ON A RAMIFICATION BOUND OF SEMI-STABLE MOD p REPRESENTATIONS OVER A LOCAL FIELD

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Abstract. Let p be a rational prime, k be a perfect field of characteristic p, $W = W(k)$ be the ring of Witt vectors, K be a finite totally ramified extension of $\mathrm{Frac}(W)$ of degree e and r be a non-negative integer satisfying $r < p-1$. In this paper, we prove the upper numbering ramification group $G_K^{(j)}$ for $j > u(K, r)$ acts trivially on the mod p representations associated to the semi-stable p-adic G_K-representations with Hodge-Tate weights in $\{0, \ldots, r\}$, where we put $u(K, 1) = 1+e(1+1/(p-1))$ and $u(K, r) = 1-1/p+e(1+r/(p-1))$ for $r \geq 2$.

1. Introduction

Let p be a rational prime, k be a perfect field of characteristic p, $W = W(k)$ be the ring of Witt vectors and K be a finite totally ramified extension of $K_0 = \mathrm{Frac}(W)$ of degree $e = e(K)$. Let the maximal ideal of K be denoted by m_K, an algebraic closure of K by $\bar{K}$ and the absolute Galois group of K by $G_K = \mathrm{Gal}(\bar{K}/K)$. We normalize the valuation v_K of K as $v_K(p) = e$ and extend this to $\bar{K}$. Let $G_K^{(j)}$ denote the j-th upper numbering ramification group in the sense of [6]. Namely, we put $G_K^{(j)} = G_K^{j-1}$, where the latter is the upper numbering ramification group defined in [12].

Let X_K be a proper smooth scheme over K and put $X_{\bar{K}} = X_K \times_K \bar{K}$. Consider the r-th p-adic étale cohomology group $H^r_{\text{ét}}(X_{\bar{K}}, \mathbb{Q}_p)$ and its G_K-stable $\mathbb{Z}_p$-lattices $\mathcal{L} \supseteq \mathcal{L}'$. In [6], Fontaine conjectured the upper numbering ramification group $G_K^{(j)}$ acts trivially on the G_K-module $\mathcal{L}/\mathcal{L}'$ for $j > e(n + r/(p-1))$ if X_K has good reduction and this module is killed by p^n. For $e = 1$ and $r < p-1$, this conjecture was solved independently by himself ([7], for $n = 1$) and Abrashkin ([1], for any n), using the theory of Fontaine-Laffaille ([8]) and the comparison theorem of Fontaine-Messing ([9]) between the p-adic étale cohomology groups of X_K and the crystalline cohomology groups of the reduction of X_K. From this result, Fontaine also showed some rareness of a proper smooth scheme over $\mathbb{Q}$ with everywhere good reduction ([7, Théorème 1]). In fact, they proved this ramification bound for the torsion representations of the crystalline p-adic representations of G_K with Hodge-Tate weights in $\{0, \ldots, r\}$ in the case where K is absolutely unramified.

On the other hand, as for a semi-stable p-adic representation V with Hodge-Tate weights in the same range, a similar ramification bound for $e = 1$ and $n = 1$ is obtained by Breuil (see [4, Proposition 9.2.2.2]). He showed, assuming Griffiths transversality which in general does not hold, that if $e = 1$ and $r < p-1$, then

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the ramification group $G_K^{(j)}$ acts trivially on the mod p representations of V for $j > 2 + 1/(p-1)$.

In this paper, we prove a version of the result of Breuil for the case where K is absolutely ramified, under the condition $r < p-1$ and $n = 1$. Our main theorem is the following.

Theorem 1.1. *Let r be a non-negative integer such that $r < p-1$. Let V be a semi-stable p-adic G_K-representation with Hodge-Tate weights in $\{0, \dots, r\}$ and $\mathcal{L} \supseteq \mathcal{L}'$ be G_K-stable $\mathbb{Z}_p$-lattices in V. Suppose that the quotient $\mathcal{L}/\mathcal{L}'$ is killed by p. Then the j-th upper numbering ramification group $G_K^{(j)}$ acts trivially on the G_K-module $\mathcal{L}/\mathcal{L}'$ for $j > u(K,r)$, where we put $u(K,1) = 1 + e(1 + 1/(p-1))$ and $u(K,r) = 1 - 1/p + e(1 + r/(p-1))$ for $r \geq 2$.*

Note that this ramification bound is sharp at least for $r = 1$, since the upper bound $1 + e(1 + 1/(p-1))$ of the ramification is obtained by a mod p representation associated to a Tate curve over K.

For the proof of the theorem, we follow the same lines as in [7]. Thanks to Liu's theorem ([11]) on G_K-stable $\mathbb{Z}_p$-lattices in semi-stable p-adic representations, it is enough to bound the ramification of the G_K-module

$$T^*_{\mathrm{st},\underline{\pi}}(\mathcal{M}) = \mathrm{Hom}_{S,\mathrm{Fil}^r,\phi_r,N}(\mathcal{M}, \hat{A}_{\mathrm{st},\infty}),$$

where $\mathcal{M}$ is a p-torsion object of the category $\underline{\mathcal{M}}^r$ of filtered (ϕ_r, N)-modules over S defined by Breuil ([2]) and $\hat{A}_{\mathrm{st},\infty}$ is a p-adic period ring. Put $\tilde{S}_1 = k[u]/(u^{ep}) \cong S/(p, \mathrm{Fil}^p S)$, $\tilde{\mathcal{M}} = \mathcal{M} \otimes_S \tilde{S}_1$, $\pi_1 = \pi^{1/p}$ and $K_1 = K(\pi_1)$. We also have a G_{K_1}-linear bijection

$$T^*_{\mathrm{st},\underline{\pi}}(\mathcal{M})|_{G_{K_1}} \cong T^*_{\mathrm{crys},\pi_1}(\mathcal{M}) = \mathrm{Hom}_{\tilde{S}_1,\mathrm{Fil}^r,\phi_r}(\tilde{\mathcal{M}}, \mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}})$$

with the natural filtered ϕ_r-module structure on $\mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}}$. Then the same argument as in [7] gives a ramification bound of the G_{K_1}-module $T^*_{\mathrm{crys},\pi_1}(\mathcal{M})$ and careful use of [6, Proposition 1.5] shows the theorem.

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2. Filtered (ϕ_r, N)-modules of Breuil

In this section, we recall the theory of filtered (ϕ_r, N)-modules of Breuil, which is developed by himself and most recently by Caruso and Liu. In what follows, we always take the divided power envelope of a W-algebra with the compatibility condition with the natural divided power structure on pW.

Let σ be the Frobenius homomorphism of W. We fix once and for all a uniformizer π of K and a system $\{\pi_n\}_{n \in \mathbb{Z}_{\geq 0}}$ of p-power roots of π such that $\pi_0 = \pi$ and $\pi_n = \pi_{n+1}^p$. Let $E(u)$ be the Eisenstein polynomial of π over W and set $S = (W[u]^{\mathrm{PD}})^\wedge$, where the divided power envelope of $W[u]$ is taken with respect to the ideal $(E(u))$ and $\wedge$ means the p-adic completion. The ring S is endowed with the σ-semilinear map $\phi : u \mapsto u^p$ and the natural filtration induced by the divided power structure. We set $\phi_t = p^{-t}\phi|_{\mathrm{Fil}^t S}$ for any non-negative integer t and

$c = \phi_1(E(u)) \in S^\times$. The W-linear derivation N on S is defined by $N(u) = -u$. We also define a filtration, ϕ, ϕ_t, N on $S_n = S/p^n S$ similarly.

Let $r \in \{0, \ldots, p-2\}$ be an integer. Set $'\underline{\mathcal{M}}^r$ to be the category consisting of the following data:

- an S-module $\mathcal{M}$ and its S-submodule $\mathrm{Fil}^r\mathcal{M}$ containing $\mathrm{Fil}^r S.\mathcal{M}$,
- a ϕ-semilinear map $\phi_r : \mathrm{Fil}^r\mathcal{M} \to \mathcal{M}$ satisfying

$$\phi_r(s_r m) = \phi_r(s_r)\phi(m)$$

  for any $s_r \in \mathrm{Fil}^r S$ and $m \in \mathcal{M}$, where we set $\phi(m) = c^{-r}\phi_r(E(u)^r m)$,
- a W-linear map $N : \mathcal{M} \to \mathcal{M}$ such that
  - $N(sm) = N(s)m + sN(m)$ for any $s \in S$ and $m \in \mathcal{M}$,
  - $E(u)N(\mathrm{Fil}^r\mathcal{M}) \subseteq \mathrm{Fil}^r\mathcal{M}$,
  - the following diagram is commutative:

$$\begin{array}{ccc} \mathrm{Fil}^r\mathcal{M} & \xrightarrow{\phi_r} & \mathcal{M} \\ {\scriptstyle E(u)N}\downarrow & & \downarrow{\scriptstyle cN} \\ \mathrm{Fil}^r\mathcal{M} & \xrightarrow[\phi_r]{} & \mathcal{M}, \end{array}$$

and the morphisms of $'\underline{\mathcal{M}}^r$ are defined to be the S-linear maps preserving Fil^r and commuting with ϕ_r and N. The category defined in the same way but dropping the data N is denoted by $'\underline{\mathcal{M}}_0^r$. These categories have obvious notions of exact sequences. The category $\underline{\mathcal{M}}^r$ (*resp.* $\underline{\mathcal{M}}_0^r$) is defined as the smallest full subcategory of $'\underline{\mathcal{M}}^r$ (*resp.* $'\underline{\mathcal{M}}_0^r$) stable under extensions and containing the objects satisfying the following condition:

- $\mathcal{M}$ is free of finite rank over S_1 and generated as an S_1-module by the image of ϕ_r.

For p-torsion objects, we have the following categories. Consider the algebra $k[u]/(u^{ep}) \cong S_1/\mathrm{Fil}^p S_1$ and let this be denoted by $\tilde{S}_1$. The algebra $\tilde{S}_1$ has the natural filtration, ϕ and N induced by those of S. Namely, $\mathrm{Fil}^t\tilde{S}_1 = u^{et}\tilde{S}_1$, $\phi(u) = u^p$ and $N(u) = -u$. Then the category $'\underline{\tilde{\mathcal{M}}}^r$ consists of the following data:

- an $\tilde{S}_1$-module $\tilde{\mathcal{M}}$ and its $\tilde{S}_1$-submodule $\mathrm{Fil}^r\tilde{\mathcal{M}}$ containing $u^{er}\tilde{\mathcal{M}}$,
- a ϕ-semilinear map $\phi_r : \mathrm{Fil}^r\tilde{\mathcal{M}} \to \tilde{\mathcal{M}}$,
- a k-linear map $N : \tilde{\mathcal{M}} \to \tilde{\mathcal{M}}$ such that
  - $N(sm) = N(s)m + sN(m)$ for any $s \in \tilde{S}_1$ and $m \in \tilde{\mathcal{M}}$,
  - $u^e N(\mathrm{Fil}^r\tilde{\mathcal{M}}) \subseteq \mathrm{Fil}^r\tilde{\mathcal{M}}$,
  - the following diagram is commutative:

$$\begin{array}{ccc} \mathrm{Fil}^r\tilde{\mathcal{M}} & \xrightarrow{\phi_r} & \tilde{\mathcal{M}} \\ {\scriptstyle u^e N}\downarrow & & \downarrow{\scriptstyle cN} \\ \mathrm{Fil}^r\tilde{\mathcal{M}} & \xrightarrow[\phi_r]{} & \tilde{\mathcal{M}}, \end{array}$$

and the morphisms are defined as before. Its full subcategory $\underline{\tilde{\mathcal{M}}}^r$ is defined by the following condition:

- As an $\tilde{S}_1$-module, $\tilde{\mathcal{M}}$ is free of finite rank and generated by the image of ϕ_r.

The categories ${}'\underline{\tilde{\mathcal{M}}}_0^r$ and $\underline{\tilde{\mathcal{M}}}_0^r$ are also defined similarly.

Let D be a weakly admissible filtered (ϕ, N)-module over K satisfying $\mathrm{Fil}^0 D_K = D_K$ and $\mathrm{Fil}^{r+1} D_K = 0$. Set $S_{K_0} = S \otimes_W K_0$ and $\mathcal{D} = D \otimes_{K_0} S_{K_0}$. Then the S_{K_0}-module $\mathcal{D}$ is endowed with the natural ϕ-semilinear map $\phi \otimes \sigma$ and K_0-linear derivation $N \otimes 1 + 1 \otimes N$, which are denoted by ϕ and N, respectively. The filtration on $\mathcal{D}$ is defined inductively by $\mathrm{Fil}^0 \mathcal{D} = \mathcal{D}$ and

$$\mathrm{Fil}^{i+1}\mathcal{D} = \{x \in \mathcal{D} \mid N(x) \in \mathrm{Fil}^i \mathcal{D} \text{ and } f_\pi(x) \in \mathrm{Fil}^{i+1} D_K\},$$

where $f_\pi : \mathcal{D} \to D_K$ is induced by the map $S \to \mathcal{O}_K$ sending u to π. An S-submodule $\hat{\mathcal{M}}$ of $\mathcal{D}$ is said to be a strongly divisible lattice of $\mathcal{D}$ if the following conditions are satisfied:

- the S-module $\hat{\mathcal{M}}$ is free of finite rank,
- $\hat{\mathcal{M}} \otimes_W K_0 = \mathcal{D}$,
- $\hat{\mathcal{M}}$ is stable under ϕ and N,
- $\phi(\mathrm{Fil}^r \hat{\mathcal{M}}) \subseteq p^r \hat{\mathcal{M}}$, where we set $\mathrm{Fil}^r \hat{\mathcal{M}} = \hat{\mathcal{M}} \cap \mathrm{Fil}^r \mathcal{D}$.

We put $\phi_r = p^{-r}\phi|_{\mathrm{Fil}^r \hat{\mathcal{M}}}$. Then the S-module $\hat{\mathcal{M}}$ is generated by $\phi_r(\mathrm{Fil}^r \hat{\mathcal{M}})$ ([2, Proposition 2.1.3]).

Let A_{crys} and $\hat{A}_{\mathrm{st}}$ be p-adic period rings. These are constructed as follows. Set

$$R = \varprojlim (\mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}} \leftarrow \mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}} \leftarrow \cdots),$$

where every arrow is the p-power map. For an element $x = (x_i)_{i \in \mathbb{Z}_{\geq 1}} \in R$, we set

$$x^{(n)} = \lim_{m \to \infty} \hat{x}_{n+m}^{p^m} \in \mathcal{O}_{\mathbb{C}} \text{ for } n \geq 0,$$

where $\hat{x}_i$ is a lift of x_i in $\mathcal{O}_{\bar{K}}$. The natural ring homomorphism θ is defined by

$$\begin{aligned} \theta : W(R) &\to \mathcal{O}_{\mathbb{C}} \\ (r_0, r_1, \ldots) &\mapsto \sum_{n \geq 0} p^n r_n^{(n)}. \end{aligned}$$

Then A_{crys} is the p-adic completion of the divided power envelope of $W(R)$ with respect to $\mathrm{Ker}(\theta)$ and $\hat{A}_{\mathrm{st}}$ is the p-adic completion of the divided power polynomial ring $A_{\mathrm{crys}}\langle X \rangle$ over A_{crys}. We set $A_{\mathrm{crys},\infty} = A_{\mathrm{crys}} \otimes_W K_0/W$ and $A_{\mathrm{st},\infty} = \hat{A}_{\mathrm{st}} \otimes_W K_0/W$. Put $\underline{\pi} = (\pi_1 \bmod p, \pi_2 \bmod p, \ldots) \in R$. These rings are considered as S-modules by the ring homomorphisms $S \to \hat{A}_{\mathrm{st}}$ and $\hat{A}_{\mathrm{st}} \to A_{\mathrm{crys}}$ which are defined by $u \mapsto [\underline{\pi}]/(1+X)$ and $X \mapsto 0$, respectively. The ring A_{crys} is endowed with the natural filtration induced by the divided power structure, the natural Frobenius ϕ and the ϕ-semilinear map $\phi_t = p^{-t}\phi|_{\mathrm{Fil}^t A_{\mathrm{crys}}}$. With these structures, A_{crys} and $A_{\mathrm{crys},\infty}$ are considered as objects of ${}'\underline{\mathcal{M}}_0^r$. Moreover, the absolute Galois group G_K acts naturally on these two rings. As for $\hat{A}_{\mathrm{st}}$, its filtration is defined by

$$\mathrm{Fil}^t \hat{A}_{\mathrm{st}} = \left\{ \sum_{i \geq 0} a_i \frac{X^i}{i!} \;\middle|\; a_i \in \mathrm{Fil}^{t-i} A_{\mathrm{crys}},\ \lim_{i \to \infty} a_i = 0 \right\}$$

and the Frobenius structure of A_{crys} extends to $\hat{A}_{\mathrm{st}}$ by

$$\begin{aligned} \phi(X) &= (1+X)^p - 1, \\ \phi_t &= p^{-t}\phi|_{\mathrm{Fil}^t \hat{A}_{\mathrm{st}}}. \end{aligned}$$

The A_{crys}-linear derivation N on $\hat{A}_{\mathrm{st}}$ is defined by $N(X) = 1 + X$. The rings $\hat{A}_{\mathrm{st}}$ and $\hat{A}_{\mathrm{st},\infty}$ are objects of $'\underline{\mathcal{M}}^r$. The G_K-action on A_{crys} naturally extends to the action on $\hat{A}_{\mathrm{st}}$. Indeed, the action of $g \in G_K$ on $\hat{A}_{\mathrm{st}}$ is defined by the formula

$$g(X) = [\underline{\varepsilon}(g)](X + 1) - 1,$$

where $g(\pi_n) = \varepsilon_n(g)\pi_n$ and $\underline{\varepsilon}(g) = (\varepsilon_1(g) \bmod p, \varepsilon_2(g) \bmod p, \dots) \in R$.

Put $K_n = K(\pi_n)$ and $K_\infty = \cup_n K_n$. For $\mathcal{M} \in \underline{\mathcal{M}}^r$, the G_K-module $T^*_{\mathrm{st},\underline{\pi}}(\mathcal{M})$ and G_{K_∞}-module $T^*_{\mathrm{crys},\underline{\pi}}(\mathcal{M})$ are defined to be

$$T^*_{\mathrm{st},\underline{\pi}}(\mathcal{M}) = \mathrm{Hom}_{S,\mathrm{Fil}^r,\phi_r,N}(\mathcal{M}, \hat{A}_{\mathrm{st},\infty}),$$
$$T^*_{\mathrm{crys},\underline{\pi}}(\mathcal{M}) = \mathrm{Hom}_{S,\mathrm{Fil}^r,\phi_r}(\mathcal{M}, A_{\mathrm{crys},\infty}).$$

Then we see as in the proof of [2, Lemme 2.3.1.1] that the natural map

$$T^*_{\mathrm{st},\underline{\pi}}(\mathcal{M}) \to T^*_{\mathrm{crys},\underline{\pi}}(\mathcal{M})$$

is bijective and G_{K_∞}-linear.

For p-torsion objects in $\underline{\mathcal{M}}^r$ and $\underline{\mathcal{M}}^r_0$, we have a simpler description of the functors $T^*_{\mathrm{st},\underline{\pi}}$ and $T^*_{\mathrm{crys},\underline{\pi}}$ ([5]). We consider $\hat{A} = (\mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}})\langle X\rangle$ as an object of $'\underline{\tilde{\mathcal{M}}}^r$ by the natural surjection

$$\hat{A}_{\mathrm{st}}/p\hat{A}_{\mathrm{st}} \to \hat{A}_{\mathrm{st}}/(p, \mathrm{Fil}^p A_{\mathrm{crys}}) \cong \hat{A},$$

where the last isomorphism is defined by $X \mapsto X$. This surjection also induces the natural G_K-action on $\hat{A}$. Similarly, the algebra $\mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}}$ is considered as an object of $'\underline{\tilde{\mathcal{M}}}^r_0$. The filtration of $\mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}}$ is given by

$$\mathrm{Fil}^r(\mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}}) = \{\hat{x} \in \mathcal{O}_{\bar{K}} |\ v_K(\hat{x}) \geq \frac{er}{p}\}/p\mathcal{O}_{\bar{K}}$$

and the Frobenius structure by

$$\phi_r(x) = \frac{\hat{x}^p}{(-p)^r} \bmod p,$$

where $\hat{x}$ denotes a lift of x in $\mathcal{O}_{\bar{K}}$. For a p-torsion object $\mathcal{M} \in \underline{\mathcal{M}}^r$, we set $T(\mathcal{M}) = \mathcal{M} \otimes_S \tilde{S}_1$. This $\tilde{S}_1$-module is naturally considered as an object of $\underline{\tilde{\mathcal{M}}}^r$. By [5, Lemme 2.3.4], we have a G_K-linear isomorphism

$$T^*_{\mathrm{st},\underline{\pi}}(\mathcal{M}) \to \mathrm{Hom}_{\tilde{S}_1,\mathrm{Fil}^r,\phi_r,N}(T(\mathcal{M}), \hat{A}).$$

Similarly, for a p-torsion object $\mathcal{M} \in \underline{\mathcal{M}}^r_0$, the $\tilde{S}_1$-module $T(\mathcal{M}) = \mathcal{M} \otimes_S \tilde{S}_1$ has a natural structure as an object of $\underline{\tilde{\mathcal{M}}}^r_0$. Then we have a G_{K_∞}-linear isomorphism

$$T^*_{\mathrm{crys},\underline{\pi}}(\mathcal{M}) \to \mathrm{Hom}_{\tilde{S}_1,\mathrm{Fil}^r,\phi_r}(T(\mathcal{M}), \mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}}),$$

where the module on the right-hand side is in fact a G_{K_1}-module. By definition, the action of $g \in G_K$ on $\hat{A}$ is defined by

$$g(X) = \varepsilon_1(g)(1 + X) - 1.$$

Thus we see that, for a p-torsion object $\mathcal{M} \in \underline{\mathcal{M}}^r$, the natural map

$$\mathrm{Hom}_{\tilde{S}_1,\mathrm{Fil}^r,\phi_r,N}(T(\mathcal{M}), \hat{A}) \to \mathrm{Hom}_{\tilde{S}_1,\mathrm{Fil}^r,\phi_r}(T(\mathcal{M}), \mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}})$$

is bijective and G_{K_1}-linear. Both sides are independent of the choice of π_n $(n \geq 2)$. We refer these modules as $T^*_{\mathrm{st},\pi_1}(\mathcal{M})$ and $T^*_{\mathrm{crys},\pi_1}(\mathcal{M})$.

Finally, let D and $\mathcal{D}$ be as above and $\hat{\mathcal{M}}$ be a strongly divisible lattice in $\mathcal{D}$. Then the G_K-module

$$\hat{T}^*_{\mathrm{st},\underline{\pi}}(\hat{\mathcal{M}}) = \mathrm{Hom}_{S,\mathrm{Fil}^r,\phi_r,N}(\hat{\mathcal{M}}, \hat{A}_{\mathrm{st}})$$

is naturally considered as a G_K-stable $\mathbb{Z}_p$-lattice in $V^*_{\mathrm{st}}(D)$. By Liu's theorem ([11, Theorem 2.3.5]), the functor $\hat{T}^*_{\mathrm{st},\underline{\pi}}$ gives an anti-equivalence of categories between the category of strongly divisible lattices in $\mathcal{D}$ and the category of G_K-stable $\mathbb{Z}_p$-lattices in $V^*_{\mathrm{st}}(D)$. The S-module $\mathcal{M} = \hat{\mathcal{M}}/p^n\hat{\mathcal{M}}$ has a natural structure as an object of $\underline{\mathcal{M}}^r$. There is an exact sequence of G_K-modules

$$0 \longrightarrow \hat{T}^*_{\mathrm{st},\underline{\pi}}(\hat{\mathcal{M}}) \xrightarrow{p^n} \hat{T}^*_{\mathrm{st},\underline{\pi}}(\hat{\mathcal{M}}) \longrightarrow T^*_{\mathrm{st},\underline{\pi}}(\mathcal{M}) \longrightarrow 0.$$

3. Galois action on $T^*_{\mathrm{crys},\pi_1}(\mathcal{M})$ over K_1

Let us fix a p-th root π_1 of π and set $K_1 = K(\pi_1)$, as before. For an algebraic extension F of K, we put

$$\mathfrak{b}_F = \{x \in \mathcal{O}_F \mid v_K(x) > \frac{er}{p-1}\}.$$

Let E be an algebraic extension of K_1. We define on the $\mathcal{O}_{K_1}/p\mathcal{O}_{K_1}$-algebra $\mathcal{O}_E/\mathfrak{b}_E$ a structure of a filtered ϕ_r-module over $\tilde{S}_1$, as follows. We consider the algebra $\mathcal{O}_E/\mathfrak{b}_E$ as an $\tilde{S}_1$-algebra by $u \mapsto \pi_1$. Define an $\tilde{S}_1$-submodule $\mathrm{Fil}^r(\mathcal{O}_E/\mathfrak{b}_E)$ of $\mathcal{O}_E/\mathfrak{b}_E$ by

$$\mathrm{Fil}^r(\mathcal{O}_E/\mathfrak{b}_E) = u^{er}(\mathcal{O}_E/\mathfrak{b}_E) = \{\hat{x} \in \mathcal{O}_E \mid v_K(\hat{x}) \geq \frac{er}{p}\}/\mathfrak{b}_E$$

and a ϕ-semilinear map ϕ_r by

$$\begin{aligned} \phi_r : \mathrm{Fil}^r(\mathcal{O}_E/\mathfrak{b}_E) &\to \mathcal{O}_E/\mathfrak{b}_E \\ x &\mapsto \frac{\hat{x}^p}{(-p)^r} \bmod \mathfrak{b}_E, \end{aligned}$$

where $\hat{x}$ is a lift of x in $\mathcal{O}_E$. We see that ϕ_r is independent of the choice of a lift $\hat{x}$ and ϕ-semilinear.

Let $\mathcal{M}$ be a p-torsion object in $\underline{\mathcal{M}}^r$. Put $\tilde{\mathcal{M}} = T(\mathcal{M})$ and

$$T^*_{\mathrm{crys},\pi_1,E}(\mathcal{M}) = \mathrm{Hom}_{\tilde{S}_1,\mathrm{Fil}^r,\phi_r}(\tilde{\mathcal{M}}, \mathcal{O}_E/\mathfrak{b}_E).$$

For finite extensions $E \subseteq E'$ of K_1, we have a natural injection of filtered ϕ_r-modules over $\tilde{S}_1$

$$\mathcal{O}_E/\mathfrak{b}_E \to \mathcal{O}_{E'}/\mathfrak{b}_{E'}$$

and this induces an injection of abelian groups

$$T^*_{\mathrm{crys},\pi_1,E}(\mathcal{M}) \to T^*_{\mathrm{crys},\pi_1,E'}(\mathcal{M}).$$

Thus we have a natural identification of abelian groups

$$T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M}) = \bigcup_{E/K_1:\text{ finite}} T^*_{\mathrm{crys},\pi_1,E}(\mathcal{M}).$$

Take an adapted basis $e_1, \ldots, e_d$ of $\tilde{\mathcal{M}}$ ([2, Proposition 2.2.1.3]) such that

$$\begin{aligned} \tilde{\mathcal{M}} &= \tilde{S}_1 e_1 \oplus \cdots \oplus \tilde{S}_1 e_d \\ \mathrm{Fil}^r \tilde{\mathcal{M}} &= u^{r_1}\tilde{S}_1 e_1 \oplus \cdots \oplus u^{r_d}\tilde{S}_1 e_d \end{aligned}$$

with some integers $r_1, \ldots, r_d$ satisfying $0 \leq r_i \leq er$ for any i. Define $\mathcal{G}(u) \in GL_d(\tilde{S}_1)$ by

$$\phi_r \begin{pmatrix} u^{r_1} e_1 \\ \vdots \\ u^{r_d} e_d \end{pmatrix} = \mathcal{G}(u) \begin{pmatrix} e_1 \\ \vdots \\ e_d \end{pmatrix}$$

and choose its lift $\hat{\mathcal{G}}(u) \in GL_d(W[[u]])$. For an ideal $I \subseteq \mathfrak{b}_E$, consider the equation in $\mathcal{O}_E/I$

$$\begin{pmatrix} \frac{\pi^{r_1}}{(-p)^r} \hat{x}_1^p \\ \vdots \\ \frac{\pi^{r_d}}{(-p)^r} \hat{x}_d^p \end{pmatrix} \bmod I = \hat{\mathcal{G}}(\pi_1) \begin{pmatrix} x_1 \\ \vdots \\ x_d \end{pmatrix}, \text{ with } x_i \in \pi_1^{er-r_i}(\mathcal{O}_E/I). \tag{1}$$

Here $\hat{x}_i$ denotes a lift of x_i in $\mathcal{O}_E$ and this equation is independent of the choice of these lifts. The elements of

$$T^*_{\mathrm{crys},\pi_1,E}(\mathcal{M}) = \mathrm{Hom}_{\tilde{S}_1,\mathrm{Fil}^r,\phi_r}(\tilde{\mathcal{M}}, \mathcal{O}_E/\mathfrak{b}_E)$$

correspond bijectively to the solutions of this equation for $I = \mathfrak{b}_E$.

Lemma 3.1. *Let E be a finite extension of K_1 and $l > e(E)r/(p-1)$. Then, every solution $(x_1, \ldots, x_d)$ of the equation (1) for $I = m_E^l$ lifts uniquely to a solution of this equation for $I = m_E^{l+1}$.*

Proof. Let $(x_1, \ldots, x_d)$ be such a solution and take a lift $\hat{x}_i$ of x_i in $\mathcal{O}_E$. Let π_E denote a uniformizer of E. Then, for some $c_1, \ldots, c_d \in \mathcal{O}_E$, we have

$$\begin{pmatrix} \frac{\pi^{r_1}}{(-p)^r} \hat{x}_1^p \\ \vdots \\ \frac{\pi^{r_d}}{(-p)^r} \hat{x}_d^p \end{pmatrix} = \hat{\mathcal{G}}(\pi_1) \begin{pmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_d \end{pmatrix} + \pi_E^l \begin{pmatrix} c_1 \\ \vdots \\ c_d \end{pmatrix}.$$

For $\hat{y}_1, \ldots, \hat{y}_d \in \mathcal{O}_E$, consider the equation

$$\begin{pmatrix} \frac{\pi^{r_1}}{(-p)^r} (\hat{x}_1 + \pi_E^l \hat{y}_1)^p \\ \vdots \\ \frac{\pi^{r_d}}{(-p)^r} (\hat{x}_d + \pi_E^l \hat{y}_d)^p \end{pmatrix} \equiv \hat{\mathcal{G}}(\pi_1) \begin{pmatrix} \hat{x}_1 + \pi_E^l \hat{y}_1 \\ \vdots \\ \hat{x}_d + \pi_E^l \hat{y}_d \end{pmatrix} \bmod \pi_E^{l+1}.$$

This is equivalent to

$$\begin{pmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_d \end{pmatrix} \equiv \hat{\mathcal{G}}(\pi_1)^{-1} \left\{ \begin{pmatrix} c_1 \\ \vdots \\ c_d \end{pmatrix} + \begin{pmatrix} \frac{\pi^{r_1}}{(-p)^r} (\sum_{t=1}^p \binom{p}{t} \hat{x}_1^{p-t} \pi_E^{l(t-1)} \hat{y}_1^t) \\ \vdots \\ \frac{\pi^{r_d}}{(-p)^r} (\sum_{t=1}^p \binom{p}{t} \hat{x}_d^{p-t} \pi_E^{l(t-1)} \hat{y}_d^t) \end{pmatrix} \right\} \bmod \pi_E,$$

where $\binom{p}{t}$ denotes the binomial coefficient $p!/(t!(p-t)!)$.

We claim that every entry of the last term of this equation is a polynomial of $\hat{y}_1, \ldots, \hat{y}_d$ whose coefficients are in the maximal ideal m_E. Indeed, the coefficient of $\hat{y}_i^p$

$$\frac{\pi^{r_i}}{(-p)^r} \pi_E^{l(p-1)}$$

has the valuation

$$e(E/K)r_i - e(E)r + l(p-1) > 0.$$

Similarly, the coefficient of $\hat{y}_i^t$ $(1 \le t \le p-1)$

$$\frac{\pi^{r_i}}{(-p)^r}\binom{p}{t}\hat{x}_i^{p-t}\pi_E^{l(t-1)}$$

has the valuation

$$\begin{aligned}&e(E/K)r_i - e(E)(r-1) + (p-t)v_E(\hat{x}_i) + l(t-1)\\&= e(E/K)r_i - e(E)(r-1) + pv_E(\hat{x}_i) - l + t(l - v_E(\hat{x}_i)).\end{aligned}$$

If $l \ge v_E(\hat{x}_i)$, then the minimum of this value for $t = 1, \dots, p-1$ is obtained by $t = 1$. This minimum value is equal to

$$\begin{aligned}&e(E/K)r_i - e(E)(r-1) + (p-1)v_E(\hat{x}_i)\\&\ge \frac{e(E)(p-r)}{p} + \frac{e(E/K)r_i}{p} > 0.\end{aligned}$$

If $l < v_E(\hat{x}_i)$, then the minimum is obtained by $t = p-1$ and is equal to

$$\begin{aligned}&e(E/K)r_i - e(E)(r-1) + (p-2)l + v_E(\hat{x}_i)\\&> e(E/K)r_i - e(E)(r-1) + \frac{e(E)r(p-2)}{p-1} + v_E(\hat{x}_i)\\&= e(E/K)r_i + e(E)(1 - \frac{r}{p-1}) + v_E(\hat{x}_i) > 0.\end{aligned}$$

Thus we can solve uniquely this equation. □

Corollary 3.2. *There is a G_{K_1}-linear isomorphism*

$$T^*_{\mathrm{crys},\pi_1}(\mathcal{M}) \cong T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M}).$$

In particular, we have $\# T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M}) = p^d$ for $d = \dim_{S_1} \mathcal{M}$.

Proof. Applying Lemma 3.1 for a sufficiently large finite extension E of K_1, we see that the natural map

$$T^*_{\mathrm{crys},\pi_1}(\mathcal{M}) \to T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M})$$

induced by $\mathcal{O}_{\bar{K}}/p\mathcal{O}_{\bar{K}} \to \mathcal{O}_{\bar{K}}/\mathfrak{b}_{\bar{K}}$ is bijective. The last assertion follows from [2, Lemme 2.3.1.2]. □

Lemma 3.3. *The fixed part $T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M})^{G_E}$ is equal to $T^*_{\mathrm{crys},\pi_1,E}(\mathcal{M})$.*

Proof. From Lemma 3.1, we see that the elements of $T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M})$ correspond bijectively to the solutions of the equation (1) for $I = 0$ in $\mathcal{O}_{\bar{K}}$. The uniqueness of the lift shows that $g \in G_{K_1}$ fixes a solution in $\mathcal{O}_{\bar{K}}$ if and only if g fixes its image in $\mathcal{O}_{\bar{K}}/\mathfrak{b}_{\bar{K}}$. This concludes the proof. □

Corollary 3.4. *Let L_1 be the finite Galois extension of K_1 corresponding to the kernel of the map $G_{K_1} \to \mathrm{Aut}(T^*_{\mathrm{crys},\pi_1}(\mathcal{M}))$. Then an algebraic extension E of K_1 contains L_1 if and only if $\# T^*_{\mathrm{crys},\pi_1,E}(\mathcal{M}) = p^d$, where $d = \dim_{S_1} \mathcal{M}$.*

Proof. An algebraic extension E of K_1 contains L_1 if and only if the action of G_E on $T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M})$ is trivial. By Lemma 3.3, this is equivalent to $T^*_{\mathrm{crys},\pi_1,\bar{K}}(\mathcal{M}) = T^*_{\mathrm{crys},\pi_1,E}(\mathcal{M})$. Then the corollary follows. □

4. Ramification bound

In this section, we prove Theorem 1.1. Take G_K-stable $\mathbb{Z}_p$-lattices $\mathcal{L} \supseteq \mathcal{L}'$ in V such that $\mathcal{L}' \supseteq p\mathcal{L}$. Since the G_K-module $\mathcal{L}/\mathcal{L}'$ is a quotient of $\mathcal{L}/p\mathcal{L}$, we may assume $\mathcal{L}' = p\mathcal{L}$. For $r = 0$, we see that the G_K-module V is unramified and the theorem is trivial. Thus we may assume $r > 0$. Then, by Liu's theorem ([11, Theorem 2.3.5]), it suffices to show the following.

Theorem 4.1. *Let $r > 0$ be an integer with $r < p-1$. For $\mathcal{M} \in \underline{\mathcal{M}}^r$ which is killed by p, $G_K^{(j)}$ acts trivially on $T^*_{\mathrm{st},\pi_1}(\mathcal{M})$ for $j > u(K,r)$.*

Let L and L_1 be the finite Galois extensions of K and K_1 corresponding to the kernels of the maps

$$G_K \to \mathrm{Aut}(T^*_{\mathrm{st},\pi_1}(\mathcal{M})) \text{ and } G_{K_1} \to \mathrm{Aut}(T^*_{\mathrm{st},\pi_1}(\mathcal{M})),$$

respectively. Similarly, put $\tilde{K}_1 = K_1(\zeta_p)$ and let $\tilde{L}_1$ denote the finite extension of $\tilde{K}_1$ corresponding to the kernel of the map

$$G_{\tilde{K}_1} \to \mathrm{Aut}(T^*_{\mathrm{st},\pi_1}(\mathcal{M})).$$

Then we have $L_1 = LK_1$ and $\tilde{L}_1 = L\tilde{K}_1 = L_1\tilde{K}_1$, where the latter is a Galois extension of K. In the following, we bound the ramification of $\tilde{L}_1$ over K.

For an algebraic extension E of K, we put

$$\mathfrak{a}^m_{E/K} = \{x \in \mathcal{O}_E \mid v_K(x) \geq m\}.$$

Proposition 4.2. *Let E be an algebraic extension of K. If $m > u(K,r)$ and there exists an $\mathcal{O}_K$-algebra homomorphism*

$$\eta : \mathcal{O}_{\tilde{L}_1} \to \mathcal{O}_E/\mathfrak{a}^m_{E/K},$$

then there exists a K-algebra injection $\tilde{L}_1 \to E$.

Proof. We mimic the proof of [7, Théorème 2]. By assumption, we have $m > er/(p-1)$ and $\mathfrak{b}_E \supseteq \mathfrak{a}^m_{E/K}$. Thus η induces an $\mathcal{O}_K$-algebra homomorphism

$$\mathcal{O}_{\tilde{L}_1} \to \mathcal{O}_E/\mathfrak{b}_E.$$

Note that $\tilde{K}_1 = K_1(\zeta_p)$ is a Galois extension of K and its greatest upper ramification break $u_{\tilde{K}_1/K}$ ([6]) is

$$u_{\tilde{K}_1/K} = 1 + e(1 + \frac{1}{p-1}) < m.$$

As η induces an $\mathcal{O}_K$-algebra homomorphism $\mathcal{O}_{\tilde{K}_1} \to \mathcal{O}_E/\mathfrak{a}^m_{E/K}$, from [6, Proposition 1.5] we get a K-linear injection $\tilde{K}_1 \to E$. Thus we see that E contains π_1 and ζ_p. More precisely, we have the following lemma.

Lemma 4.3. $\eta(\pi_1) \equiv \pi_1\zeta_p^i \bmod \mathfrak{b}_E$ *for some i.*

Proof. As η is $\mathcal{O}_K$-linear, the equality $\eta(\pi_1)^p = \pi$ holds in $\mathcal{O}_E/\mathfrak{a}^m_{E/K}$. Set $\hat{x}$ to be a lift of $\eta(\pi_1)$ in $\mathcal{O}_E$. Then we have

$$v_K(\hat{x}^p - \pi) = \sum_{i=0}^{p-1} v_K(\hat{x} - \pi_1\zeta_p^i) \geq m.$$

Take i such that $v_K(\hat{x}-\pi_1\zeta_p^i)\geq v_K(\hat{x}-\pi_1\zeta_p^{i'})$ holds for any i'. From the equality

$$v_K(\hat{x}-\pi_1\zeta_p^i)=v_K(\hat{x}-\pi_1\zeta_p^{i'}+\pi_1(\zeta_p^{i'}-\zeta_p^i)),$$

we see that $v_K(\hat{x}-\pi_1\zeta_p^{i'})\leq 1/p+e/(p-1)$ and

$$v_K(\hat{x}-\pi_1\zeta_p^i)\geq m-e-\frac{p-1}{p}.$$

Since

$$m-e-\frac{p-1}{p}>\frac{er}{p-1}$$

by assumption, we have $\hat{x}\equiv\pi_1\zeta_p^i \bmod \mathfrak{b}_E$ and the lemma follows. □

Lemma 4.4. *The $\mathcal{O}_K$-algebra homomorphism η induces an $\mathcal{O}_K$-algebra injection*

$$\bar{\eta}:\mathcal{O}_{\tilde{L}_1}/\mathfrak{b}_{\tilde{L}_1}\to\mathcal{O}_E/\mathfrak{b}_E.$$

Proof. We write the Eisenstein polynomial of a uniformizer $\pi_{\tilde{L}_1}$ of $\tilde{L}_1$ over $\mathcal{O}_K$ as

$$P(T)=T^{\tilde{e}}+c_1T^{\tilde{e}-1}+\cdots+c_{\tilde{e}-1}T+c_{\tilde{e}},$$

where $\tilde{e}=e(\tilde{L}_1/K)$. Then $t=\eta(\pi_{\tilde{L}_1})$ satisfies

$$t^{\tilde{e}}=-(c_1t^{\tilde{e}-1}+\cdots+c_{\tilde{e}-1}t+c_{\tilde{e}})$$

in $\mathcal{O}_E/\mathfrak{a}_{E/K}^m$. Let $\hat{t}$ be a lift of t in $\mathcal{O}_E$. Since $m>1$, we have $v_E(\hat{t})=e(E/K)/\tilde{e}$. The condition $n>e(\tilde{L}_1)r/(p-1)$ is equivalent to the condition

$$v_E(\hat{t}^n)>\frac{e(\tilde{L}_1)r}{p-1}\cdot\frac{e(E/K)}{\tilde{e}}=\frac{e(E)r}{p-1}.$$

Thus the lemma follows. □

We give the $\mathcal{O}_K$-algebras $\mathcal{O}_{\tilde{L}_1}/\mathfrak{b}_{\tilde{L}_1}$ and $\mathcal{O}_E/\mathfrak{b}_E$ natural structures of filtered ϕ_r-modules over $\tilde{S}_1$ as follows. The $\mathcal{O}_K$-algebra $\mathcal{O}_{\tilde{L}_1}/\mathfrak{b}_{\tilde{L}_1}$ is considered as an $\tilde{S}_1$-module by $u\mapsto\pi_1$. Put

$$\mathrm{Fil}^r(\mathcal{O}_{\tilde{L}_1}/\mathfrak{b}_{\tilde{L}_1})=\{\hat{x}\in\mathcal{O}_{\tilde{L}_1}\mid v_K(\hat{x})\geq\frac{er}{p}\}/\mathfrak{b}_{\tilde{L}_1}$$

and for $x\in\mathrm{Fil}^r(\mathcal{O}_{\tilde{L}_1}/\mathfrak{b}_{\tilde{L}_1})$, set

$$\phi_r(x)=\frac{\hat{x}^p}{(-p)^r} \bmod \mathfrak{b}_{\tilde{L}_1},$$

as in Section 3. On the other hand, we consider $\mathcal{O}_E/\mathfrak{b}_E$ as an $\tilde{S}_1$-module by $u\mapsto\pi_1\zeta_p^i$ and define Fil^r and ϕ_r of $\mathcal{O}_E/\mathfrak{b}_E$ in the same way as $\mathcal{O}_{\tilde{L}_1}/\mathfrak{b}_{\tilde{L}_1}$. From the definition, we see that $\bar{\eta}$ is $\tilde{S}_1$-linear. We can also check that $\bar{\eta}$ preserves Fil^r and ϕ_r of both sides, just as in the proof of [7, Théorème 2].

Then the injection $\bar{\eta}$ induces an injection of abelian groups

$$T^*_{\mathrm{crys},\tilde{L}_1,\pi_1}(\mathcal{M})\to T^*_{\mathrm{crys},E,\pi_1\zeta_p^i}(\mathcal{M}).$$

Since $\tilde{L}_1$ contains L_1, the abelian group $T^*_{\mathrm{crys},\tilde{L}_1,\pi_1}(\mathcal{M})$ is of order p^d by Corollary 3.4, where $d=\dim_{S_1}\mathcal{M}$. This implies $\#T^*_{\mathrm{crys},E,\pi_1\zeta_p^i}(\mathcal{M})=p^d$. Let L_1' denote the finite Galois extension of $K_1'=K(\pi_1\zeta_p^i)$ corresponding to the kernel of the map

$$G_{K_1'}\to\mathrm{Aut}(T^*_{\mathrm{crys},E,\pi_1\zeta_p^i}(\mathcal{M})).$$

Then, again by Corollary 3.4, we see that $E \supseteq L'_1$. However, by definition, the finite extension L'_1 is the conjugate field $g(L_1)$ of L_1 over K by some element $g \in G_K$. Since $\tilde{L}_1 = L_1\tilde{K}_1$ is Galois over K, the Galois group $G_{L'_1\tilde{K}_1} = G_{L'_1} \cap G_{\tilde{K}_1}$ is equal to

$$gG_{L_1}g^{-1} \cap G_{\tilde{K}_1} = g(G_{L_1} \cap G_{\tilde{K}_1})g^{-1} = gG_{\tilde{L}_1}g^{-1} = G_{\tilde{L}_1}.$$

Hence $\tilde{L}_1$ is equal to $L'_1\tilde{K}_1$. Since E contains $\tilde{K}_1$, we conclude that E also contains $\tilde{L}_1$. □

Corollary 4.5. *we have the inequality*

$$u_{L/K} \leq u_{\tilde{L}_1/K} \leq u(K,r).$$

Proof. This follows from [10, Proposition 5.6]. □

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