2022.11.30 Lecture 1

13:30 – 15:00

量子ドット：閉じ込めと開放の物理学

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1. 量子ドットとは，量子ドットの製法
2. 単電子効果
3. 量子閉じ込め効果
4. 量子ドットを通した電気伝導

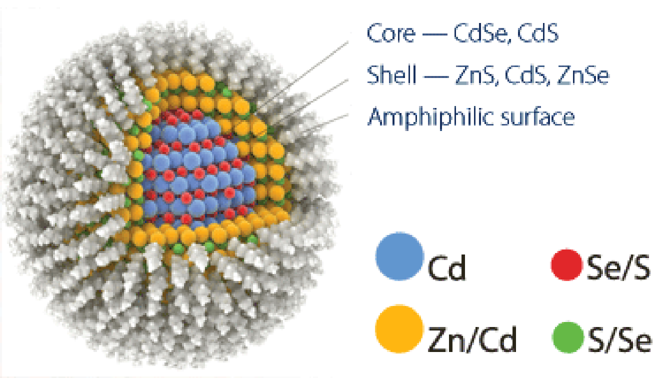
量子ドットとは

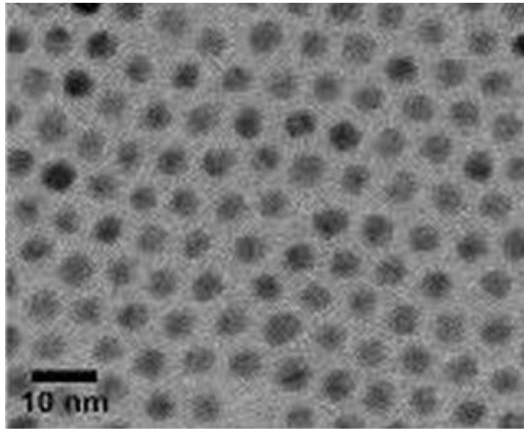
量子ドット (quantum dot)：電子系を3次元的に閉じ込めたもの (0次元系)

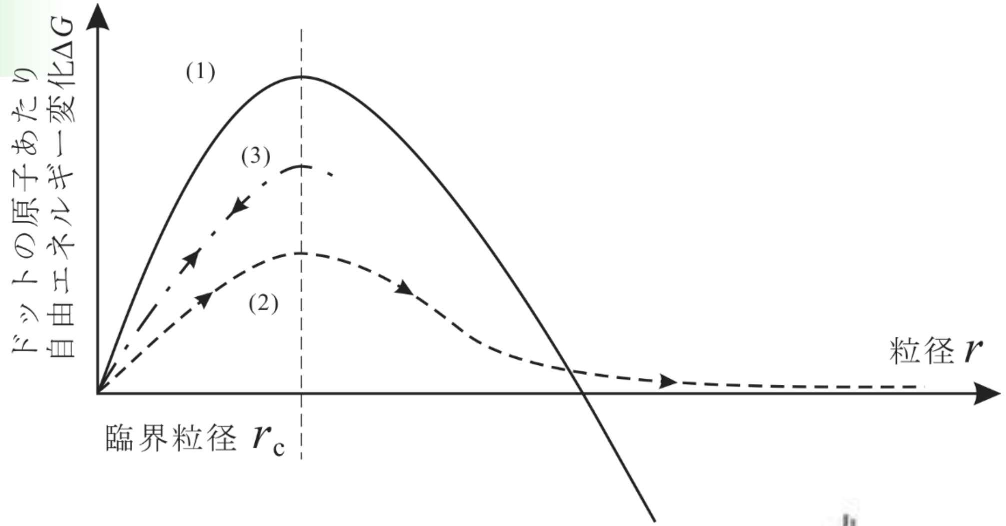
電子系の性質による分類
- 金属量子ドット (金属超微粒子)
- 半導体量子ドット

製法による分類
- 天然型
  - 自己集積　真空蒸着，ガス中蒸着
  - コロイド　飽和液→析出
- 人工型
  - 形状加工
    - リソグラフィー
    - やすり掛け，機械的製法
  - 電気的　金属ゲート→半導体

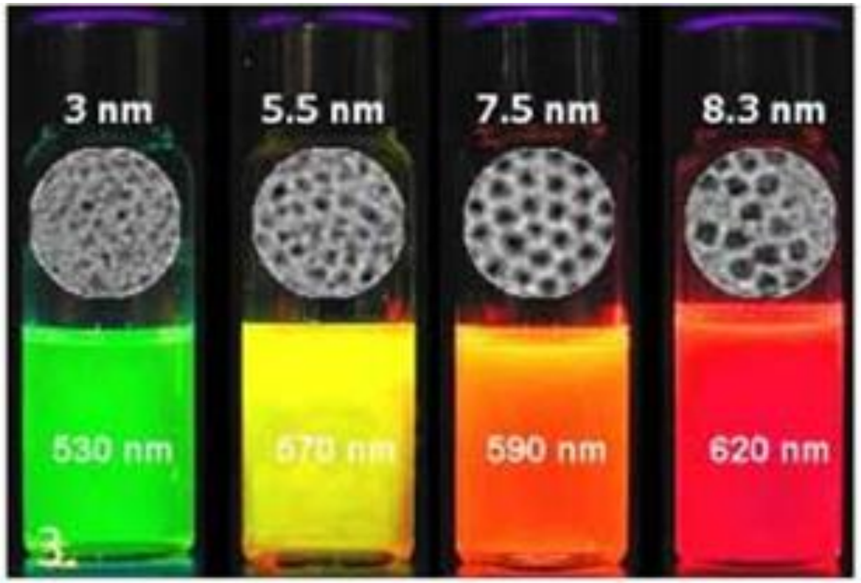
天然型製法の量子ドット：コロイド

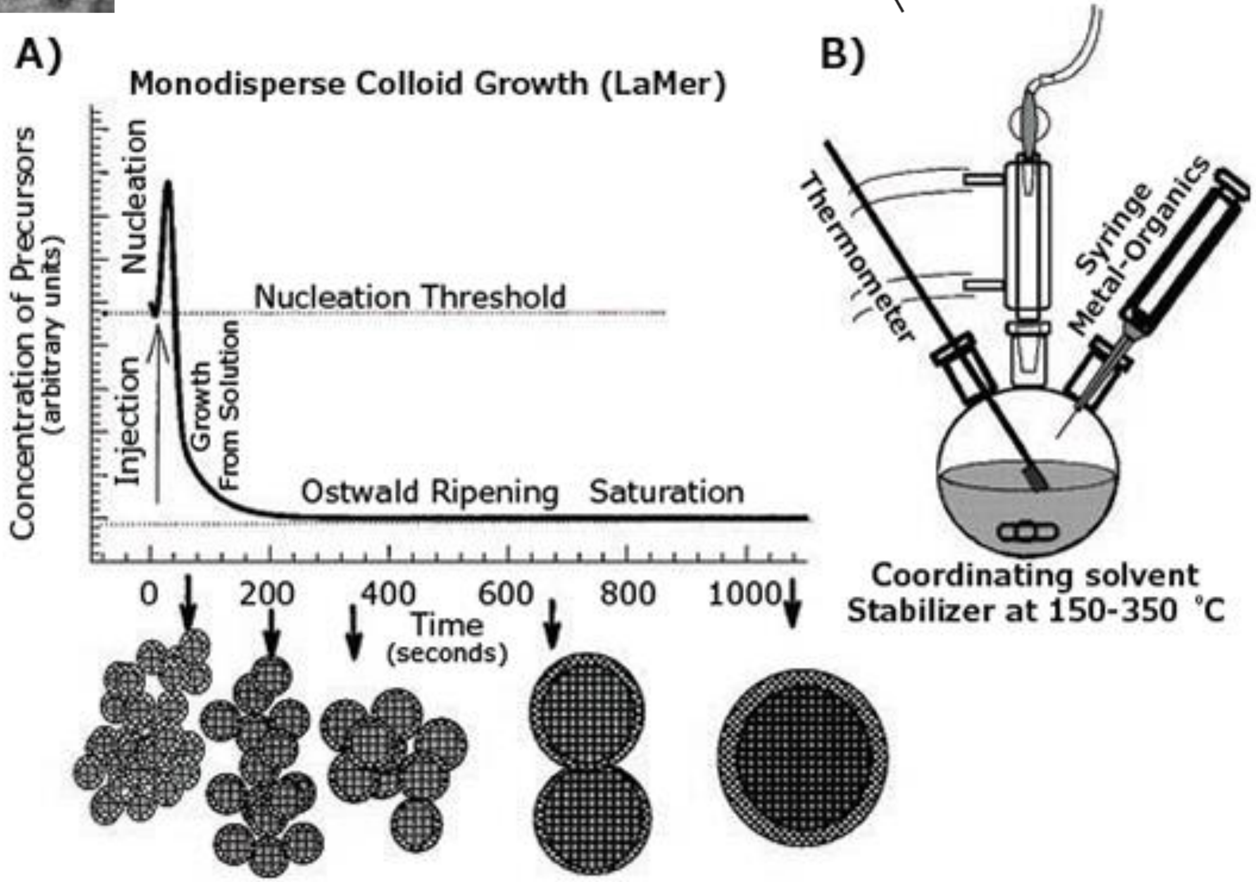






コア-シェル型



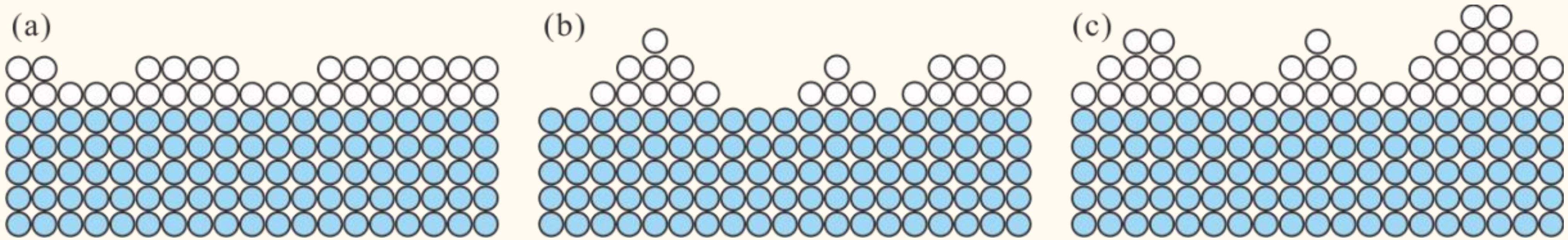


Klimov “Nanocrystal Quantum Dots” (2010, CRC)

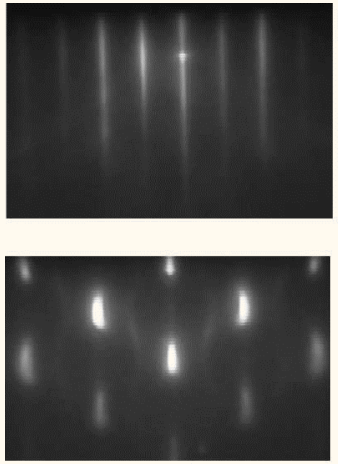
天然型製法の量子ドット：自己集積型

Molecular beam epitaxy (MBE) growth modes

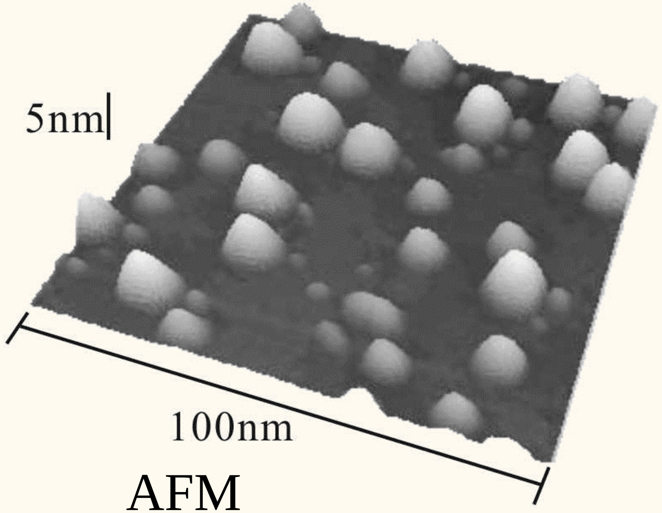
Frank-van der Merve　　Volmer-Weber　　Stranski-Krastanow



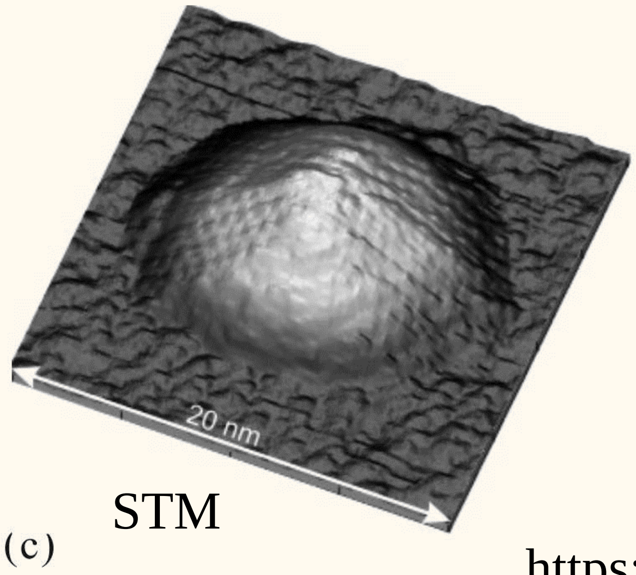
InAs on GaAs substrate



RHEED



AFM



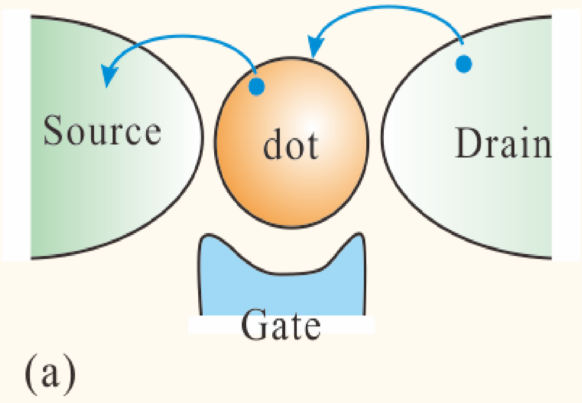
STM

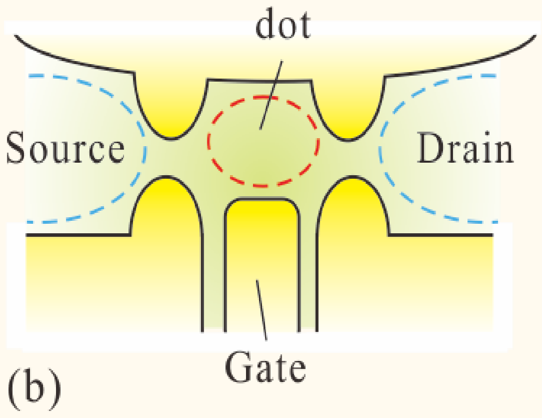
QD LASER

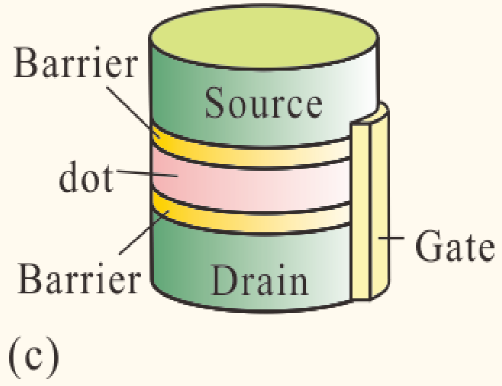
https://www.qdlaser.com/

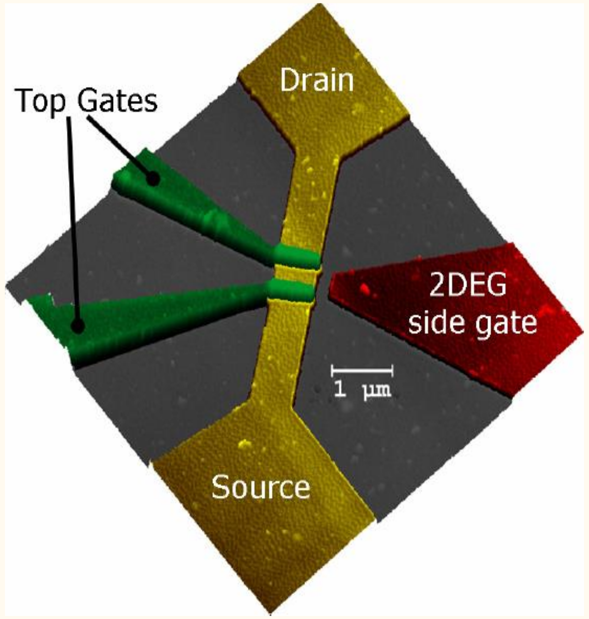
金属ゲートを用いて形成する量子ドット

Quantum dots with nano-fabrication techniques

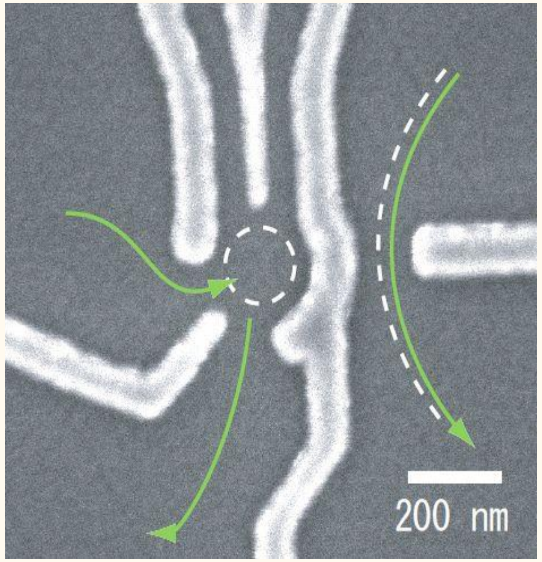






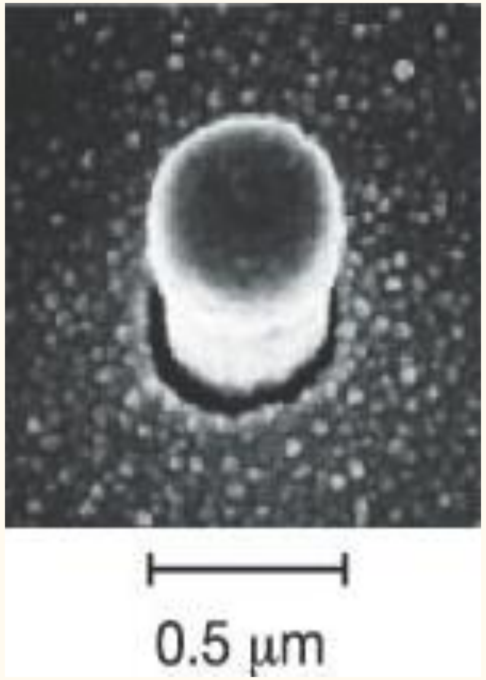


wrap gate



split gate

with charge detector



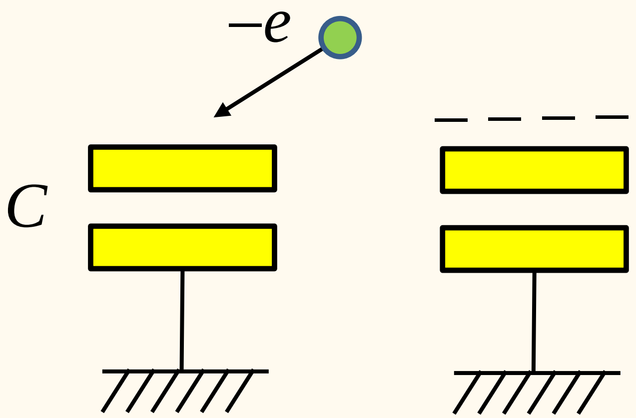
vertical type

量子ドットの基礎

単電子効果と量子閉じ込め効果

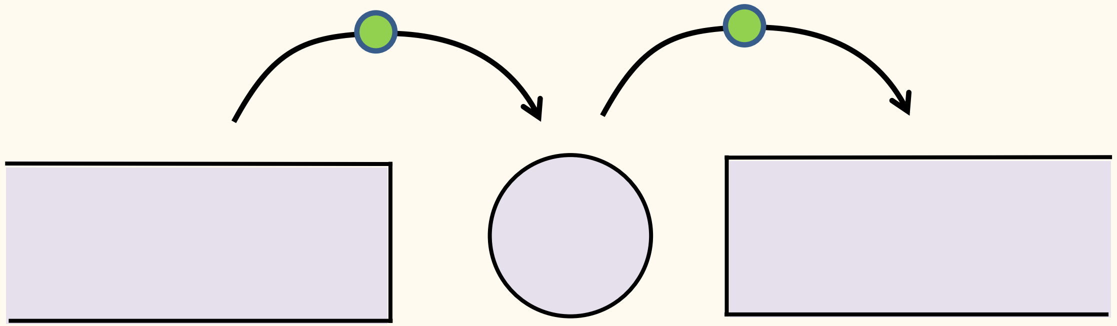
Quantum dot + Micro LED

Single electron effect simplest view

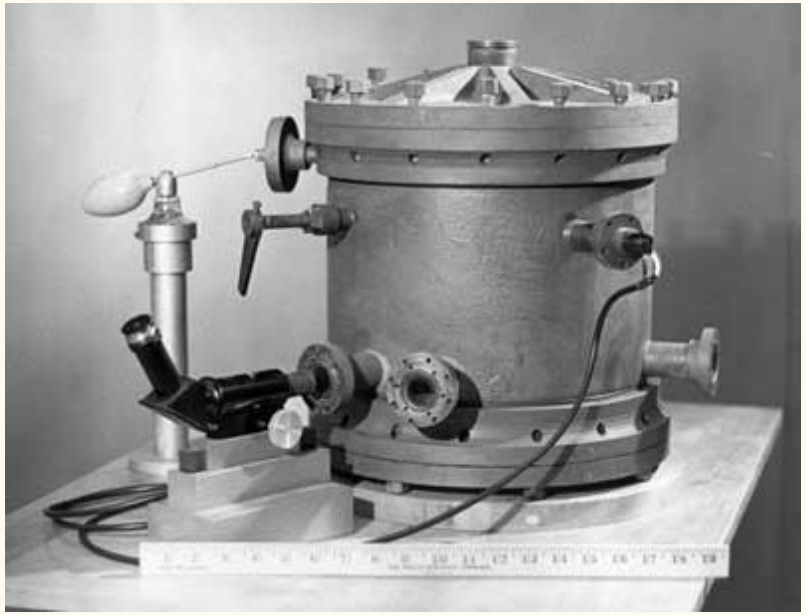


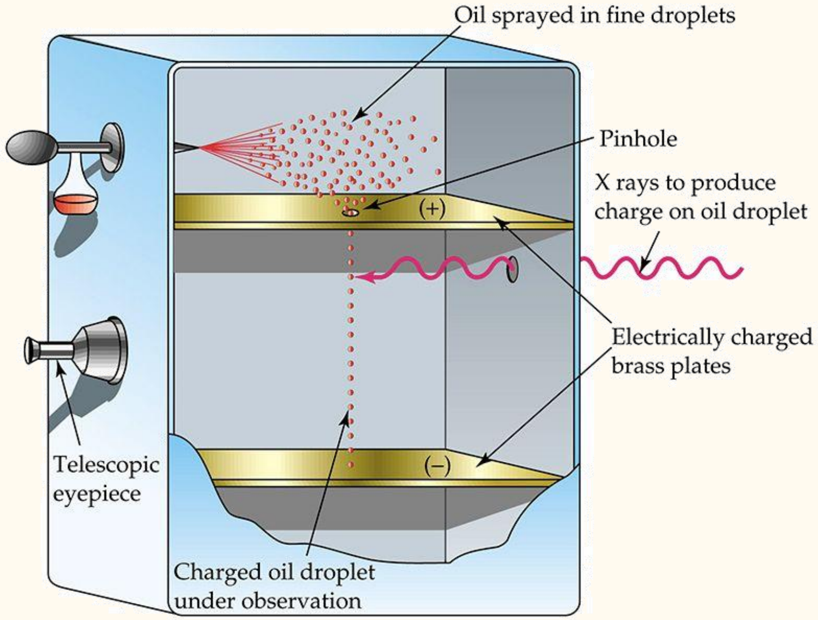
$$E_{\rm c} = \frac{e^2}{2C} > k_{\rm B}T$$

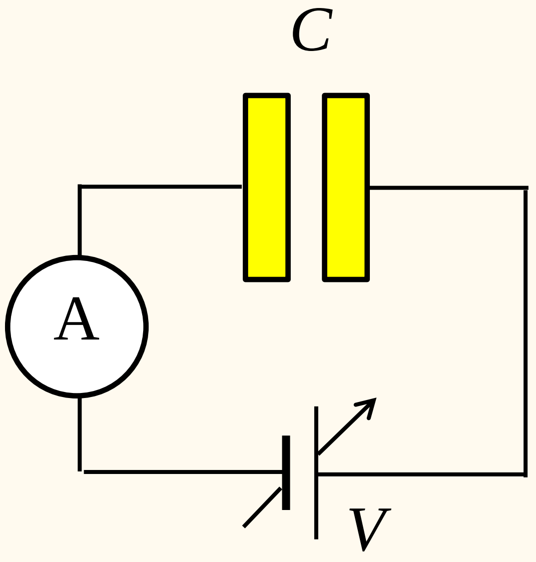
Coulomb blockade

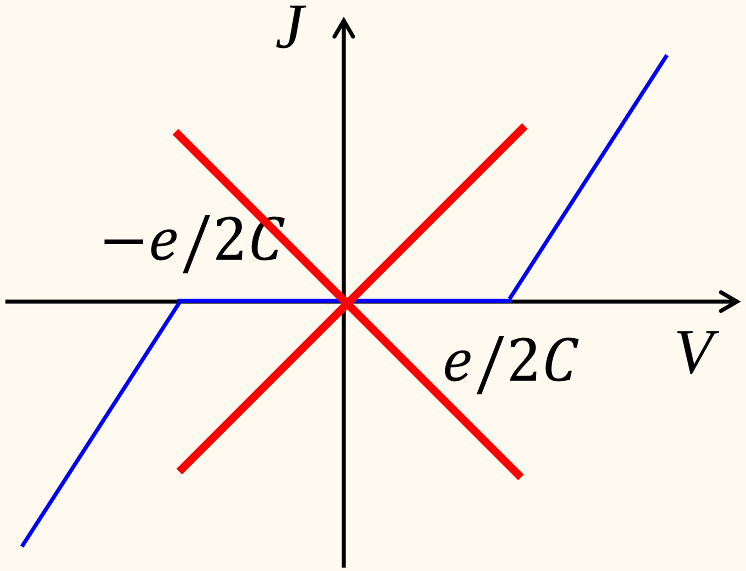


Millikan's oil droplet experiment









Power sources: Automatically supply energy.

Energy → Enthalpy $H = U - PV$

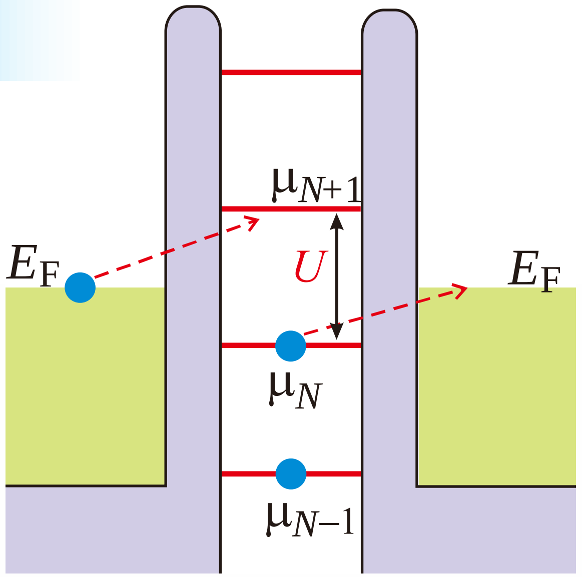
Constant interaction model, capacitor model

Constant interaction: U

Electron number: N

Interaction energy $$E_{cN} = {}_N C_2 U = \frac{N(N-1)U}{2} = \frac{U(N-1/2)^2}{2} - \frac{U}{8}$$

Chemical potential $$\Delta E_+(N) = (N-1)U$$

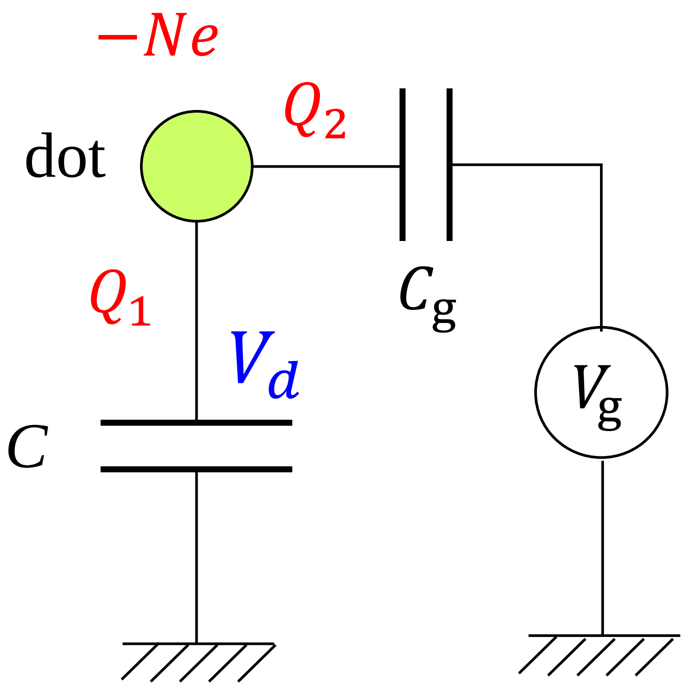


Charge relations $$Q_1 + Q_2 = -eN, \quad Q_1 = CV_d,$$ $$Q_2 = C_g(V_d - V_g)$$

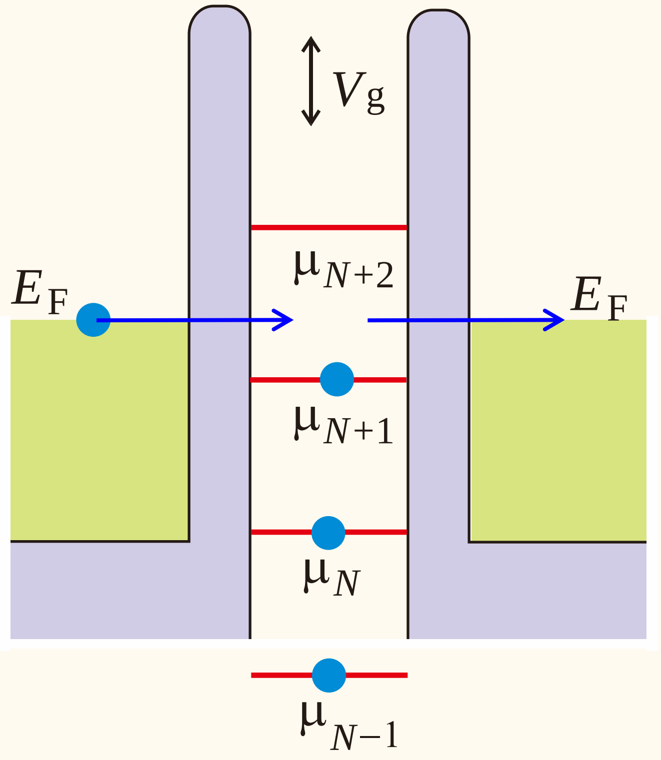
Electrostatic energy $$E = \frac{1}{2}CV_d^2 + \underline{\frac{1}{2}C_g(V_d - V_g)^2}$$

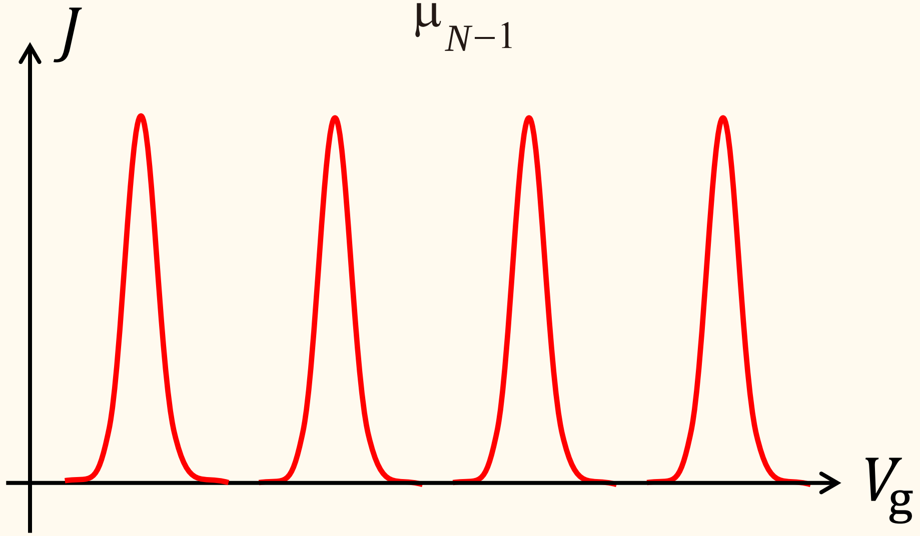
Enthalpy $$H(N, V_g) = \frac{(Ne - C_gV_g)^2}{2(C+C_g)} \equiv \frac{(Ne - C_gV_g)^2}{2C_s}$$

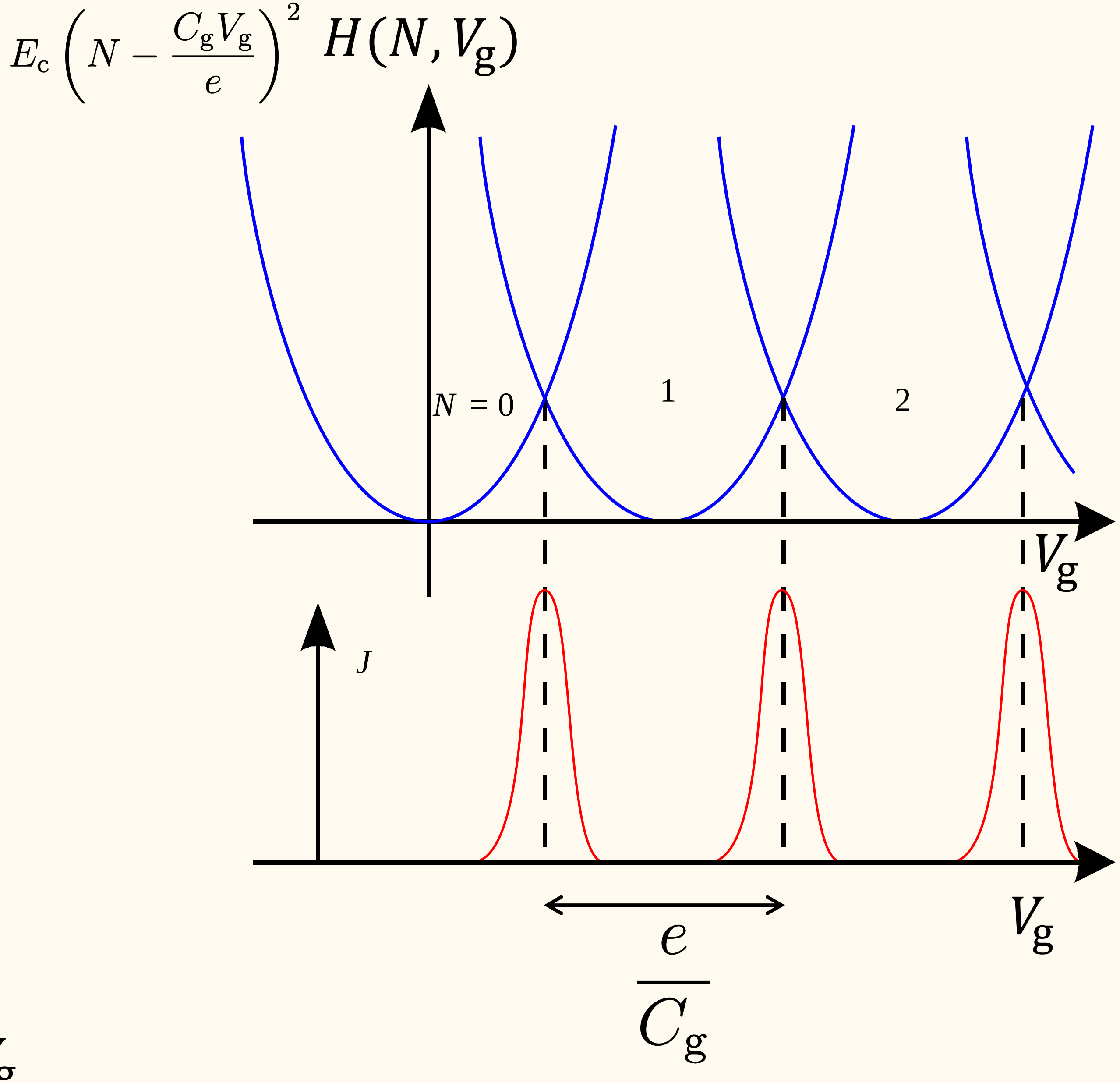
Chemical potential $$\mu_N \approx \frac{dH}{dN} = \frac{e(Ne - C_gV_g)}{C_s} = 2E_c\left(N - \frac{C_gV_g}{e}\right)$$



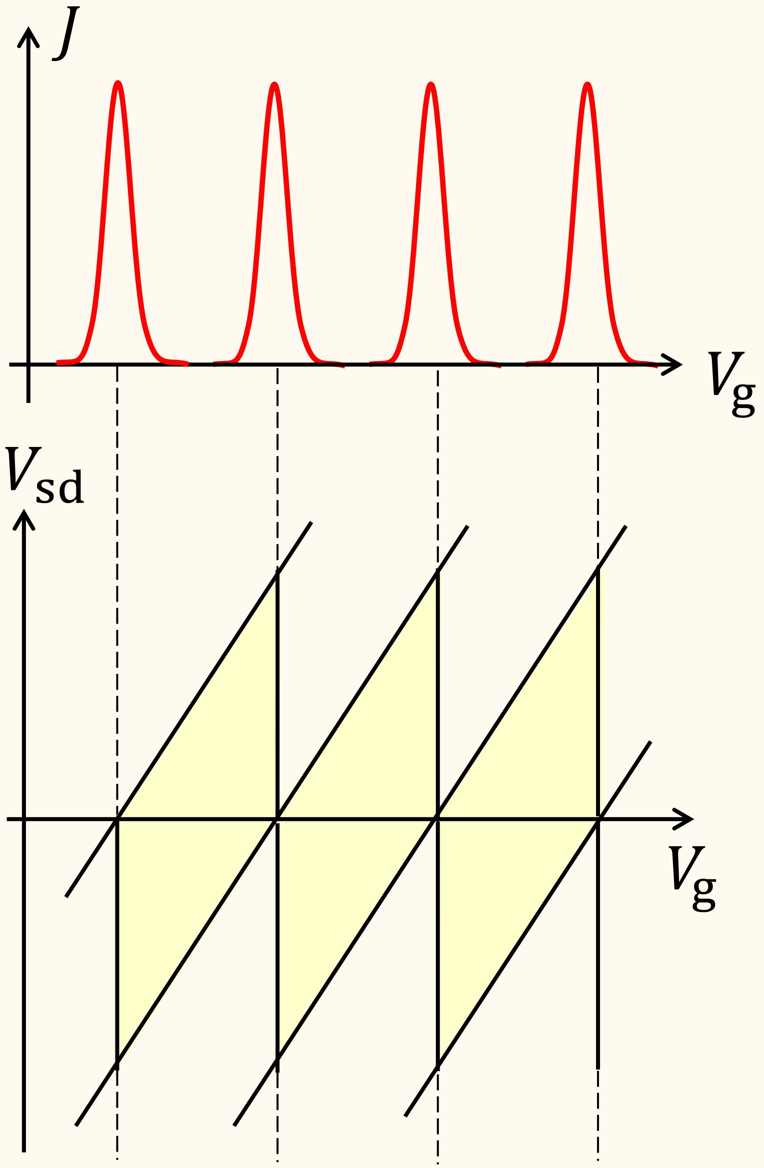
Coulomb oscillation

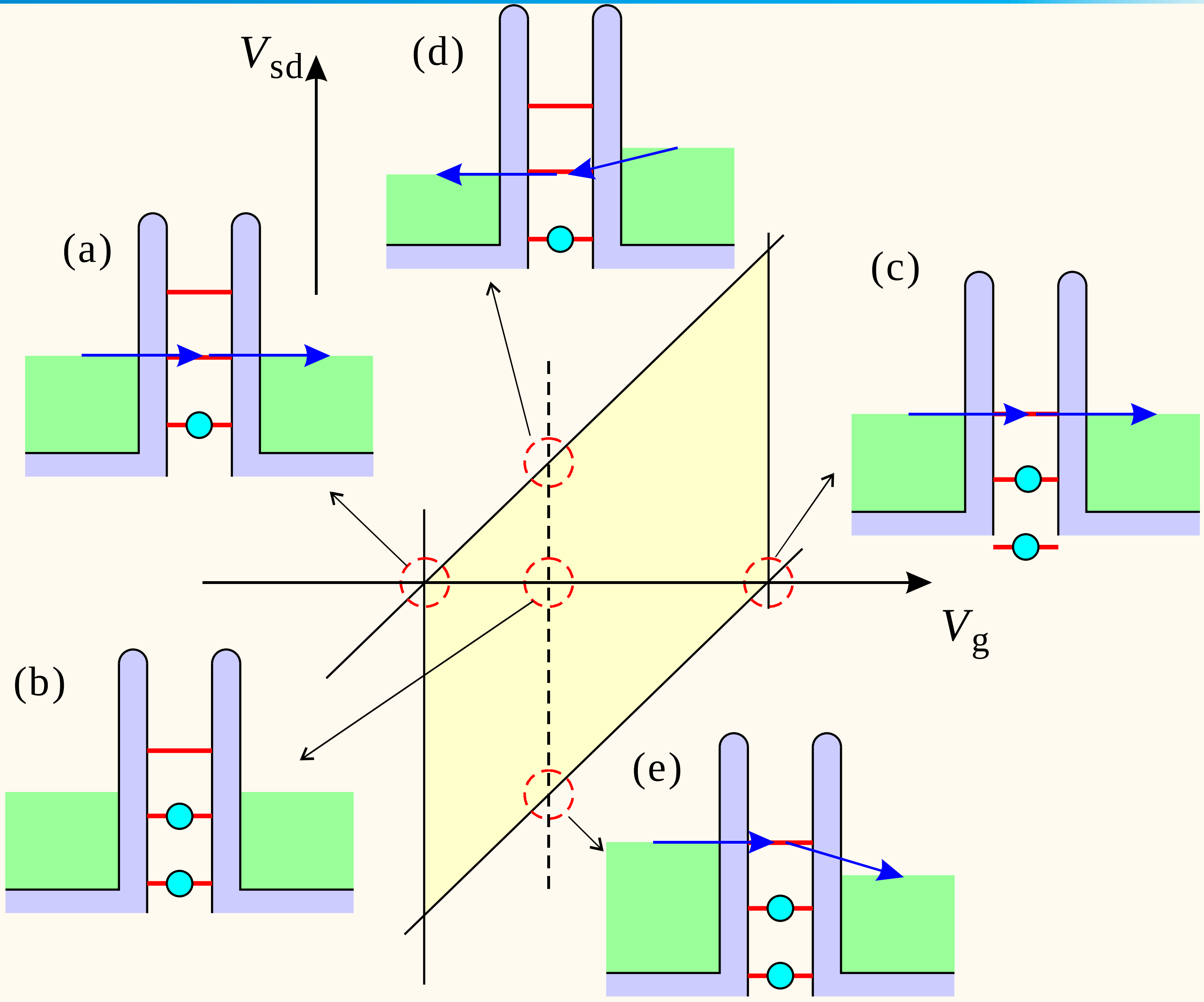




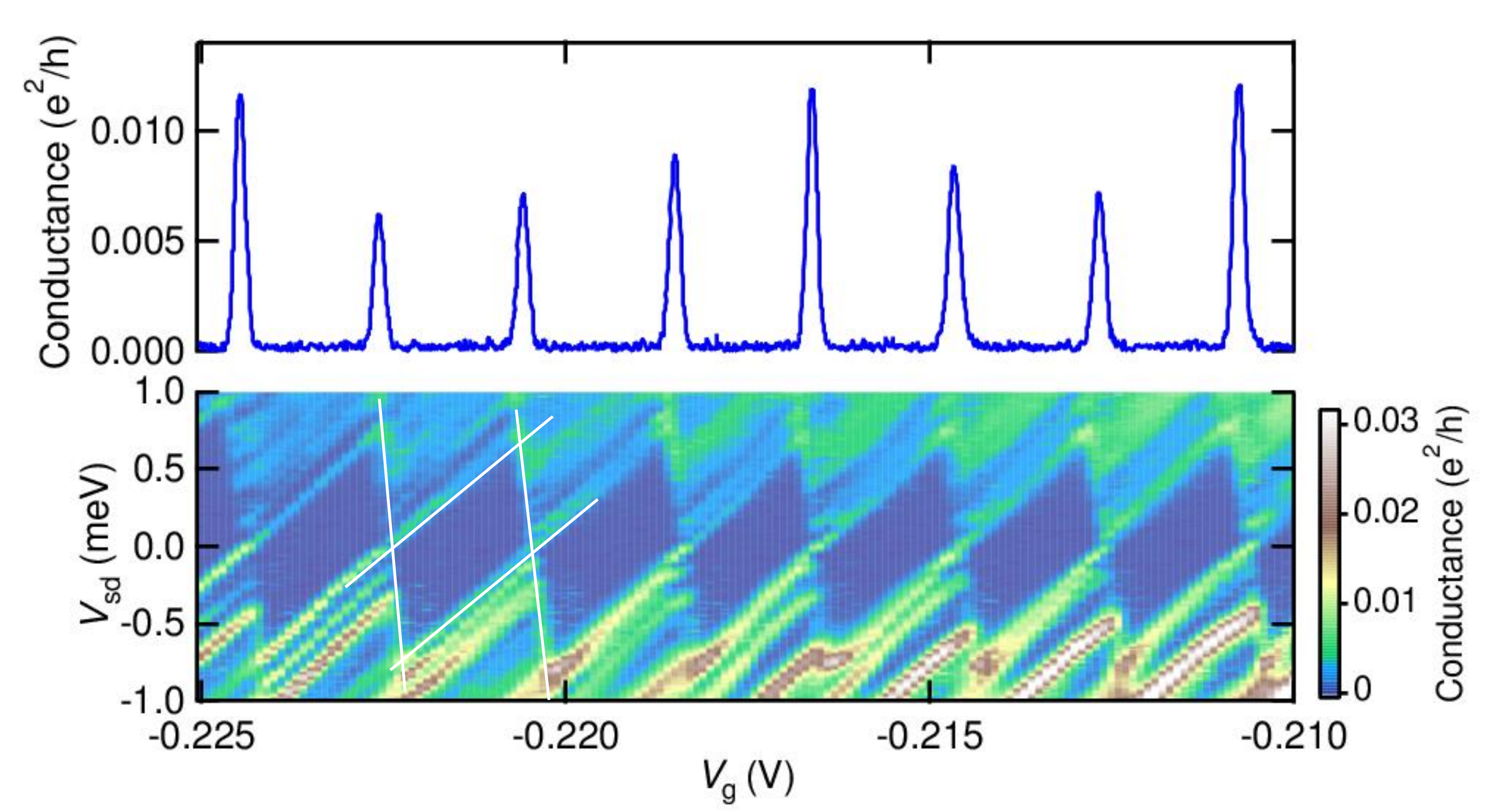


Coulomb diamond



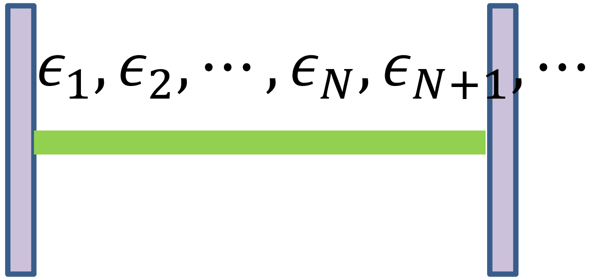


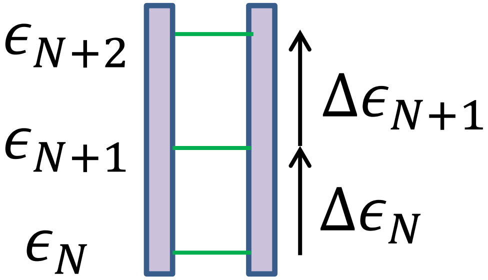
Coulomb oscillation and diamonds

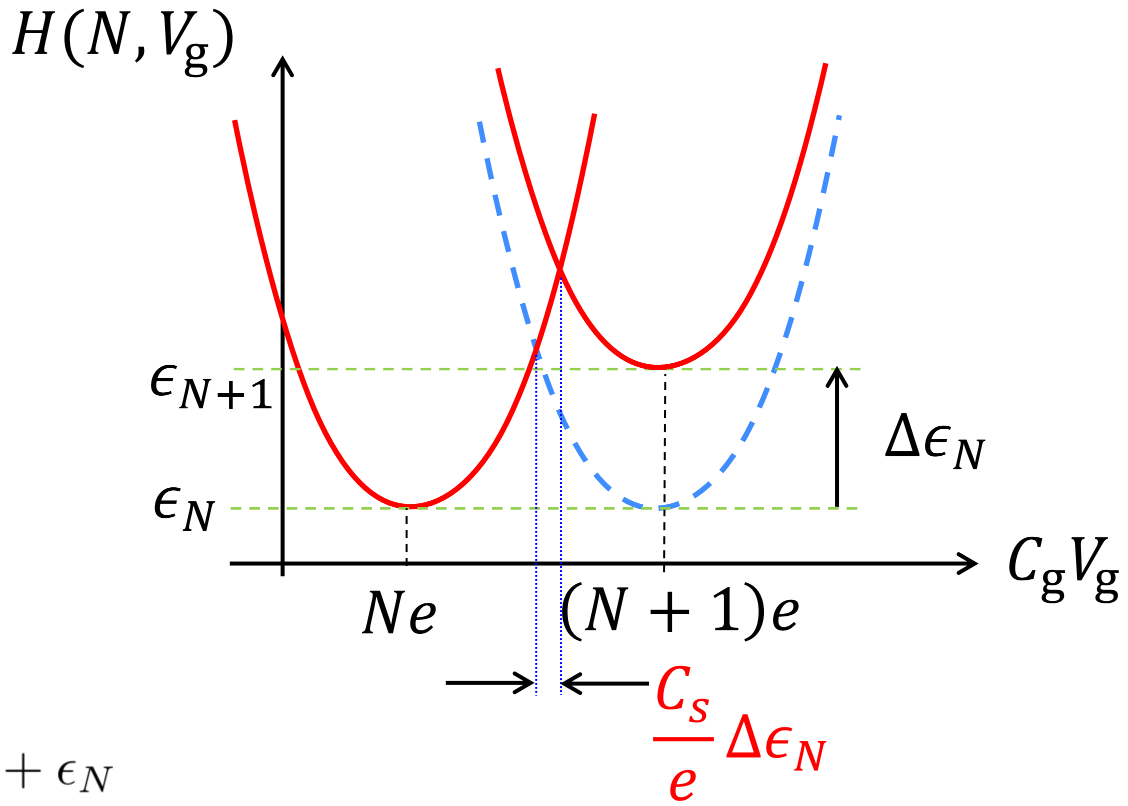


Quantum confinement

Zero-dimensional confinement to a quantum dot gives shifts in Coulomb peak positions.







Enthalpy shift by quantum confinement

$$H(N) = \frac{(Ne - C_g V_g)^2}{2C_s} + \epsilon_N$$

Chemical potential shift

$$\Delta H(N, N+1) = H(N+1) - H(N)$$

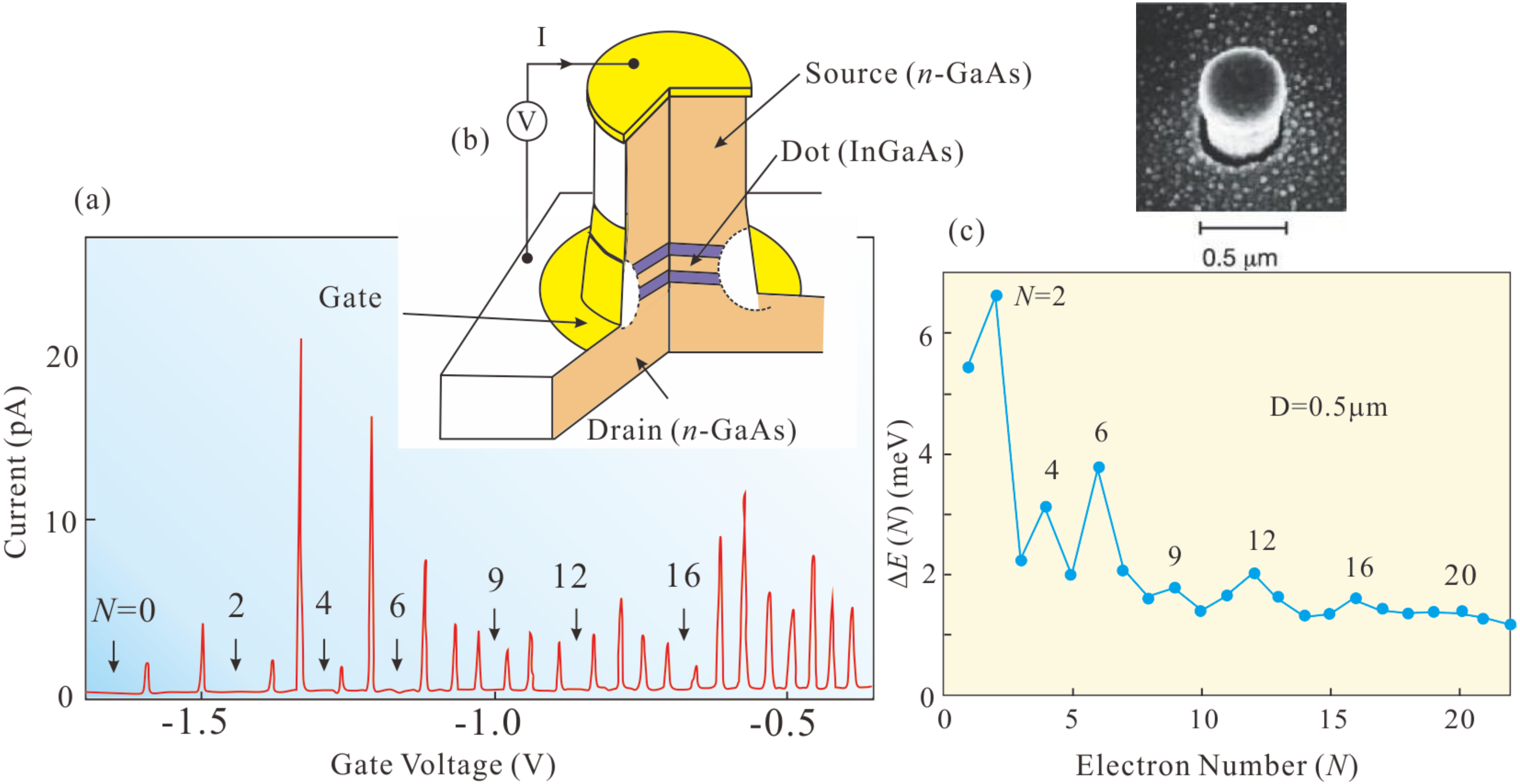
$$= \frac{e}{C_s}\left\{\left(N + \frac{1}{2}\right)e - C_g V_g\right\} + \Delta\epsilon_N$$

$$\Delta\epsilon_N \equiv \epsilon_{N+1} - \epsilon_N$$

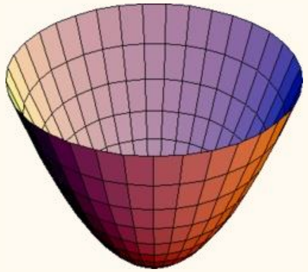
Shift in gate voltage

$$V_{gX}(N, N+1) = \frac{1}{C_g}\left\{\left(N + \frac{1}{2}\right)e + \frac{C_s}{e}\Delta\epsilon_N\right\}$$

Quantum confinement effect in a vertical quantum dot



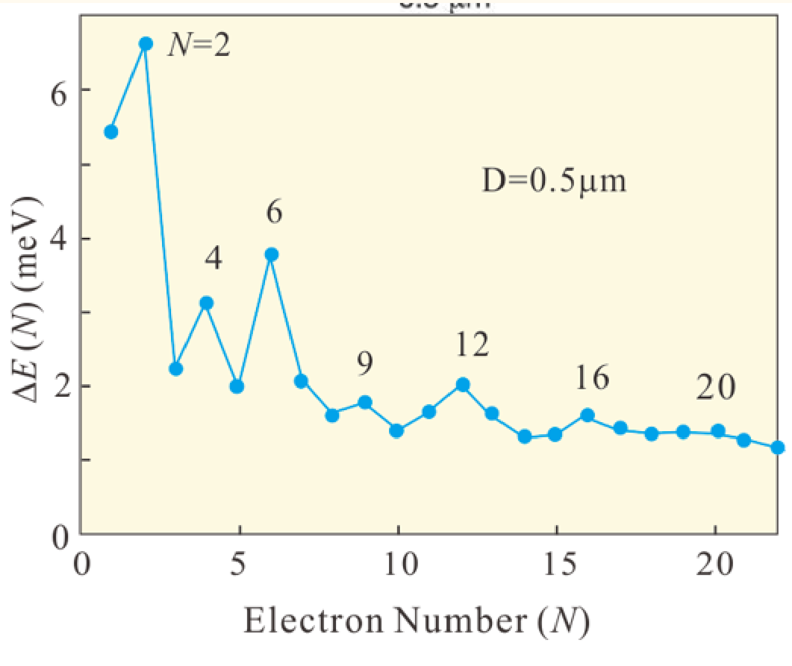
Two-dimensional harmonic potential

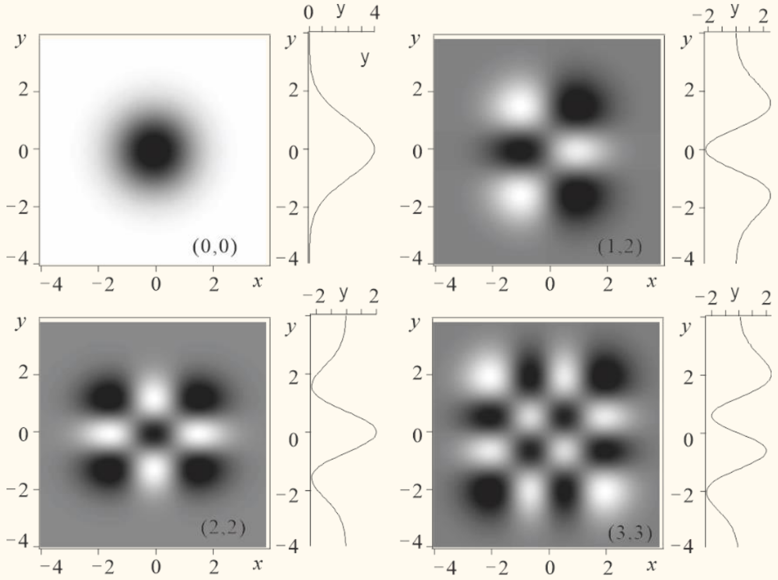


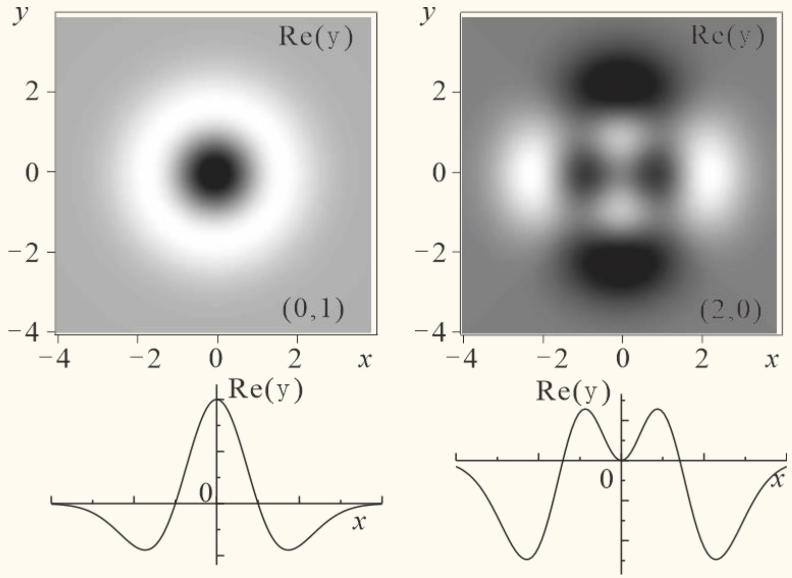
Potential shape: $V(x,y) = \frac{m\omega}{2}(x^2 + y^2)$

Easy solutions from 1d harmonic potential $\psi_{n_x n_y} = A \exp\left[-\frac{m\omega(x^2+y^2)}{2\hbar}\right] H_{n_x}\left[\sqrt{\frac{m\omega}{\hbar}}x\right] H_{n_y}\left[\sqrt{\frac{m\omega}{\hbar}}y\right]$

Eigen energies: $E(n_x, n_y) = (n_x + n_y + 1)\hbar\omega = (n_t + 1)\hbar\omega \quad n_x + n_y \equiv n_t = 0, 1, 2, \cdots$







For fixed $n_t \quad n_x = 0, 1, 2, \cdots, n_t$ $\qquad n_t + 1$ degeneracy

With spin degeneracy: 2, 4, 6, 8, ⋯

$N = 2, 6, 12, 20, \cdots, (n+1)(n+2), \cdots$

Quantum dot in magnetic field

Hamiltonian with $\boldsymbol{B} = (0,0,B)$

$$\mathscr{H} = \frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} + \frac{m}{2}\omega^2(x^2 + y^2) \quad \boldsymbol{A} = \left(-\frac{By}{2}, \frac{Bx}{2}, 0\right)$$

Expansion of the kinetic energy term

$$\frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{ie\hbar B}{2m}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2B^2}{8m}(x^2 + y^2)$$

Cyclotron frequency and composite harmonic confinement potential frequency

$$\omega_\mathrm{c} = \frac{eB}{m} \quad \Omega \equiv \sqrt{\omega^2 + (\omega_\mathrm{c}/2)^2}$$

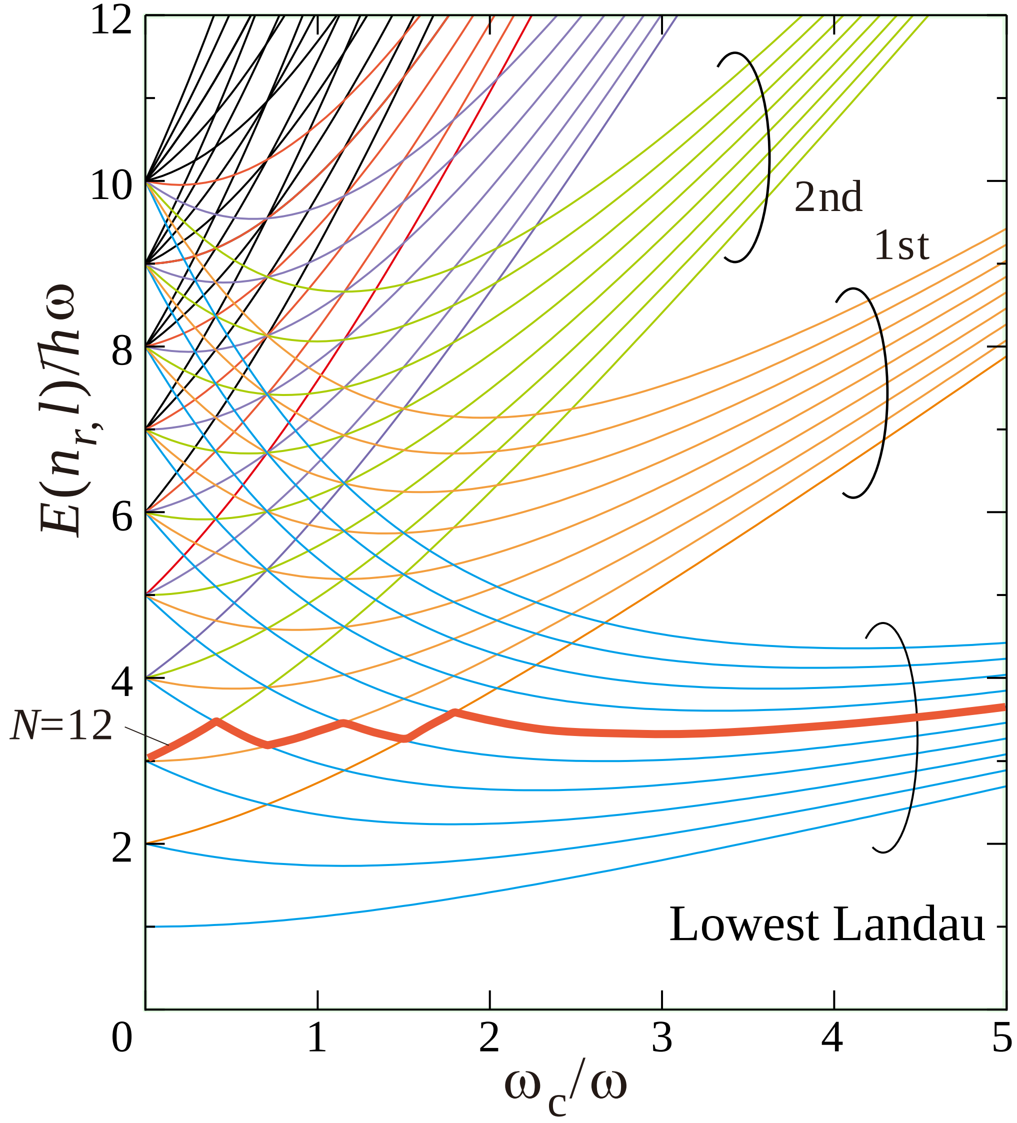
The Hamiltonian is rewritten as

$$\mathscr{H} = \frac{\hbar^2\nabla^2}{2m} + \frac{\Omega}{2m}(x^2 + y^2) + \frac{\omega_\mathrm{c}\hat{L}_z}{2} = \mathscr{H}_\Omega + \frac{\omega_\mathrm{c}\hat{L}_z}{2}$$

Fock-Darwin state eigen energies

$$E(n_r, l) = \hbar\Omega(2n_r + |l| + 1) + \hbar\omega_\mathrm{c} l/2$$

Quantum dot in magnetic field



$$\mathscr{H} = \frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} + \frac{m}{2}\omega^2(x^2 + y^2) \quad \boldsymbol{A} = \left(-\frac{By}{2}, \frac{Bx}{2}, 0\right)$$

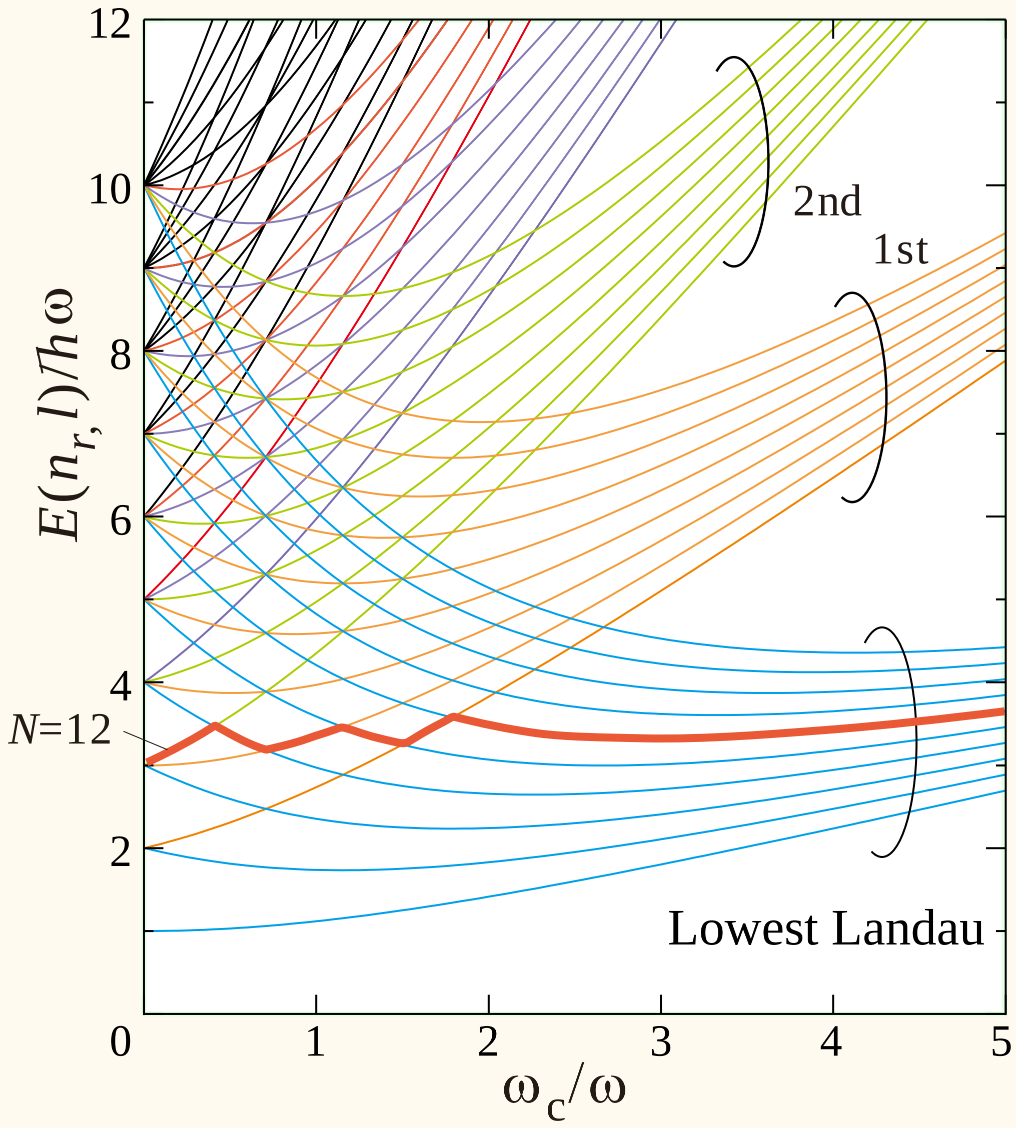
$$\frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{ie\hbar B}{2m}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2B^2}{8m}(x^2 + y^2)$$

$$\omega_c = \frac{eB}{m} \quad \Omega \equiv \sqrt{\omega^2 + (\omega_c/2)^2}$$

$$\mathscr{H} = \frac{\hbar^2\nabla^2}{2m} + \frac{\Omega}{2m}(x^2 + y^2) + \frac{\omega_c \hat{L}_z}{2} = \mathscr{H}_\Omega + \frac{\omega_c \hat{L}_z}{2}$$

$$E(n_r, l) = \hbar\Omega(2n_r + |l| + 1) + \hbar\omega_c l/2$$

Fock-Darwin states

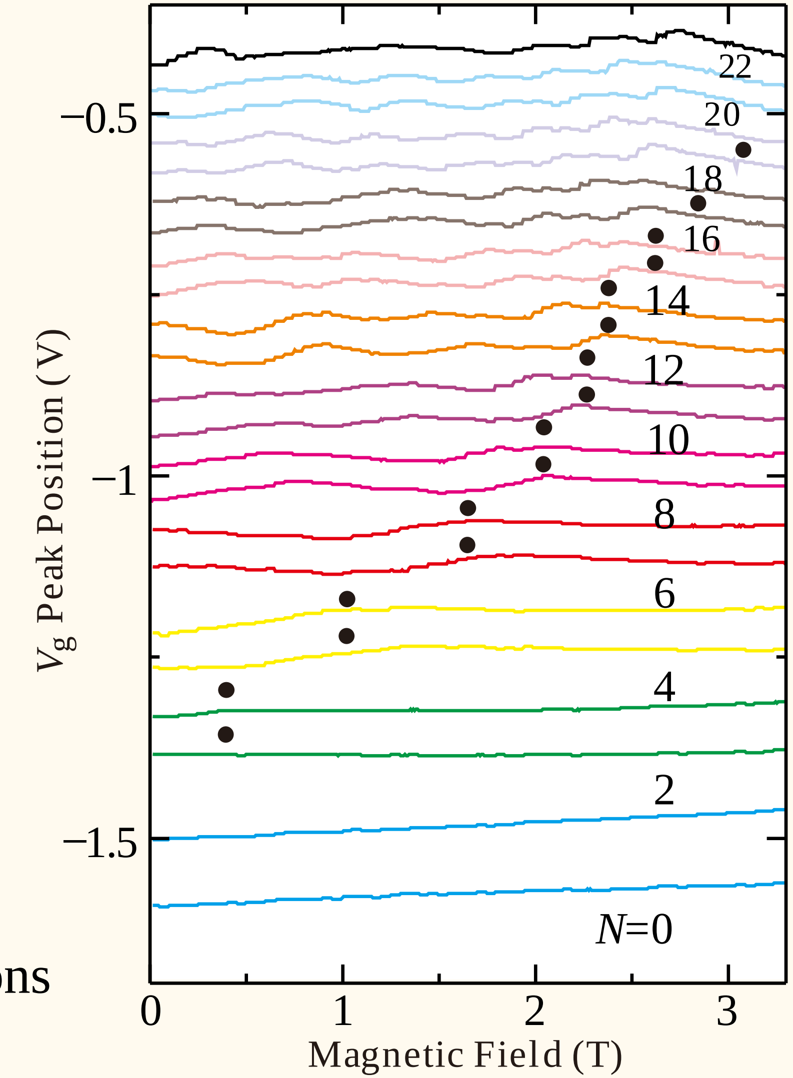


Level crossing points

$$\left(\frac{\omega_c}{\omega}\right)^2 = n_L - 2 + \frac{1}{n_L}$$

n_L: Landau index
= 1, 2, ….

• : Solutions



Transport though quantum dots

Quantum confinement

Cascade connection of T-matrices

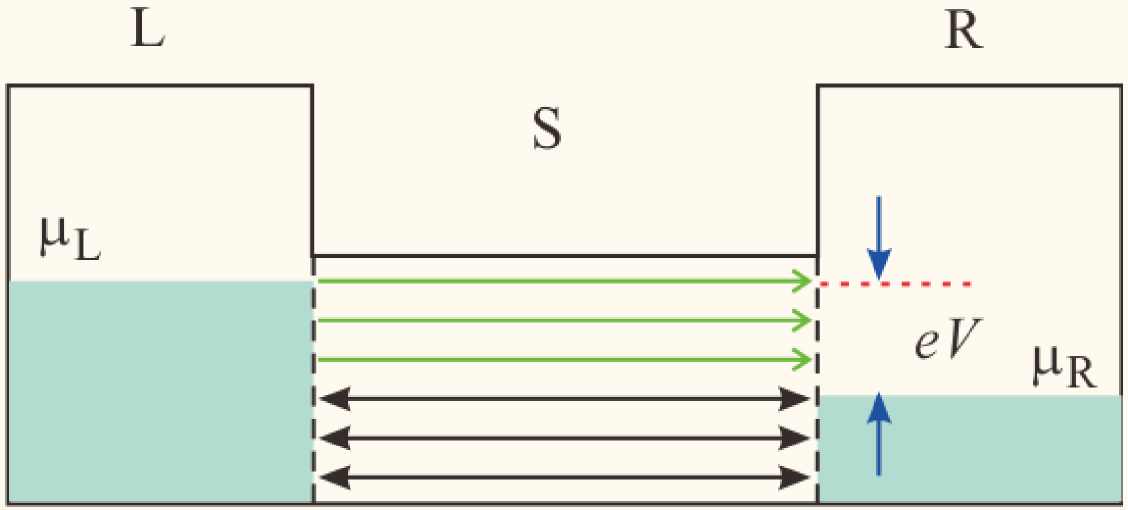
Single electron

Master equation

Landauer formula: conductance quantum

Transport through a 1d wire without scattering

Landauer, IBM Res. Dev. **1**, 223 (1957)



L, R : Particle reservoirs

Thermal equilibrium:
well defined chemical potentials

Instantaneous thermalization:
particles loose quantum coherence

$$j(k) = \frac{e}{L} v_{\mathrm{g}} = \frac{e}{\hbar L} \frac{dE(k)}{dk}$$

L: wavefunction normalization length

$$J = \int_{k_{\mathrm{R}}}^{k_{\mathrm{L}}} j(k) \frac{L}{2\pi} dk = \frac{e}{h} \int_{\mu_{\mathrm{R}}}^{\mu_{\mathrm{L}}} dE = \frac{e}{h}(\mu_{\mathrm{L}} - \mu_{\mathrm{R}}) = \frac{e^2}{h} V$$

$$G = \frac{J}{V} = \frac{e^2}{h} \equiv G_{\mathrm{q}}$$ Conductance quantum $$\left(\frac{2e^2}{h} \equiv G_{\mathrm{q}} \quad \text{spin freedom} \right)$$

Conductance quantum as uncertainty relation

Space coordinate-wavenumber

Wave packet: $\Delta k \rightarrow \Delta x = \dfrac{2\pi}{\Delta k}, \quad v_{\mathrm{g}} = \dfrac{\Delta E}{\hbar \Delta k}$

Fermion statistics: electron charge concentration $= \dfrac{e}{\Delta x} = \dfrac{e\Delta k}{2\pi}$

$$J = e n_{\mathrm{electron}} v_{\mathrm{g}} = \frac{e}{\Delta x}\frac{\Delta E}{\hbar \Delta k} = \frac{e^2}{h} V$$

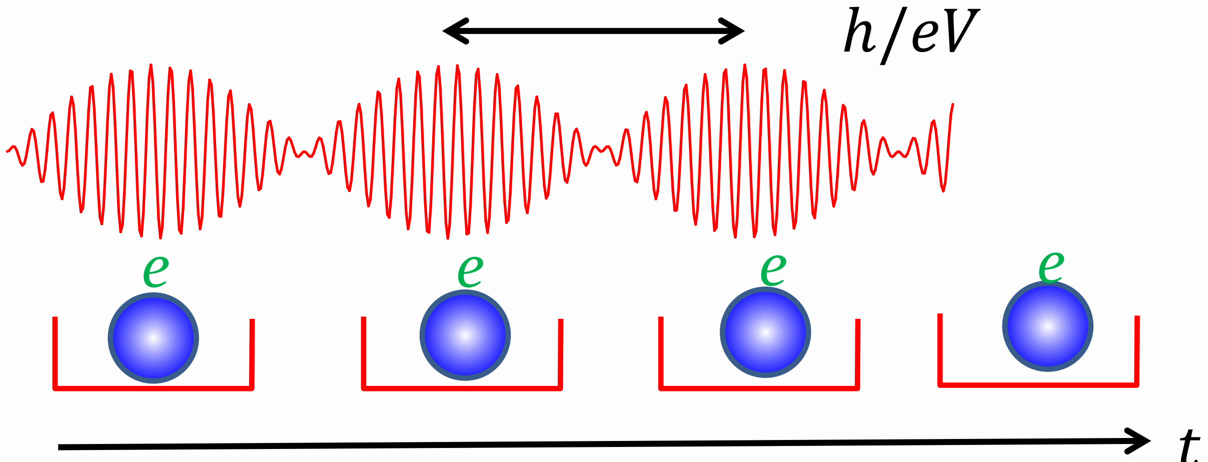
Energy-time

Energy width: $\Delta E = eV$

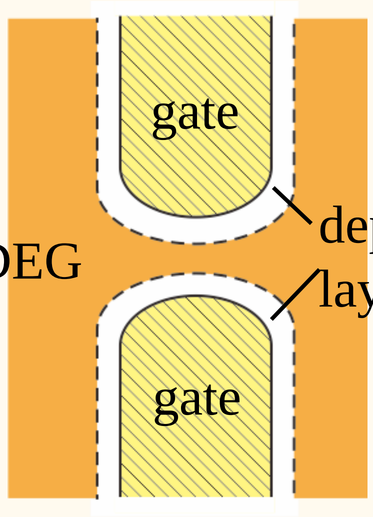
Wave packet width in time: $\Delta t = \dfrac{h}{\Delta E} = \dfrac{h}{eV}$

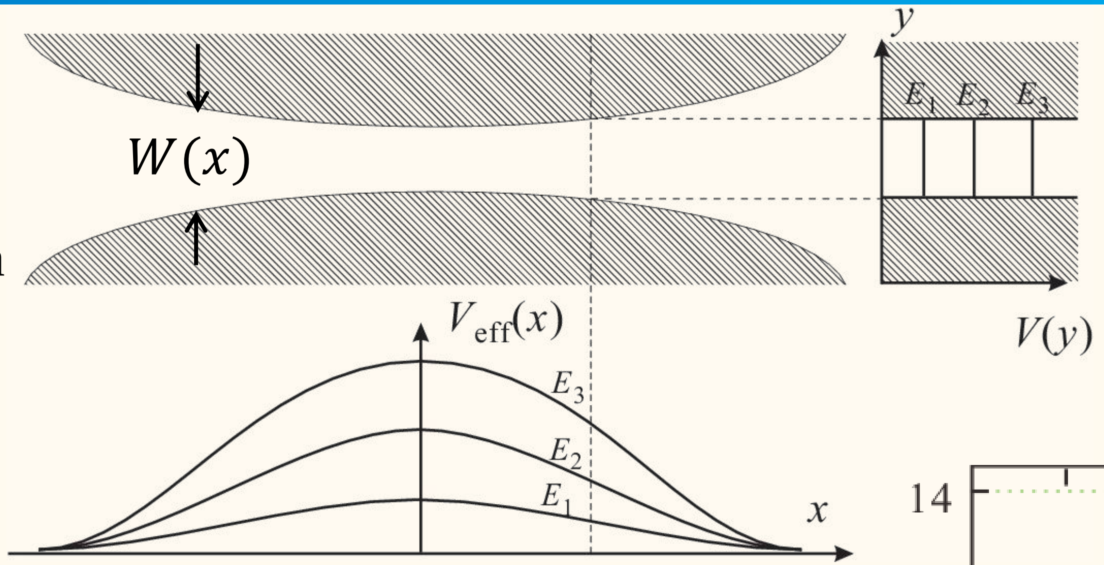
Fermion anti-bunching effect: $J = \dfrac{e}{\Delta t} = \dfrac{e^2}{h} V$

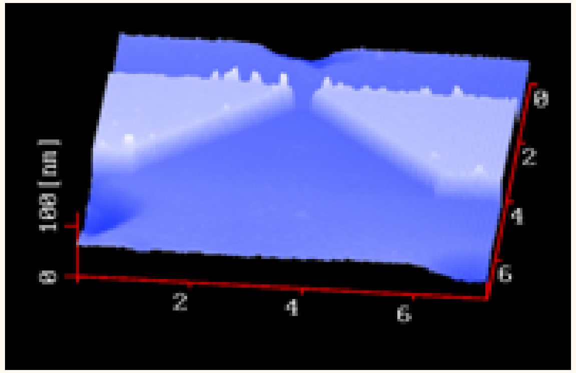
Conductance quantum comes from fermion statistics of electrons



Landauer formula: multipath and quantum point contact (QPC)



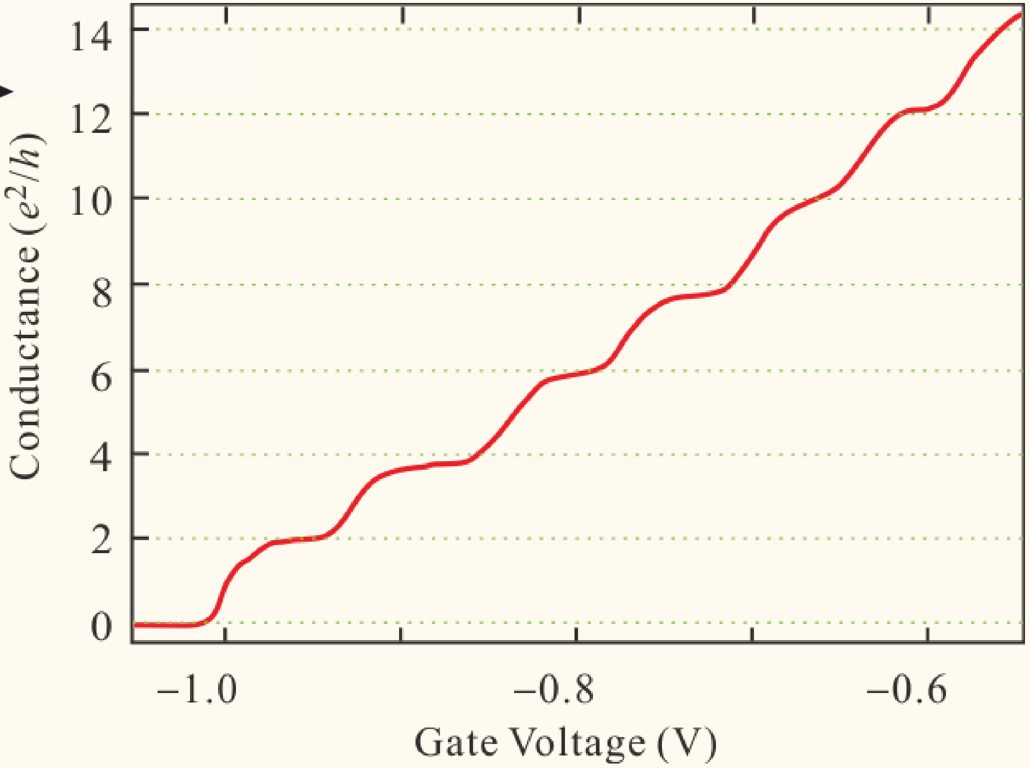




$$H\psi(x,y) = \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi_n(y)\phi(x)$$

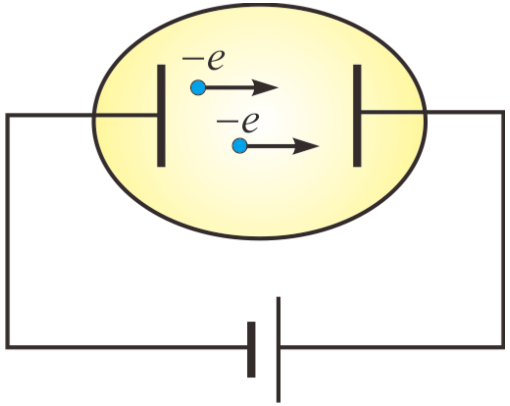
$$= \varphi_n(y)\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \left(\frac{n\pi}{2W}\right)^2\right)\phi(x) = E\varphi_n(y)\phi(x)$$

$$V_{\text{eff}}(n,x) = \frac{\hbar^2}{2m}\left(\frac{n\pi}{2W(x)}\right)^2$$



Transmissible one-dimensional system: Conductance Channel

Sensitivity to gate voltage → used as a remote charge (single electron) sensor



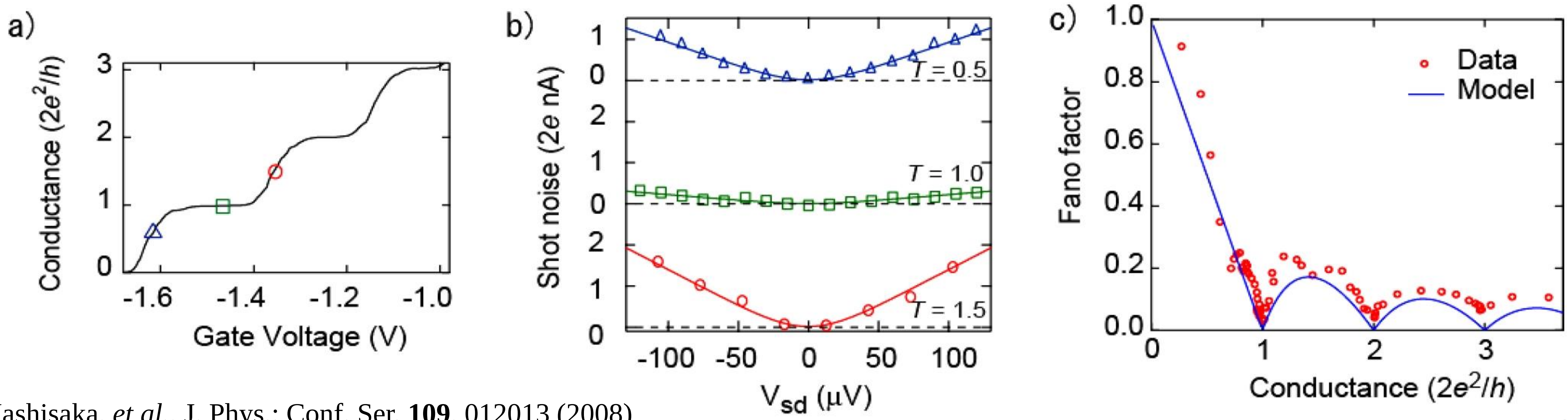
Flow of single electron: Time domain: δ-function approximation

$$J_e(t) = e\delta(t - t_0) = e\int_{-\infty}^{\infty} e^{2\pi i f(t-t_0)} df = 2e\int_0^{\infty} \cos\left[2\pi f(t - t_0)\right] df$$

Current fluctuation density for infinitesimal band df $\quad \delta J = d\sqrt{\langle J_e^2 \rangle} = \frac{2e}{\sqrt{2}} df = \sqrt{2} e df$

$$\overline{\langle \delta j^2 \rangle} = (j_p + j_q e^{i\phi})(j_p + j_q e^{-i\phi}) = j_p^2 + j_q^2 + 2 j_p j_q \cos\phi = j_p^2 + j_q^2 = 2 \times (\sqrt{2} e)^2 df$$

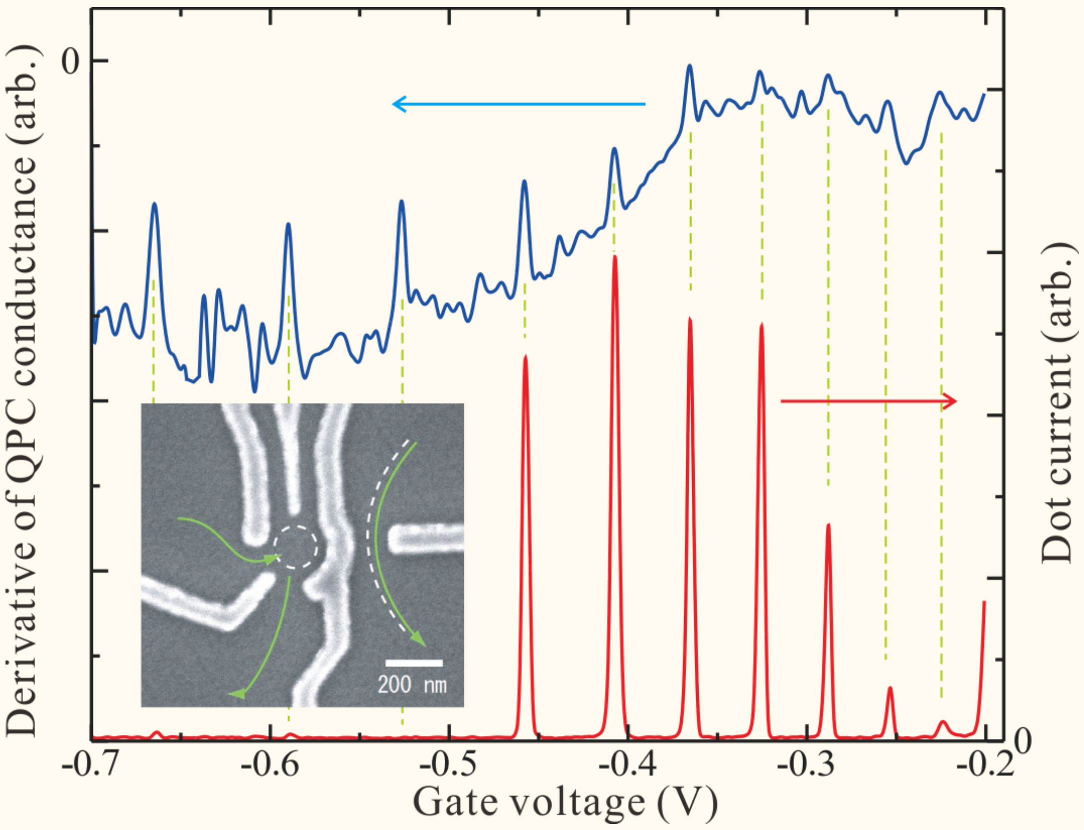
Flow of N-electrons $\quad \overline{\langle \delta J^2 \rangle} = N \times 2e^2 df = 2e\bar{J} df \quad (\bar{J} = eN)$: Poissonian noise



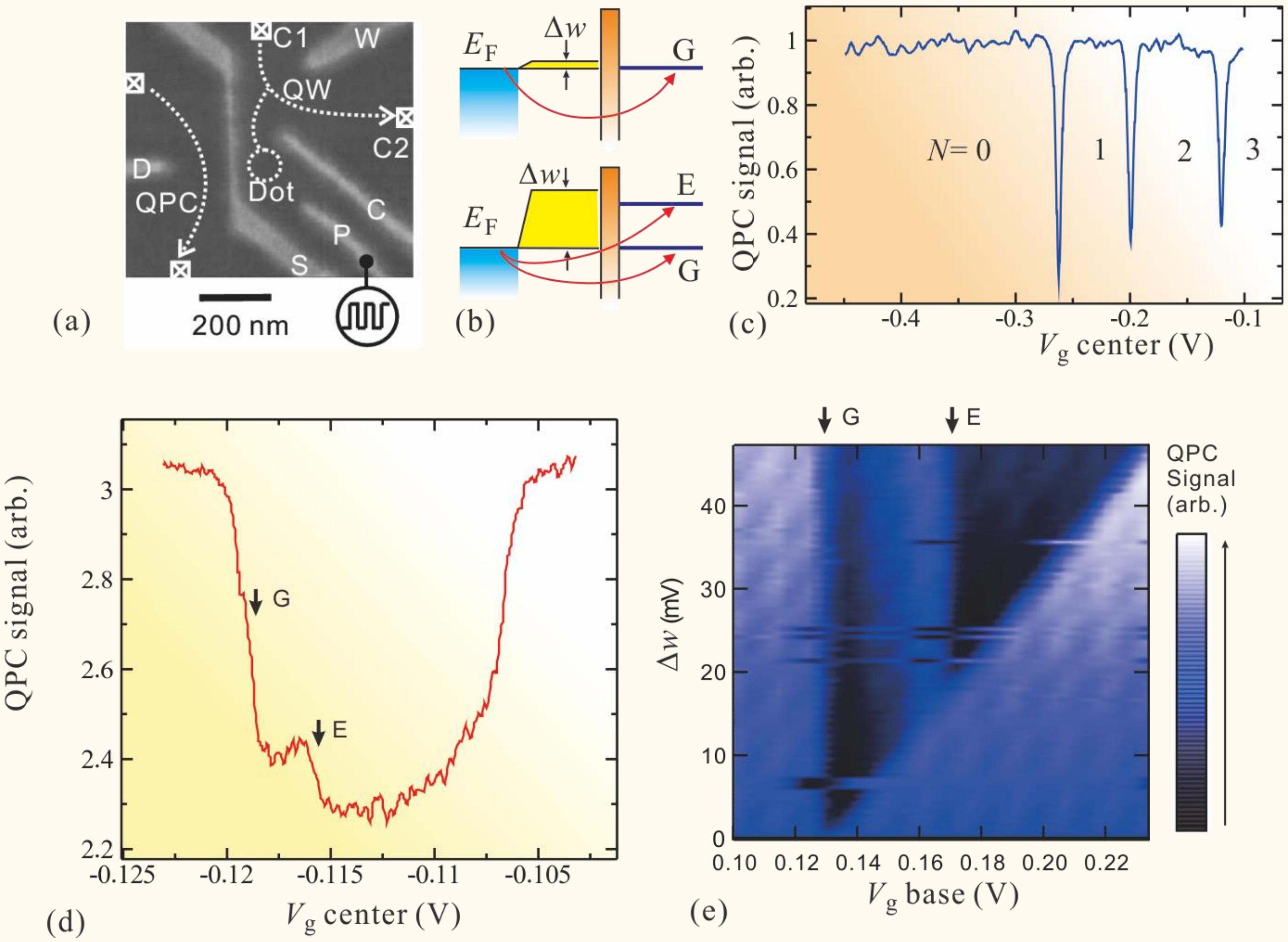
M. Hashisaka, *et al.*, J. Phys.: Conf. Ser. **109**, 012013 (2008)

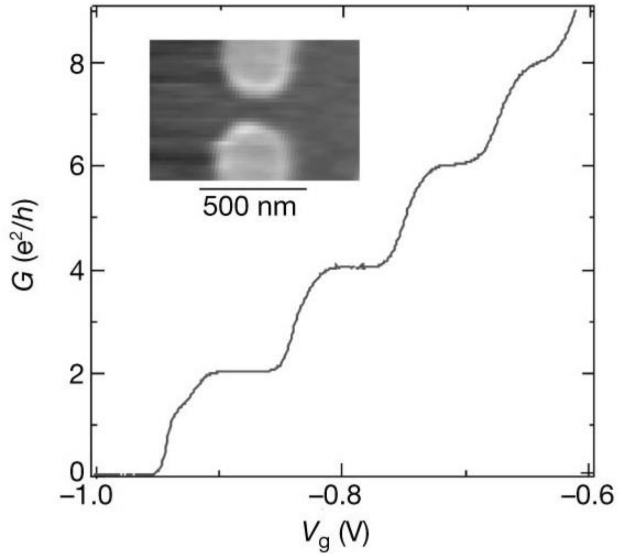
QPC charge sensor and excited states spectroscopy

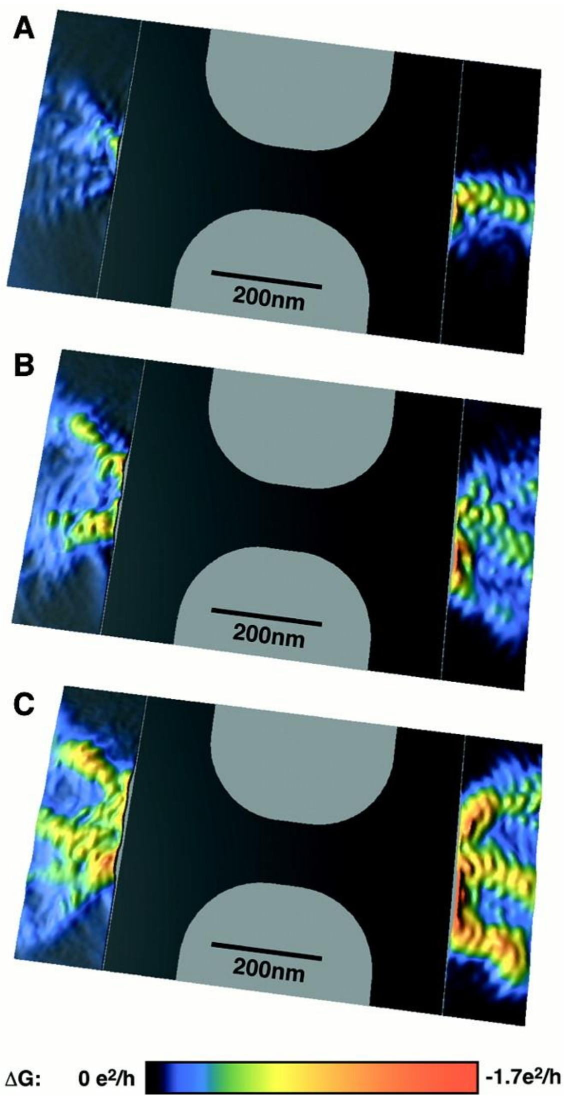
Charge sensor

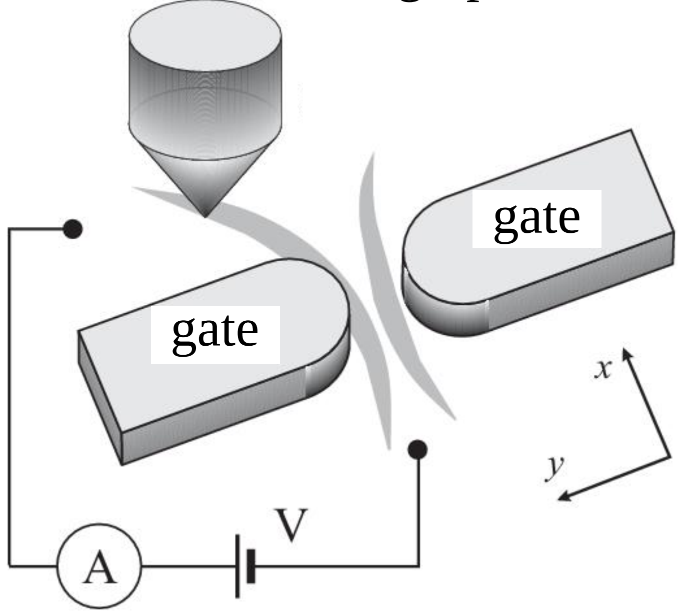


Excited states spectroscopy









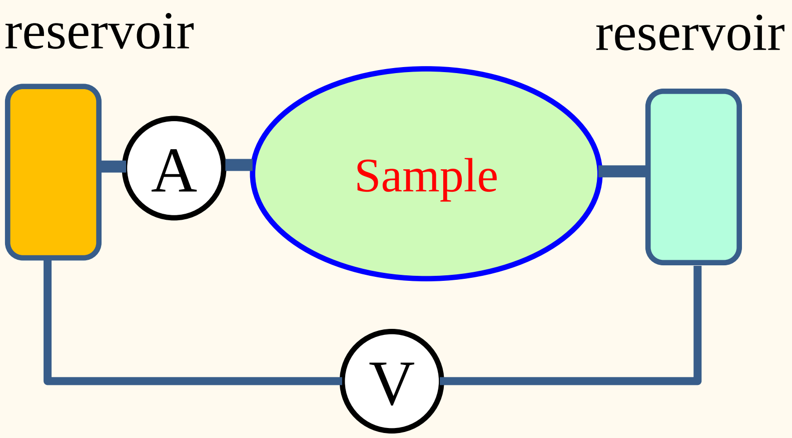
Tip image potential scatters electrons

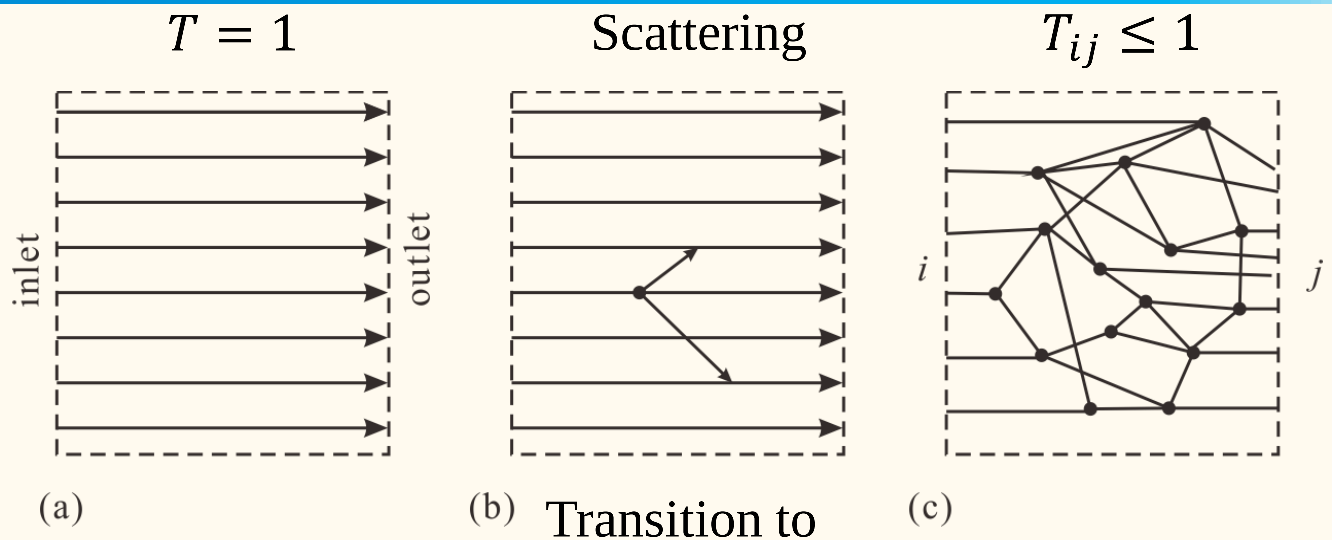
→ conductance shifts from quantized value

Scattering amplitude ∝ $|\psi|^2$

M. A. Topinka et al.,
Nature **410**, 183 (2001)

Landauer formula: multipath, s-matrix





Transition to other channels

Electron spin

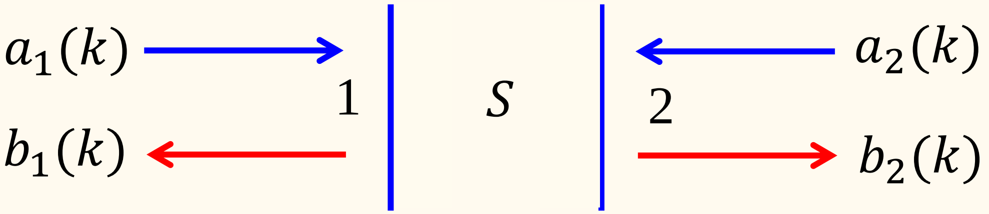
$$G = 2\frac{e^2}{h}\sum_{i,j} T_{ij}$$

Fermion antibunching

Waveguide connection

Waveguide connection

Scattering matrix   S-matrix



incoming

outgoing

$$\begin{pmatrix} b_1(k) \\ b_2(k) \end{pmatrix} = S \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} = \begin{pmatrix} r_\mathrm{L} & t_\mathrm{R} \\ t_\mathrm{L} & r_\mathrm{R} \end{pmatrix} \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix}$$

S-matrices: Unitary

Complex probability density flux $a_i(k) = \sqrt{v_{\mathrm{F}i}}\psi_{ai}(k_\mathrm{F})$

T matrix and transport through a quantum dot

$$\begin{pmatrix} b_1(k) \\ b_2(k) \end{pmatrix} = S \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} = \begin{pmatrix} r_\mathrm{L} & t_\mathrm{R} \\ t_\mathrm{L} & r_\mathrm{R} \end{pmatrix} \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix}$$

Transmission, reflection: $\mathcal{T} = |t|^2, \quad \mathcal{R} = |r|^2$

Conservation of probability $\mathcal{T} + \mathcal{R} = |t|^2 + |r|^2 = 1$ Subscripts L, R are not shown.

Time reversal symmetry → $t_\mathrm{L} = t_\mathrm{R}$ Unitary → $r_\mathrm{R} = -\dfrac{t}{t^*} r_\mathrm{L}^*$

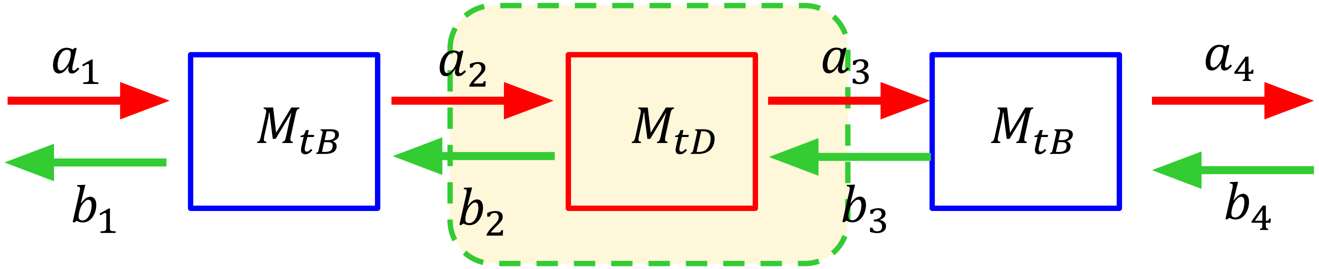
Formally matrix M_t which satisfies $M_t \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ can be obtained as

T matrix

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1/t^* & -r^*/t^* \\ -r/t & 1/t \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \equiv M_\mathrm{t} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

Quantum dot T matrix model

Quantum dot traverse → phase kL



Cascade connection

→ Product of T-matrices

$$\begin{pmatrix} 1/t^* & -r^*/t^* \\ -r/t & 1/t \end{pmatrix} \begin{pmatrix} \exp(ikL) & 0 \\ 0 & \exp(-ikL) \end{pmatrix} \begin{pmatrix} 1/t^* & -r^*/t^* \\ -r/t & 1/t \end{pmatrix}$$

Resonant transmission

Total transmission coefficient

$$\boxed{\mathcal{T}_{\rm tot}(k) = \frac{1}{1+4(|r|^2/|t|^4)\cos^2(\varphi+kL)}, \quad \varphi \equiv \arg(t)}$$

Peak position, energy

$$k_n = \frac{(n-1/2)\pi - \varphi}{L} \equiv \frac{\theta_n}{L}, \quad E'_n = \frac{\hbar^2 k_n^2}{2m} \quad (n = 1, 2, \cdots)$$

$$\delta k_n \equiv k - k_n \qquad 4\frac{|r|^2}{|t|^4}\cos^2(\varphi+kL) \approx 4\frac{|r|^2}{|t|^4}(L\delta k_n)^2$$

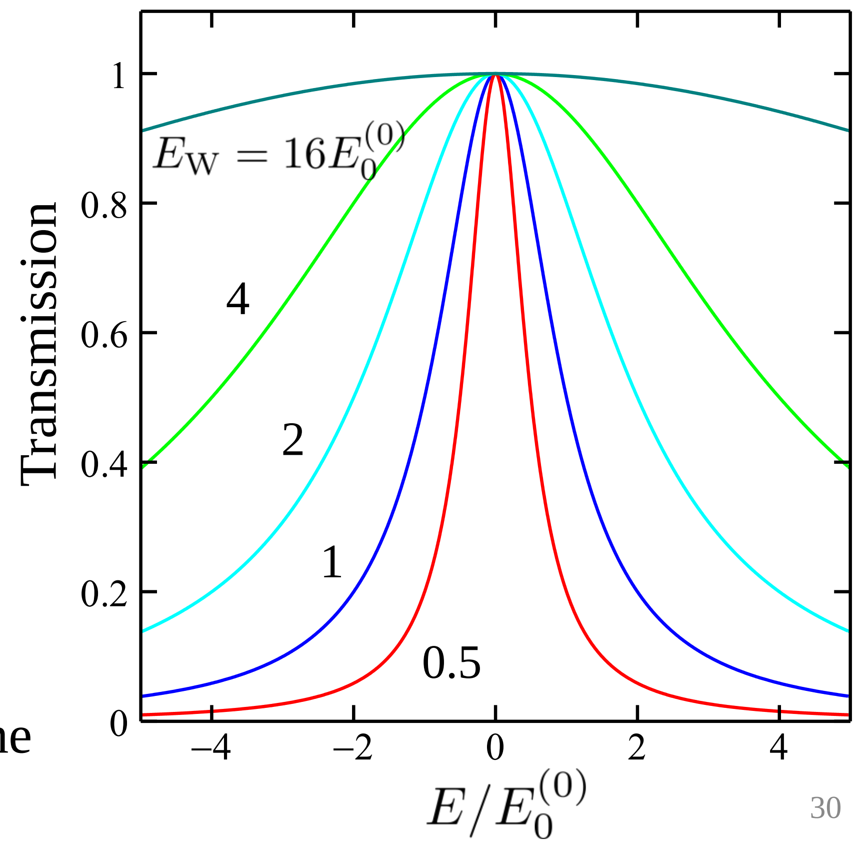
Breit-Wigner or Lorentz form

$$\boxed{\mathcal{T}_{\rm tot}^{(n)}(E) = \frac{E_{\rm w\it n}^2}{(E-E'_n)^2 + E_{\rm w\it n}^2},}$$

$$E_{\rm w\it n} \equiv \frac{|t|^2}{|r|}\frac{E'_n}{\theta_n}$$

$$2E_{\rm w\it n} = \frac{|t|^2}{|r|}\hbar\frac{\hbar k_n}{m}\frac{k_n}{\theta_n} = \frac{|t|^2}{|r|}\hbar\frac{v_n}{L} = \frac{\mathcal{T}}{\sqrt{\mathcal{R}}}\frac{\hbar}{\tau_n}$$

Dot traverse time



Transport via discrete chemical potential levels

The concept of transmission coefficient is applicable even to finite source-drain voltage transport.

T-matrix method, though, at least needs some modifications.

Occupation numbers of energy levels in the quantum dot: $\{n_i\} = \{n_1, n_2, \cdots\}$

Partition function $$Z = \sum_{\{n_i\}} \exp\left[-\frac{1}{k_{\rm B}T}\left(\sum_{i=1}^{\infty} E_i n_i + U(N) - NE_{\rm F}\right)\right] \quad N = \sum_i n_i$$

Distribution function $$P(\{n_i\}) = Z^{-1} \exp\left[-\frac{1}{k_{\rm B}T}\left(\sum_{i=1}^{\infty} E_i n_i + U(N) - NE_{\rm F}\right)\right]$$

The joint probability $$P_{\rm eq}(N, n_p = 1) = \sum_{\{n_i\}} P(\{n_i\})\delta_{N,\sum_i n_i}\delta_{n_p,1}$$

$$G = \frac{e^2}{k_{\rm B}T}\sum_{p=1}^{\infty}\sum_{N=1}^{\infty}\frac{\Gamma_p^{\rm l}\Gamma_p^{\rm r}}{\Gamma_p^{\rm l}+\Gamma_p^{\rm r}} P_{\rm eq}(N, n_p = 1)[1 - f(E_p + U(N) - U(N-1) - E_{\rm F})]$$

Transport via discrete chemical potential levels (2)

At high temperatures $\delta E,\ \dfrac{e^2}{C} \ll k_{\rm B}T \ll E_{\rm F}$ $\quad G = (G_{\rm r}^{-1} + G_{\rm l}^{-1})^{-1} = \dfrac{G_{\rm l}G_{\rm r}}{G_{\rm l} + G_{\rm r}}$

From the Landauer formula $G_{\rm l,r} = -\dfrac{e^2}{h}\displaystyle\int_0^\infty dE\mathcal{T}_{\rm l,r}(E)\dfrac{df}{dE}$

$$\Gamma_{\rm l,r}(E) = \nu(E)\mathcal{T}_{\rm l,r}(E) = \frac{\mathcal{T}_{\rm l,r}(E)}{h\rho(E)}$$

$$\mathcal{T}_{\rm l,r}(E) = h\Gamma_{\rm l,r}(E)\rho(E)$$

$$G_{\rm l,r} = \frac{e^2}{h}\mathcal{T}_{\rm l,r}(E_{\rm F}) = e^2\Gamma_{\rm l,r}(E_{\rm F})\rho(E_{\rm F})$$

$$G = e^2\rho\frac{\Gamma_{\rm l}\Gamma_{\rm r}}{\Gamma_{\rm l}+\Gamma_{\rm r}} = \frac{e^2}{h}\frac{\mathcal{T}_{\rm l}\mathcal{T}_{\rm r}}{\mathcal{T}_{\rm l}+\mathcal{T}_{\rm r}} \equiv G_\infty$$

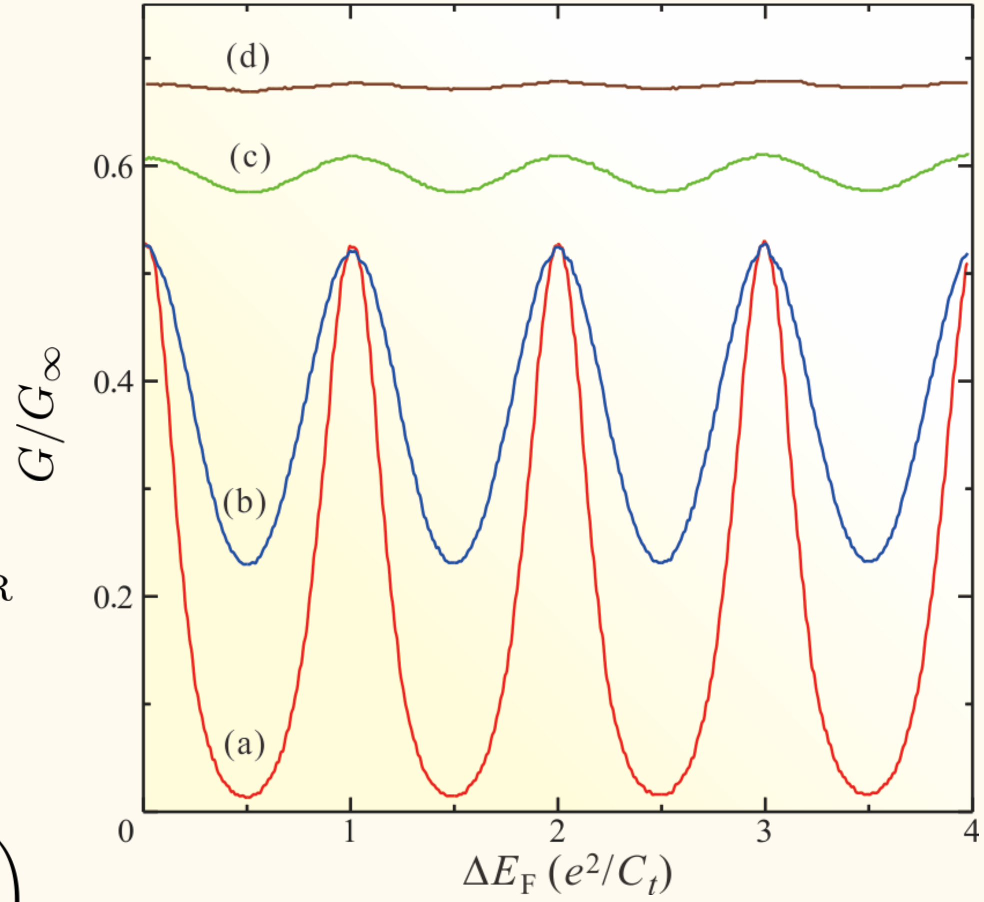
Effective tunnel barrier $\Delta(N) = U(N) - U(N-1) + \mu_{\rm dot} - E_{\rm R}$

$\Delta_{\rm min} \equiv \Delta(N_{\rm min})$ (Possible minimum value of Δ

$\delta E < k_{\rm B}T < e^2/C$

$$\frac{G}{G_{\rm max}} = \frac{(\mu_N - E_{\rm F})/k_{\rm B}T}{\sinh((\mu_N - E_{\rm F})/k_{\rm B}T)} \approx \cosh^{-2}\left(\frac{\mu_N - E_{\rm F}}{2.5k_{\rm B}T}\right)$$

Thermal energy (a)-(d) 0.075, 0.15, 0.3, 0.4 in



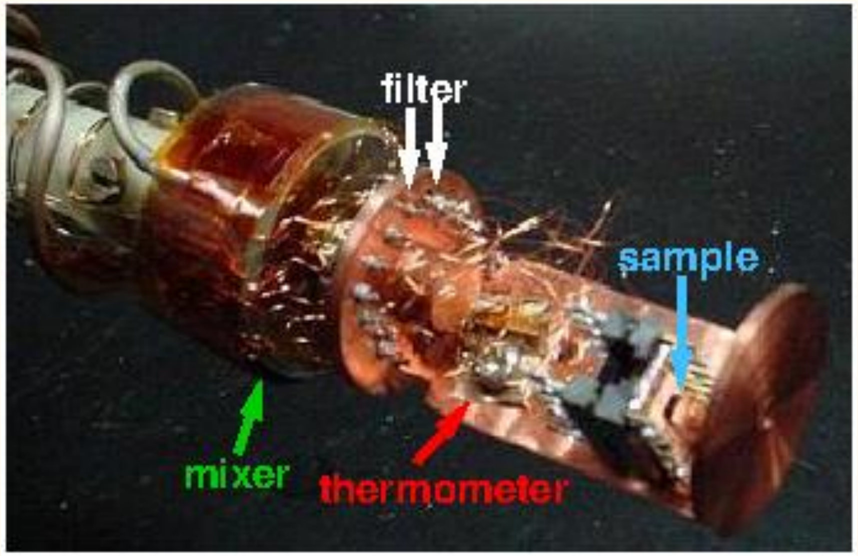
Coulomb peak and temperature sensing

$k_{\mathrm{B}}T \ll \delta E$

incoherent case $$\frac{G}{G_{\max}} = \cosh^{-2}\left(\frac{\mu_0 - E_{\mathrm{F}}}{2k_{\mathrm{B}}T}\right)$$

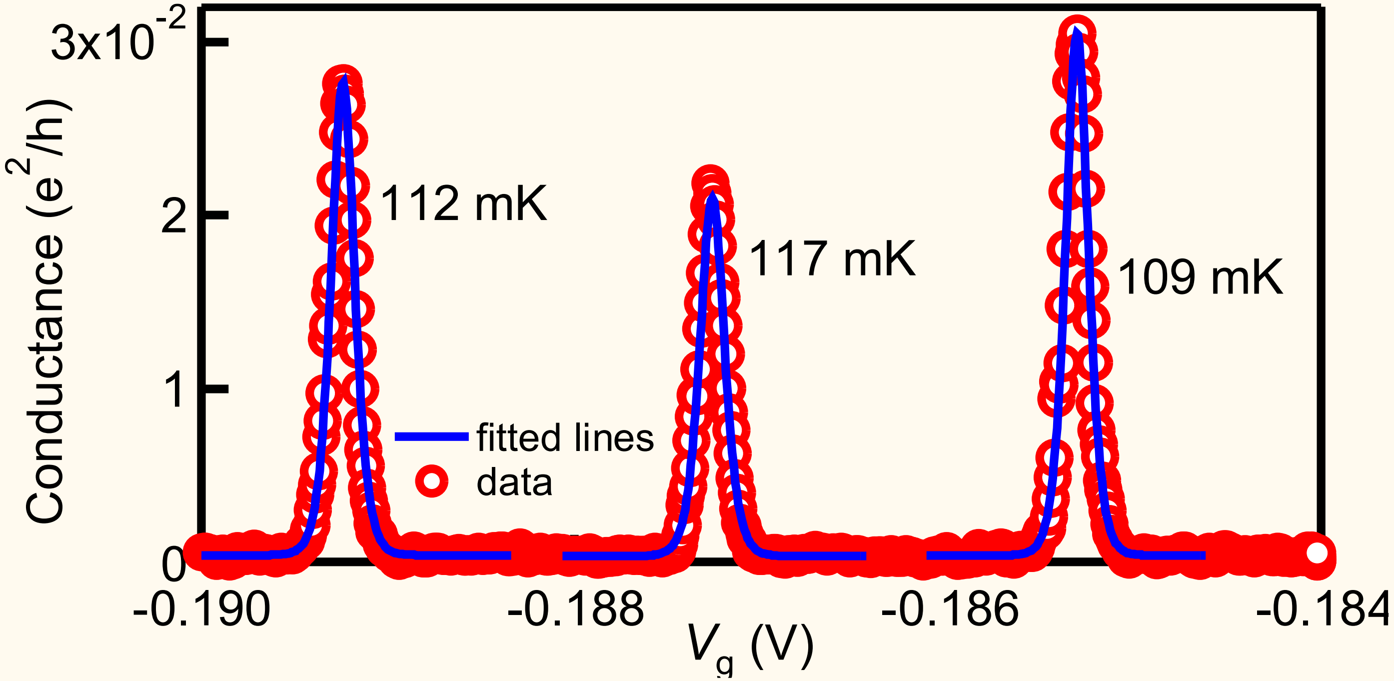
Coherent case $$\mathcal{T}(E) = \frac{(h\Gamma_{\mathrm{l}})(h\Gamma_{\mathrm{r}})}{(E-E_0)^2 + (h\Gamma_{\mathrm{l}})^2 + (h\Gamma_{\mathrm{r}})^2}$$

$$G = -\frac{e^2}{h}\int_0^{\infty} dE\,\mathcal{T}(E)\frac{df}{dE}$$



This can be used to measure the electron temperature of the cooled sample.

(Primary thermometry)



マスター方程式

力学系の状態　$\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$

時間発展が確率論的過程→ $x_1, x_2, \cdots, x_n$　確率変数

分布関数　$P(\boldsymbol{x}, t)$

マルコフ (Markov) 過程　$\underbrace{(\boldsymbol{x}(t_1), \boldsymbol{x}(t_2), \cdots, \boldsymbol{x}(t_j))}_{\text{有限個}}$ ⇒ t_j以降の$P(\boldsymbol{x}, t)$が決定される

(非マルコフ過程：無限個の初期条件を必要とする)

単純マルコフ過程　$\dfrac{\partial P(\boldsymbol{x}, t)}{\partial t} = \Gamma[P(\boldsymbol{x}, t)]$　のように偏微分方程式で表せる

状態遷移　$\boldsymbol{x} \to \boldsymbol{y}$　単位時間当たりの遷移確率　$W(\boldsymbol{x} \to \boldsymbol{y})$　で記述される場合

レート方程式

$$\frac{\partial P(\boldsymbol{x}, t)}{\partial t} = -\int d\boldsymbol{y} W(\boldsymbol{x} \to \boldsymbol{y}) P(\boldsymbol{x}, t) + \int d\boldsymbol{y} W(\boldsymbol{y} \to \boldsymbol{x}) P(\boldsymbol{y}, t)$$

マスター方程式 (master equation)

量子ドット(単電子効果)伝導とマスター方程式

量子ドット電荷状態指数 i，その確率をp_i　全確率条件 $\sum_i p_i = 1$

$i \to j$ の遷移確率 $R_{i,j}$　$$\frac{dp_i}{dt} = \sum_j R_{i,j} p_j - \sum_j R_{j,i} p_i$$ マスター方程式

- 量子ドット電荷状態として基底状態のみ考慮→電子数Nで決まる
- 遷移は隣接電荷状態間のみ

$$\frac{dp_N}{dt} = R_{N,N-1} p_{N-1} + R_{N,N+1} p_{N+1} - (R_{N-1,N} + R_{N+1,N}) p_N$$

遷移確率 (D:ドレイン，S：ソース)　$R_{N,N-1} = f_{\rm D}(\mu(N), \mu_{\rm D})\Gamma_{\rm D} + f_{\rm S}(\mu(N), \mu_{\rm S})\Gamma_{\rm S}$

分布関数 ($\mu(N)$位置の空隙確率)　$$f_i(\mu(N), \mu_i) = \frac{1}{\exp[(\mu(N) - \mu_i)/k_{\rm B}T] + 1}, \quad i = {\rm D, S}$$

その他，同様に
$$\begin{cases} R_{N,N+1} = (1 - f_{\rm D}(\mu(N+1), \mu_{\rm D})\Gamma_{\rm D} + (1 - f_{\rm S}(\mu(N+1), \mu_{\rm S}))\Gamma_{\rm S}, \\ R_{N+1,N} = f_{\rm D}(\mu(N+1), \mu_{\rm D})\Gamma_{\rm D} + f_{\rm S}(\mu(N+1), \mu_{\rm S})\Gamma_{\rm S}, \\ R_{N-1,N} = (1 - f_{\rm D}(\mu(N), \mu_{\rm D})\Gamma_{\rm D} + (1 - f_{\rm S}(\mu(N), \mu_{\rm S}))\Gamma_{\rm S} \end{cases}$$

量子ドット伝導とマスター方程式(2)

定常状態 $$\frac{dp_i}{dt} = R_{i,i-1}p_{i-1} + R_{i,i+1}p_{i+1} - (R_{i-1,i} + R_{i+1,i})p_i = 0 \qquad \sum_i p_i = 1$$

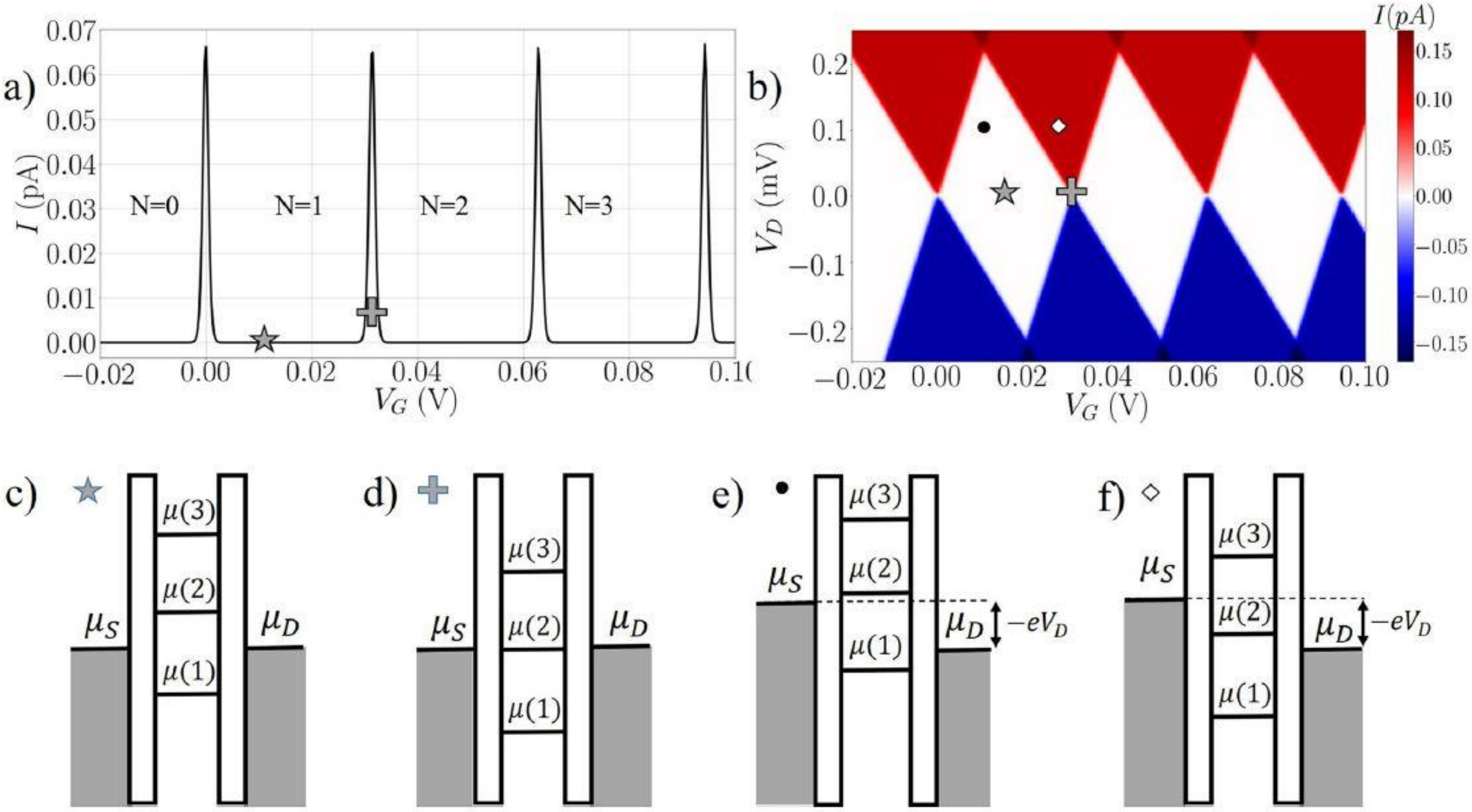
行列表現 $$A = \begin{pmatrix} -R_{1,0} & R_{0,1} & \cdots & R_{0,N} \\ R_{1,0} & -R_{0,1} - R_{2,1} & \cdots & R_{1,N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N-1,0} & R_{N-1,1} & \cdots & -R_{N-1,N} \\ 1 & 1 & \cdots & 1 \end{pmatrix} \quad C = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \quad P = \begin{pmatrix} p_0 \\ p_1 \\ \vdots \\ p_N \end{pmatrix}$$

Pを数値的に求める $$C = AP, \quad \text{i.e.,} \quad P = A^{-1}C$$

電流 $$I = -e\left(\sum_i p_i \Gamma_{\mathrm{D}}(1 - f_{\mathrm{D}}(\mu(i), \mu_{\mathrm{D}})) - \sum_i (1 - p_i)\Gamma_{\mathrm{D}} f_{\mathrm{D}}(\mu(i), \mu_{\mathrm{D}})\right)$$

量子ドット伝導とマスター方程式(3)

Fundamentals of quantum dots : Master equation



Summary

1. 量子ドットとは，量子ドットの製法
2. 単電子効果
3. 量子閉じ込め効果
4. 量子ドットを通した電気伝導