\[ x_u = g \left[ p \phi(u) \right] \phi(u) \] .................................................................(31)

where \( x_u \) : demand by consumer at distance \( u \)

\( g \) : the inverse of function \( f' \)

In a similar way to the spatial model, the demand for individual product is,
\[ x = D \int_0^R g \left[ p \phi(u) \right] \phi(u) H(u) \, du \] .......................................................(32)

then,
\[ \frac{\partial \log x}{\partial \log R} = RH(R) \frac{x}{x} g \left[ p \phi(R) \right] \phi(R) = \tau \frac{x_M}{x_A} \] .......................................................(33)

\[ \lim_{R \to 0} \frac{\partial \log x}{\partial \log R} = \tau \] .......................................................(34)

Since the following " most preferred equivalent " are perfect substitutes,
\[ \frac{x}{\phi(u)} \] .................................................................(35)

the radius of the catchment area, \( R \) is a function of \( p^* \), \( p \) as the following:
\[ p \phi( R ) = p^* \phi( u^* - R ) \] .......................................................(36)

By differentiating this at symmetric equilibrium, we get
\[ \frac{\partial \log R}{\partial \log p^*} = \frac{\phi}{2 \phi'(R) R} \] .......................................................(37)

Thus, the cross p-elasticity is
\[ \xi_C = \frac{\partial \log R}{\partial \log p^*} \frac{\partial \log x}{\partial \log R} = \frac{\tau \phi}{2 \phi'(R) R} \frac{x_M}{x_A} \] .......................................................(38)

Since \( \phi(0) = 1 \), we can construct the same argument for the limit property as in the case of spatial competition and confirm that
\[ \lim_{n \to \infty} \eta = \infty \] .......................................................(39)

But, again, this does not necessarily mean that \( \xi_c \) monotonously increases as \( R \) decreases.
Note that, as \( n R^\tau \) is constant,

\[
d \log n + \tau \, d \log R = 0
\]

Thus, (19) and (33) can be re-written as

\[
-1 \leq \frac{\partial \log x}{\partial \log n} = - \frac{x_M}{x_A} < 0
\]

Now, let us summarize what we have so far seen on spatial and quasi-spatial competition. Under the assumption of unchanged (total) market area,

(a) The cross price elasticity may increase or decrease with the radius.

(b) However as the number of firms approaches infinity, it approaches infinity.

(c) The number-of-firms elasticity of demand is related to an equality index defined as the ratio of marginal to average consumption.

Among the above, (b) implies that crowding will make a market converge to perfect competition. Of course, one cannot claim that this holds product for any localized competition. But, given the generality of our two models, we may claim that this property holds for many localized competition. However, as (a) says, it does not necessarily follow that cross elasticity always increase when the number of firm increases. In the next section, let us see an example with which, up to a certain level of \( n \), the cross p-elasticity is constant.

5 Constant Cross Price Elasticity Case (CPE)

With the framework of the quasi-spatial model, assume a set of utility and compensation function as the following.

\[
U = x_0 + \left[ \frac{x}{\phi(u)} \right]^r
\]

where \( 0 < r < 1 \) and

\[
\phi(u) = \begin{cases} 
1, & \text{if } u \leq 1 \\
u^\alpha, & \text{if } u > 1
\end{cases}
\]

The above assumption on the “compensation function“ implies that there is a certain indifference range of specification of products within which people are just as satisfied as when they are offered their most preferred products. To put it another way, this assumption implies that the subjective value of product does not decrease within a certain distance.

Applying (30) and (31), the demand function by consumer at distance \( u \) is
$$x_u = \left[ \frac{r}{p} \frac{1}{\phi(u)^r} \right]^{\frac{1}{1-r}}$$  \hspace{1cm} \text{(44)}$$

The demand for individual product is an integral of this over its catchment area.

$$\frac{x}{D} = \int_0^R x_u H(u) \, du = \int_0^1 \left[ \frac{r}{p} \right]^{\frac{1}{1-r}} H(u) \, du + \int_1^R \left[ \frac{r}{u^\alpha p} \right]^{\frac{1}{1-r}} H(u) \, du .... \text{(45)}$$

In one dimensional market, for example, $H(u)=2$, and

$$x = 2D \left[ \frac{r}{p} \right]^{\frac{1}{1-r}} \left[ 1 + \frac{1-r}{1-r-\alpha r} \left( \frac{R^{1-r-\alpha r}}{1-r} - 1 \right) \right] \hspace{1cm} \text{(46)}$$

If $R > 1$ and $\alpha < (1-r)/r$, we can approximate this by

$$x = \frac{2D(1-r)}{1-r-\alpha r} \left[ \frac{r}{p} \right]^{\frac{1}{1-r}} \frac{R^{1-r-\alpha r}}{1-r} \hspace{1cm} \text{(47)}$$

As we assume that the total market area is constant, $R$ is inversely proportional to $n$. Thus,

$$X = nx \propto p \frac{1}{1-r} R^{\frac{\alpha r}{1-r}} \hspace{1cm} \text{(48)}$$

More generally, as for a $\tau$ dimensional space, it can be shown that if

$$R >> 1 \hspace{1cm} \text{(49)}$$

and

$$\alpha << \frac{\tau(1-r)}{r} \hspace{1cm} \text{(50)}$$

$$X = nx \propto p \frac{1}{1-r} R^{\frac{\alpha r}{1-r}} \hspace{1cm} \text{(51)}$$

(49) implies that the actual radius of the catchment area is large in comparison with the indifference distance. (50) implies that when the market area of a firm is divided into rings of different diameter, $\alpha$ is small enough (but not necessarily smaller than 1) to ensure that the sales share of an outer ring is larger than that of an inner one. The boundary condition is obtained by substituting (43) into (37).

$$\frac{\partial \log R}{\partial \log p^*} = \frac{1}{2 \alpha} \hspace{1cm} \text{(52)}$$

$$\xi_c = \frac{\partial \log x}{\partial \log p^*} = \frac{\partial \log R}{\partial \log p^*} \frac{\partial \log x}{\partial \log R} = -\frac{\alpha r + \tau(1-r)}{2 \alpha (1-r)} \hspace{1cm} \text{(53)}$$

Thus, in this case, the cross elasticity does not depend on the number of products in the market. This case is hereafter referred to as constant cross price elasticity case or CPE.
6 Overall Competition

This is a completely symmetric case in the sense $\xi_c$ is the same for any pair of different products.

A notable difference from the localized competition is that $\xi_o$ and the perceived price elasticity can remain finite even in the limit. One such example is the CES market in Appendix 2-(ii). There, from (9), the perceived own price elasticity approaches

$$\lim_{n \to \infty} \eta_o = \xi + \frac{(1 - \lambda)(\xi - 1)(1 - \delta)}{\delta}$$

In overall competition, each firm competes directly with every other. This requires that, as the number of firms increases, the number of dimension of market must increase sufficiently rapidly, as Archbald and Rosenbluth (1975) argued. Thus, every new entrant adds a new dimension, and hence a new portion to the market.

7 Market Demand Function with Variety Elasticity

Once the symmetric assumption is accepted, we can define the market price elasticity of demand -- the DD curve by Chamberlin (1962). In order to capture an important aspect of heterogeneous market, let us introduce a new parameter to a market demand function, variety elasticity, which is an index of potential heterogeneity of the market demand. It is defined as the degree of the proportional increase in total market demand in response to an increase in the number of products available in the market, under a given price level. Broadly speaking, there are two reasons why a market appreciates large variety. One is that variety is desirable in individual's preference, and the other is that different people have different tastes and a market with larger variety can extract more of the potential demand.

As for intra-personal appreciation of variety, variety elasticity is small if each consumer sticks to a particular product. On the other hand, as for inter-personal appreciation, the variety elasticity is large if consumers are either widely spread or sticky to their most preferred specification. In the spatial context, a set up of a new seller will reduce transportation costs born by consumers and the whole market demand will generally increase under a given mill price.

Clearly, in a homogeneous market this elasticity is zero. If this elasticity is small, the total demand increases little with the number of the products and the market quickly becomes satiated. The other extreme is the case where this elasticity is unity. There, no congestion occurs. Indeed, in this case, a new entrant brings in a new piece of market exactly as large as that of an existing product, and this new piece is independent from the rest. Thus, if variety elasticity is unity, it can be viewed as a bundle of independent markets rather than a heterogeneous market.

From above discussion, we require that variety elasticity lies between 0 and 1. Our market demand function is written as

$$X = nx = F( p, n)$$
\[ \frac{\partial \log X}{\partial \log p} = -\xi \] \hspace{1cm} \text{................................................................................................................................(56)}

\[ \frac{\partial \log X}{\partial \log n} = \delta, \quad 0 < \delta < 1 \] \hspace{1cm} \text{................................................................................................................................(57)}

where

\( X \) : total market demand, \hspace{0.5cm} \( p \) : price

\( x \) : demand for each product, \hspace{1cm} \( \xi \) : price elasticity

\( n \) : number of products (\( X = n \, x \)), \hspace{1cm} \( \delta \) : variety elasticity

Note that, in spatial and quasi-spatial competition, the variety elasticity is directly related to the equality in the market. This is because, from (41),

\[ \delta = \frac{\partial \log X}{\partial \log n} - 1 + \frac{\partial \log X}{\partial \log R} \frac{\partial \log x}{\partial \log R} x_M - \frac{x_M}{x_A} \] \hspace{1cm} \text{..................................................................................................................(58)}

This holds regardless of the shape of the demand, transportation-cost or compensation function. Interestingly, the variety elasticity turns out to be a measure of inequality with an emphasis on the least-conveniently-located consumers. Market appreciates variety to the extent that consumption differs due to locative inequality.

8 Inverse Problem

An interesting question concerning the market demand function just introduced is, how much, under the assumption of symmetry, we can deduce the underlying micro-economic structure from a market demand function, in particular, the cross price elasticity. To be more precise,

Given the following market demand function :

\[ X = F(p, n) \] : DD curve

How much can one know about the underlying demand function for individual product, namely,

\[ x = f(p, p^*, n) \] : dd curve

such that \( X = F(p, n) = n \, f(p, p, n) \) ?

where \( x \) : demand for individual product

\( p \) : own price

\( p^* \) : price of all the others

\( n \) : number of products in the market

Note that the market demand function, DD, is observable, at least theoretically, whereas the individual demand function, dd, cannot be observed unless \( p \) and \( p^* \) move differently. In order to investigate this inverse problem, let us examine three concrete examples, of which detail is shown in Appendix 2, and see how the cross price elasticity of micro-economic demand is related to the two elasticities of market demand \( \xi \), and \( \delta \).
(i) CPE market (localized)

In a CPE market introduced before, $\xi$ and $\delta$ are both constant as is seen by equation (51) and the cross elasticity is,

$$\xi_c = \frac{(\xi-1)(1-\delta)}{2m\delta} \tag{59}$$

where $m$ : number of direct rivals

(ii) CES market (overall)

If the demand system is generated by a CES utility function as was assumed in Dixit and Stiglitz (1977), $\xi$ and $\delta$ are both constant, and the cross price elasticity is the following:

$$\xi_c = \frac{(\xi-1)(1-\delta)}{n\delta} \tag{60}$$

where $n$ : number of products in the whole market

(iii) Quadratic case (overall)

If the demand system is generated by a symmetric quadratic utility function, the cross price elasticity is the following:

$$\xi_c = \frac{1-\beta}{\beta} \tag{61}$$

where $\beta = \frac{n(n^\delta-1)}{n^\delta(n-1)}$

From the above, following observation can be made:

First, in all the above three examples, $\xi_c$ is negatively related to $\delta$ such that if $\delta=0$, $\xi_c$ is infinite, and if $\delta=1$, $\xi_c$ is zero. Given the definition of $\delta$, this property seems to be general, although the exact relationship between the two is model-dependent. Moreover, as is seen in Appendix 2, if the specification of the underlying micro-economic model is known but only the parameters are unknown, they can be deduced from the two elasticities of the market demand, $\xi, \delta$, and $n$, the number of products.

Secondly, there can be more than one different underlying micro-economic models that give the same market demand function, since among our three examples, (i) and (ii) have the same form of market demand function despite the fact that the micro-economic structure and the cross price elasticity is substantially different. Thus, the inverse problem can not be uniquely solved, at least in general terms.

9 Assumptions

Based on the above, let us see how much can be said about the properties of monopolistic competition.
We assume the following:

A. constant price elasticity of market demand
B. constant variety elasticity of market demand
C. fixed set up cost F
D. constant marginal cost c
E. one firm produces one product unless otherwise noted

A and B are particularly restrictive. But there are cases that these hold good as is already discussed. Moreover, what we really require is not necessarily the strict constancy over the whole region, but sufficient constancy in the relevant region. The market demand function is:

\[ X = nx = K p^{-\xi} n^\delta \] ...........................................(62)

where \( K \): constant
\[ \xi > 1 \] ...........................................(63)
\[ 0 < \delta < 1 \] ...........................................(64)

C and D implies economies of scale. We assume this for two reasons. One is that, in most industries, there are economies of scale at least to a certain extent. The other reason is that, if there were no economies of scale, every consumer can be served with his most preferred product and there is no trade-off between production efficiency and variety. C and D leads to a simple cost function as the following:

\[ C(x) = F + cx \] ...........................................(65)

In this regard, let us define an index, h, by the following:

\[ h = Fc^{\xi-1} / K \]

As h is the ratio of the fixed cost to the sales at marginal cost pricing, it is an index of economies of scale, or degree of scale merit. Note, however, that the total cost increases with h.

10 Structure of the Analysis

The market regimes to be compared are the following:

--- First best (F)

This is the case where everything is managed so as to maximize social surplus. The social surplus is defined as the sum of consumer surplus and profits of firms. Although the social surplus approach has two weak points: neglect of distribution aspects and income effects, it does enable us to isolate the efficiency aspect of the problem. As Willig (1976) showed, this approximation is not bad if income elasticity is small or if the price and quantity change is small in relation to the consumers' whole income. We assume that the approximation is good enough to show us the order of magnitude of changes in welfare. Indeed, we will see later that welfare difference can be so substantial that a rough measure is sufficient for our purpose.
Given the demand and cost functions, the social surplus is:

$$SS(p, n) = \int_{0}^{X} p \, dX - c \, X - n \, F = \frac{\xi}{\xi - 1} \, p \, X - c \, X - n \, F$$ ................................(66)

In the first best, this is maximized with respect to $p$ and $n$. This results in the familiar marginal cost pricing. But under the assumption of economies of scale, this implies deficits for firms, and requires some lump sum transfer, which may not be practical in real world.

--- Second best (S)

If the lump-sum transfer is not practical, we have to seek a constrained optimum. Here, the social surplus is maximized subject to the constraint that the sales must cover the production cost.

$$\pi = px - cx - F = 0$$ .........................................(67)

If this is combined with the demand function (62), the break-even condition can be rewritten as,

$$(k - 1)k^{-\xi} = n^{1-\delta}h$$ .........................................(68)

where $k$: price cost ratio ($k = p/c$)

As is shown by Figure 2, this is an inverse-U shaped curve in the $k$-$n$ plane, and the second best is given by its tangency with a social surplus isoquant.

--- Monopolistic Competition (MP)

Here, we confine ourselves to competition with free entry. This does not necessarily imply that the break-even condition is binding. Especially in the case of localized competition, if product "location" does not change, it may happen that a vacant part of market is not large enough to support a new product, and excess profit exists even under free entry. However, in the following, we take up the cases where excess profit is soaked up by free entry.

Even under this simplification, we can not have a unique solution for monopolistic competition. As is already seen, the equilibrium depends on, among other things, whether or not competition is localized, which variable (price or quantity) is used as a mode of competition, and how much collusion the firms expect of their rivals.

To cope with this difficulty, we will do two things. First, we will examine two extreme cases which will give upper or lower bounds to the variables we are concerned. One is the second best market introduced above, which gives an upper-bound of social surplus. The other is the collusive market, which gives an upper-bound for the number of products and lower-bound for the social surplus as will be seen shortly. Second, where useful, we will use an example of micro-economic model.

--- Collusive market (C)

Here we assume that firms have complete collusion under free entry. In other words, the conjectural coefficient is assumed to be unity. Then, the distinction between the price and quantity competition does not matter. The perceived own price elasticity of demand is the same as price elasticity of market demand, and we do not worry about the micro-economic detail. Note that this
case is what is called Löshian in spatial competition. The equilibrium is characterized by the break-even condition and the monopolistic pricing. Since this corresponds the peak of the break-even curve in the k-n plane (see Figure 2), the number of products in collusive market is largest among monopolistic competition. As for the social surplus, combination of (66) and (68) enables us to write the social surplus as a function of k. Since this is a decreasing function of k, when k is larger than its second best level, the social surplus in the collusive market is lowest among monopolistic competition.

--- Monopoly (M)

For the sake of comparison, we include monopoly, in which all the products are served by a single monopolist in the conventional sense. This is characterized by monopolistic pricing and the optimal number of products from the viewpoint profit maximization.

![Equilibria in heterogeneous market](image)

**Fig.2** Equilibria in heterogeneous market

11 Homogeneous Market and Comparison

Heterogeneous market is different from homogeneous market in a number of important respects. As for the welfare loss due to the break-even condition (the ratio of social surplus in the second best to that in the first best) in heterogeneous market,

A: the welfare loss is independent from h, economies of scale
B: the welfare loss is negatively related to price elasticity
C: the welfare loss can be very substantial even if the market is served
D: the number of products is not a good proxy measure
where the opposite is the case in homogeneous market.

As regards the variety elasticity,
E: the welfare loss increases with variety elasticity

A is because, the number of product is endogenously determined, reflecting economies of scale. Thus, even a very small degree of scale economies can cause a significant welfare loss.

As for B, there are two reasons. First, if price elasticity is small, consumer surplus is large relative to the sales. If the market is under supplied, relatively large consumer surplus is lost. To put it another way, one suffers a lot if necessities are under-consumed. Second,

B is rather complicated. In homogeneous market, in order to cover the given set up cost, the price has to be set above the marginal cost. If the price elasticity is large, this leads to a relatively large decrease in demand, which in turn pushes price even higher, resulting in a large decrease in consumer surplus. Thus, the price in the second best market is an increasing function of the price elasticity. This is the reason why the loss is an increasing function of the price elasticity. In contrast, in heterogeneous market, the fixed costs are endogeneous as well as the number of products, and the price in the second best market turns to a decreasing function of the price elasticity. The fact that products of small price elasticity is “necessities”, becomes dominant. Consumer surplus on such products is large relative to the sales, and one suffers a lot if such commodities are under-supplied.

To see C and D, a numerical example would be helpful. If $\xi = 1.1$ and $\delta = 0.9$, the social surplus under the second best is almost non-existent as compared to that in the first best, despite the fact that a large number of products are supplied to the market. In contrast, in homogeneous market, the second best is not so bad so long as the market is served.

For the welfare loss in homogeneous market, there have been a number of analyses on its magnitude, and their conclusions are different from one another. For example, Harberger (1954) estimated that it was of the order of 0.1% of the national income for the US economy in the 1920s, based on the assumption of unit price elasticity. Bergson (1973) argued that, if one takes account of the factors such as larger price elasticity, interaction among markets and income effect, it may be much larger. Although he did not show his own empirical estimates, his illustrative calculation seems to suggest the order up to 10 to 30% of the national income. But this conclusion is criticized by Carson (1975) and by Worcester (1975) for an inconsistency between the profit maximizing price mark-up ratio and the price elasticity of demand. If this point is modified, according to Carson, the welfare loss would be at maximum 3.2% of the national income. These arguments are not very different from what we see for homogeneous markets based on our model, but quite different from the order of magnitude of possible welfare loss our model suggests for heterogeneous markets.

E is related to the tendency of monopolistic competition to supply too few products when the variety elasticity is large. This will be shown later.

It follows that, we should be concerned with markets with a small number of firms and a large price elasticity, since there the performance of the monopolistic competition is seriously unsatisfactory.
12 Number of products in monopolistic competition

Let us examine whether or not monopolistic competition leads to too many products or too few products in comparison to the first best. This is the question to which most diversified answers have been given. Since the number is largest in collusive market, we have a following sufficient condition:

\[ n_{MP} < n_F \quad \text{if} \quad n_C^{1-\delta} = \frac{1}{h(\xi - 1)} \left(\frac{\xi}{\xi - 1}\right)^{-\xi} < \frac{\delta}{h(\xi - 1)} = n_F^{1-\delta} \] \hspace{1cm} \text{(69)}

This implies that if potential heterogeneity is sufficiently large in comparison to the market price elasticity, monopolistic competition leads to too few products regardless of the inner structure of the market. On the other hand, if the potential heterogeneity is small, monopolistic competition may lead to sufficient or too many number of products depending on the inner competitive structure of the market. The situation is illustrated in Figure 3.

![Figure 3: Variety in monopolistic competition](image)

13 What can be done?

If the welfare loss can be very substantial, two important questions would naturally arise: How can we distinguish markets with large welfare loss from others, and what should be done for them? For the first question, we have seen that the welfare loss is substantial in markets with a small price elasticity and a large variety elasticity. The price elasticity can be measured with reasonable accuracy. As for the variety elasticity, it is one minus the ratio of the marginal to average consumption, in localized competition. It is interesting that it is an index of inequality as we have already seen, but can also be used as an index for market inefficiency. Subsidies to facilities in local areas are often justified on the equity ground, but seldom on the efficiency ground. Indeed, it is generally believed that there is an important trade-off between equity and efficiency. But what the forgoing analysis suggests is that the two considerations can be quite consistent in the world of
heterogeneity. To sum up, if a market of necessity (with large potential consumer surplus) is unevenly served, it is highly desirable to increase the supply variety, both from equity and efficiency point of view.

One way of addressing such a market is to try to achieve the first best directly, through such means as nationalization. However, as is emphasized by Vickers and Yarrow (1988), lack of incentive for efficiency improvement makes such an option unattractive, and this is the main reason why privatization of nationalized firms in many developed countries is called for as an important method of structural reform and of economic revitalization. In order to guarantee incentives for improvement, some element of competition has to be introduced. On the other hand, competition in heterogeneous market is subject to monopolistic factor to a certain extent. Thus there is a trade-off.

However, if the welfare loss due to insufficient variety is much lager than the loss due to monopolistic factor, we can gain by decreasing the former loss at the cost of the latter. There are two market-conforming options. One is **privatized competition** and the other is **subsidized competition**. Of course, both are inferior to the first best, as some monopolistic factor is induced, but can be much superior to the second best. Let us examine exactly how such operations can bring a welfare gain over the second best or over the free market competition. We will first examine the constant cross price elasticity case (CPE), which we discussed before, and then consider other cases.

----- Bertland-Nash free competition

First let us see what free market competition looks like in the CPE market. We assume that firms believe that their rivals do not change prices in response to there price change. In other words, firms have Bertland conjecture, and \( \lambda \), the conjectural coefficient is assumed to be zero. Then, since the price cost ratio is dependent on the perceived price elasticity of demand, from (7), we have,

\[
k = \frac{n}{\eta - 1} \frac{\xi + m \xi_c}{\xi + m \xi_c - 1}
\]

(70)

If we combine this and (59) for CPE market, we have the price-cost ratio,

\[
k_b = \frac{(\xi - 1)(1 + \delta) - 2}{(\xi - 1)(1 + \delta)}
\]

(71)

Since this does not depend on \( n \), this is a perpendicular line in the k-n plane as in Figure 2. Since this line is based on the structure of competition, let us call it competition curve. The equilibrium is determined by its intersection with the break-even curve, which is expressed by point B in Figure 2.

----- Privatized competition

This operation involves (a) public provision of supply diversification (large \( n \)) which seeks to realize more equality among people than the private competition would achieve, and (b) privatization of production units by competitive bidding. The financial burden for the government is not as large as in the first best. This is because, in a competitive bid for the production units, their price would be bid up to eliminate the excess profits. Since the cross price elasticity is independent from the number of products in CPE market, the price cost ratio is the same as (71). Subject to this constraint, the government chooses the optimal \( n \). This implies that the resulting equilibrium is point P in Figure 2, which is a tangent point between the perpendicular line and one of social-surplus

80
----- Subsidized competition

Alternatively, government can subsidize for the production. We assume linear subsidy such that the amount of subsidy is in proportion to the production quantity. For firms, this is identical to a reduction in marginal cost, and the price is lowered proportionally, resulting in a leftward shift of the competition curve in Figure 2. At the same time the break-even curve is pushed up. Thus, for a given subsidy rate, the equilibrium is characterized, in the k-n plane, by an intersection of the new competition curve and the new break-even curve. If we change the subsidy rate, such a point moves, and its locus is presented by the subsidy curve in Figure 2. The optimal subsidy rate is determined by the tangency of this curve to one of social surplus isoquants, as point U.

As to which of the above two operations is preferable depends on the properties of the market. For CPE market, Figure 4 illustrates the situation. If the variety elasticity is large and the price elasticity is small, which is the case that the performance of the second best is worst, privatized competition is the best regime. The reason why subsidized competition is inferior is that, if demand is price inelastic, the price reduction due to the subsidization leads to only a limited increase in production quantity, and does not help much to cover the fixed cost. Thus, in order to increase the variety to a given level, a large subsidy is needed, and this will lead to a pricing under the marginal cost, which impinges efficiency. The low price elasticity also implies that consumers do not change the consumption pattern much, and the subsidy is rather like a transfer to the consumers. In comparison, if the price elasticity is large, subsidized competition is recommendable. The second best is superior to these operations, only when the variety elasticity is small.

----- non-CPE market

What if the cross price elasticity increases with the number of products? Then the competition curve is not perpendicular in the k-n plane but downward sloping. The welfare gain of the privatized competition will be larger than the constant price elasticity case. This is because the increase in the
number of products will have a beneficial effect through strengthening competitive pressure. This is illustrated by P' in Figure 2. On the other hand, subsidized competition becomes less attractive as is illustrated by U' in Figure 2. Thus, we can say that, if the cross price elasticity increases with the number of products, privatized competition is relatively more advantageous than subsidized competition.

To summarize the property of these operations, there are three factors. First, an increase in number requires additional resources. Second, there is a gain from increased variety, to an extent which is dependent on variety and price elasticities. Third, there is a gain if competitive pressure increases and the price cost ratio is lowered as the number of products increases. In heterogeneous markets, all the three factors can be at work, and a welfare gain is expected by these market-conforming operations, if the second or the third is large. Especially, the second can be very significant. In a homogeneous market, in contrast, the second factor can not be at work if the market is already served, and the third factor may fail to outweigh the first factor.

14 Conclusion

----- If we assume symmetry, we can express the perceived own price elasticity by the market price elasticity and the cross price elasticity. This enables us to examine the properties of competition in a heterogeneous markets, which are quite different between localized and overall competition.

----- In localized competition, because it is a chain of local oligopolies, firms have only a few direct rivals, and they tend to be oligopolistic even when there are a large number of competitors in the whole market. On the other hand, in overall competition, oligopolistic factor quickly diminishes as the number of competitors increases.

----- However, the limit property is rather opposite. In localized competition, as the number of firms approaches infinity, the cross price elasticity almost always converges to infinity and so does the perceived own price elasticity, making the equilibrium converge to perfect competition. In overall competition, on the other hand, firms may remain monopolistic even in the limit. This twisted situation may be one of the reasons of the confused arguments on monopolistic competition.

----- The welfare implications of the market competition under economies of scale are substantially different, and often opposite between homogeneous and heterogeneous market. Conventional economic theories based on homogeneous market may be misleading.

----- In a market of large heterogeneity and small price elasticity, the welfare loss due to the break-even condition is so large that market-conforming intervention, such as “privatized competition” or “subsidized competition”, is desirable. To identify such a market, a measure of inequality will help. Efficiency and equity is no trade-off in such a market.

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Appendix 1: Proof of the limit property of spatial competition

Since we have assumed that $\phi'(R)R$ is monotonous with respect to $R$ where $R$ is sufficiently small ($R < R^*$), it must be the case that, as $R$ decreases

\[
\begin{cases}
- increases \\
- decreases and converges to a positive value $\varepsilon$ \\
- decreases and converges to zero
\end{cases}
\]

Suppose the last is not the case. Then, for sufficiently small $R$,

$\phi'(R)R > \varepsilon > 0$

and if we define

$\varepsilon^* = \text{Min}(\varepsilon, R^*)$

we have
\[ \phi'(R)R > \varepsilon > 0, \text{ for } R \leq \varepsilon \]

Now, define a function \( \phi(R) \) as
\[ \phi(R) = \varepsilon \ln R - \varepsilon \ln \varepsilon + \psi(\varepsilon) \]

Then, since
\[ \phi(\varepsilon) = \psi(\varepsilon) \]
and
\[ \phi'(R) = \varepsilon R^{-1} < \psi'(R) \text{ for } R < \varepsilon \]

It has to be the case that
\[ \psi(R) < \phi(R) \text{ for } R < \varepsilon \]

But as \( R \to 0 \), \( \phi(R) \to -\infty \) thus \( \psi(R) \to -\infty \) too. But on the other hand, \( \phi(0) = 0 \) at (30). This inconsistency shows that such a function does not exist. Q.E.D.

**Appendix 2: Three Models of Heterogeneous Market**

(i) **CPE**

The market demand function is given by
\[ X = nx = K p^{\frac{1}{1-r}} n^{\frac{ar}{r(1-r)}} \]

Thus, the variety elasticity is positively related to the stickiness of people, and inversely proportional to the number of dimension. Parameters are related in the following way:
\[ \alpha = \frac{\delta \tau}{\xi-1}, \quad r = \frac{\xi-1}{\xi} \]
\[ \xi_o = \frac{\xi \delta + \xi + \delta - 1}{2 \delta}, \quad \text{and} \quad \xi_c = \frac{(\xi - 1)(1-\delta)}{2 \delta} \]

(ii). **CES market**

Suppose a utility function of the form of
\[ U(x_0, x_1, \ldots, x_n) = x_0 + (a \sum_{i=1}^{n} x_i^\beta)^r \]

where \( 0 < \beta < 1, \quad r < 1 \), \( x_0 \) : numeraire

\[ x_i : \text{market goods}, \quad a \text{ : constant} \]

The market demand function when \( m \) goods are supplied is
\[ X = nx = (r \beta a^r p^{\frac{1}{1-\beta r}} n^{\frac{r-\beta r}{1-\beta r}}) \]

Since \( r < 1 \), the variety elasticity is a decreasing function of the elasticity of substitution, \( 1/(1-\beta) \), but independent of \( m \). \( \beta \) and \( r \) can be expressed by the following.
\[ \beta = \frac{\xi - 1}{\delta + \xi - 1}, \quad \text{and} \quad r = \frac{\delta + \xi + 1}{\xi} \]
Using these, all the price elasticities are expressed as the following:

$$\xi_o = 1 + \frac{\xi - 1}{\delta} - \frac{(\xi - 1)(1 - \delta)}{n \delta}, \quad \text{and} \quad \xi_c = \frac{(\xi - 1)(1 - \delta)}{n \delta}$$ ...................................(7)

(iii) Quadratic case

With a utility function of the form of

$$U = x_0 + \frac{1}{2} a \sum_{i=1}^{n} x_i^2 + \frac{1}{2} a (1 - \beta) \sum_{i=1}^{n} \sum_{j \neq i} x_i x_j + \alpha \sum_{i=1}^{n} x_i$$ ...................................(8)

The inverse demand function is

$$p_i = \alpha - a x_i - a \sum_{j \neq i} (1 - \beta) x_j$$ ...........................................(9)

The market demand when n goods are supplied symmetrically is

$$X = \sum x = \frac{n(\alpha - p)}{a[n - (n - 1)\beta]}$$ .................................................(10)

Thus, the variety elasticity is given by

$$n^\delta = \frac{n}{n - (n - 1)\beta}$$ ........................................................................(11)

In particular, if \(\beta = 0\), then \(\delta = 0\), and goods are homogeneous each other with a demand function as the following:

$$p_i = \alpha - a \sum x_i = \alpha - a X$$ .................................................................(12)

If \(\beta = 1\), then \(\delta = 1\), and goods are independent from each other, with the individual demand function same as when \(\delta = 0\), i.e.,

$$p_i = \alpha - a x_i$$ .................................................................(13)

As for \(\xi\),

$$\xi = - \frac{dx}{dp} \frac{p}{X} = \frac{p}{\alpha - p}$$ .................................................................(14)

With these, a, \(\alpha\) and \(\beta\) are expressed by the following:

$$a = \frac{n^\delta p}{\xi X}, \quad \alpha = \frac{\xi + 1}{\xi} p, \quad \beta = \frac{n(n^\delta - 1)}{n^\delta (n - 1)}$$ ........................................................................(15)

The price elasticities are as follows:

$$\xi_o = \xi \left[1 + \frac{(n - 1)(1 - \beta)}{\beta}\right], \quad \text{and} \quad \xi_c = \frac{1 - \beta}{\beta}$$ ........................................................................(16)