2022.12.01 Lecture 3
10:30 – 12:00

量子ドット：閉じ込めと開放の物理学

Institute for Solid State Physics, University of Tokyo

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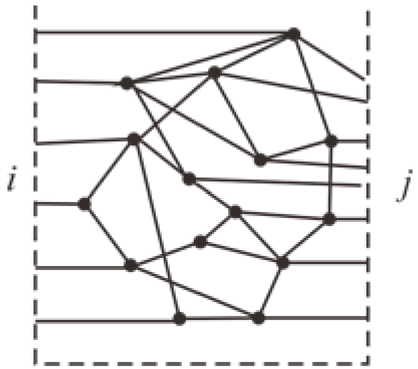
1. Origin of “scar” wavefunction
2. S-matrix connection for makeup of interference circuits
3. Quantum dots in interference circuits
4. Phase rigidity problem
5. Strongly coupled state (originates from scar wavefunction)

Review: 量子伝導の基礎, Feynman経路積分

Landauerの２端子公式　$G = 2\frac{e^2}{h}\sum_{i,j} T_{ij}$

Waveguide connection:
S行列　$\begin{pmatrix} b_1(k) \\ b_2(k) \end{pmatrix} = S\begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} = \begin{pmatrix} r_\mathrm{L} & t_\mathrm{R} \\ t_\mathrm{L} & r_\mathrm{R} \end{pmatrix}\begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix}$



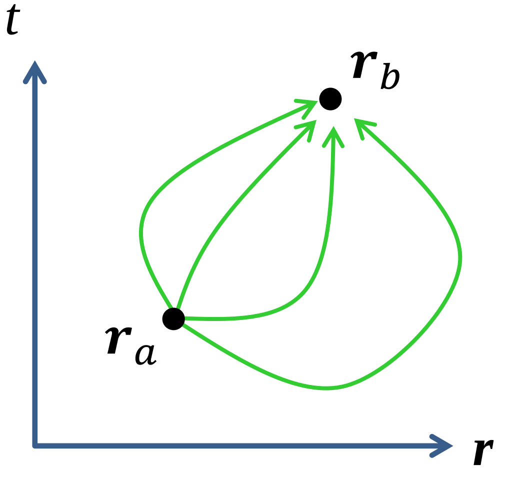
経路積分 (一粒子問題)

$(\boldsymbol{r}_a, t_a) \rightarrow (\boldsymbol{r}_b, t_b)$ の遷移：空間のあらゆる可能な経路をたどる

経路の重み付け：複素数　その位相をθ とする

系のLagrangian：　$\mathcal{L}(\dot{\boldsymbol{r}}, \boldsymbol{r}, \xi)$

作用 (action)　$S\{\boldsymbol{r}(t)\} \equiv \int_{t_a}^{t_b} \mathcal{L}(\dot{\boldsymbol{r}}, \boldsymbol{r}, \xi) d\xi$　　$\theta = \frac{S}{\hbar}$　　($\boldsymbol{r}(t)$ 経路)

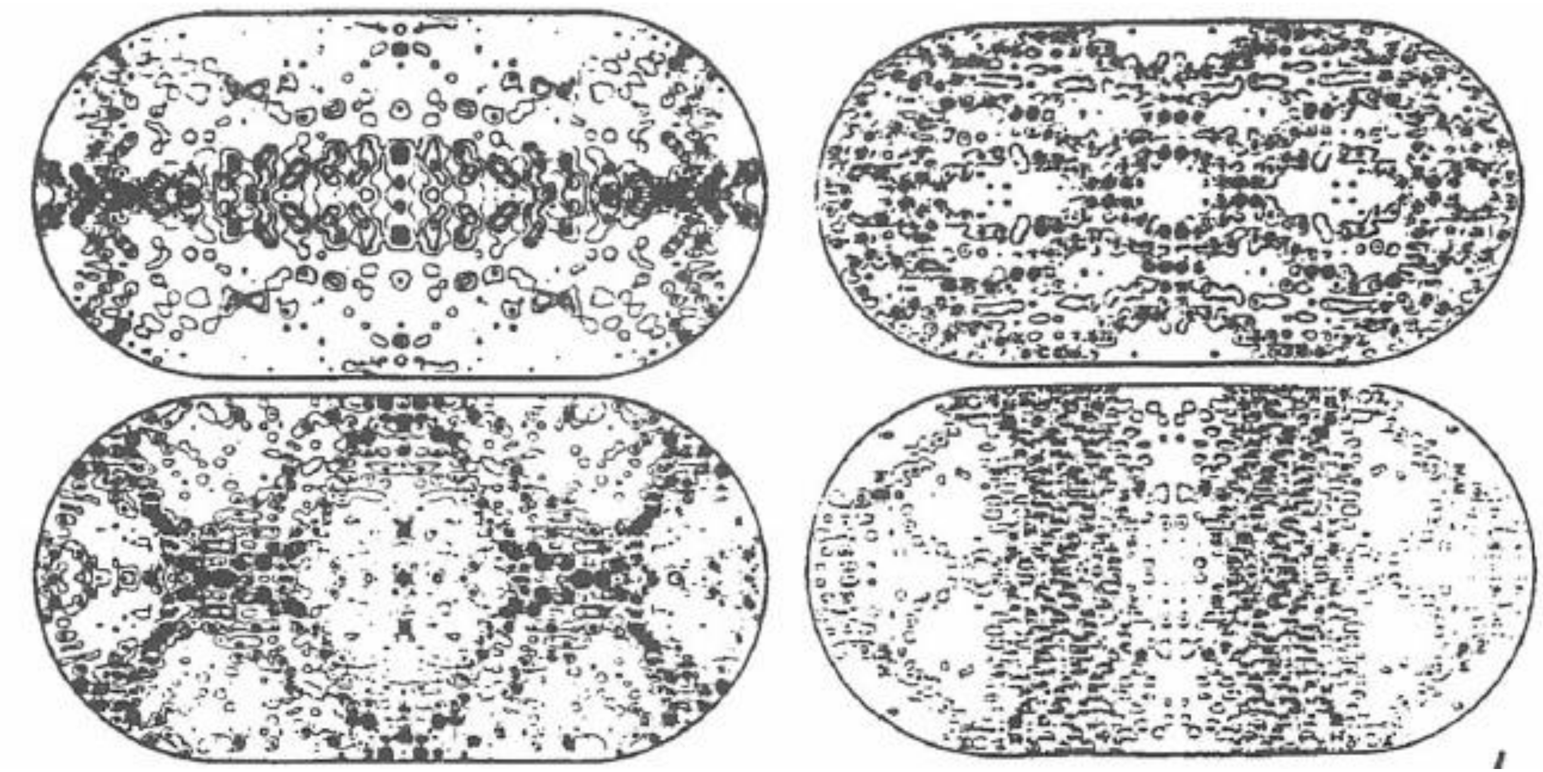


$(\boldsymbol{r}_a, t_a) \rightarrow (\boldsymbol{r}_b, t_b)$　遷移確率　$\int_{\boldsymbol{r}_a}^{\boldsymbol{r}_b} \mathcal{D}\{\boldsymbol{r}(t)\} \exp\left[i\frac{S\{\boldsymbol{r}(t)\}}{\hbar} \right]$　経路積分 ⇨ Schrödinger 方程式

寄与の大きな経路　$\frac{\delta S}{\delta \boldsymbol{r}(t)} = 0 = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{r}}} - \frac{\partial \mathcal{L}}{\partial t}$

Scar 波動関数

量子カオスに特有な現象：Scar波動関数の出現



Heller, Phys. Rev. Lett. **53**, 1515 (1984).

古典系の不安定周期軌道(KAMトーラス) → 量子系では大きな波動関数振幅 Scar 波動関数

Scar波動関数 (2)

Bogomolny, Physica D **31**, 169 (1988), Berry Proc. Royal Soc. London **423**, 219 (1989),
Adam & Fishman, J. Phys. A **26**, 2113 (1993).

Energy window $w(E)$

$$\langle|\psi(q)|^2\rangle = \int w(E)\sum_n |\psi(q_n)|^2\delta(E-E_n)dE$$

これをGreen関数

$$\begin{cases} G(q_A,q_B,E) = \sum_n \dfrac{\psi_n^*(q_B)\psi(q_A)}{E-E_n} \\ \sum_n \delta(E-E_n)\psi_n^*(q_A)\psi_n(q_B) = -\dfrac{1}{\pi}\mathrm{Im}[G(q_A,q_B,E)] \end{cases}$$

を使って

$$\langle|\psi(q)|^2\rangle = -\frac{1}{\pi}\mathrm{Im}\left[\int w(E)G(q,q,E)dE\right]$$

と書く.

Green関数を滑らかな成分と振動成分に分け

$$G(q,q,E) = G_0(q,q,E) + G_{\rm osc}(q,q,E)$$

滑らか成分

$$G_0(q,q,E) = \left(\frac{1}{2\pi i\hbar}\right)^d \int dp\frac{1}{E-H(p,q)}$$

その状態密度

$$\rho_0(q,E) \equiv \left(\frac{1}{2\pi i\hbar}\right)^d \int dp\delta[E-H(p,q)]$$

Scar波動関数 (3)

$$\langle|\psi(q)|^2\rangle_0 = \langle\rho_0(q,E)\rangle$$

振動成分

$$G_{\rm osc}(q,q,E) = -\frac{i}{\hbar}\left(\frac{1}{2\pi i\hbar}\right)^{(d-1)/2}\sum_r |\Delta_r|^{1/2}\exp\left[\frac{i}{\hbar}S_r(q,q,E) - i\frac{\nu_r\pi}{2}\right]$$

和はすべての閉軌道
(周期的/非周期的)

ν_r ：マスロフ指数 (方向を変える散乱回数)

作用を展開

$$S_r(q,q,E) = S_r(q,q,E_0) + \underline{T_r}(q,E_0)(E-E_0) + \cdots .$$

軌道周回時間

$$\langle G_{\rm osc}(q,q,E)\rangle = -\frac{i}{\hbar}\left(\frac{1}{2\pi i\hbar}\right)^{(d-1)/2}\sum_r |\Delta_r|^{1/2}\exp\left[\frac{i}{\hbar}S_r(q,q,E_0) - i\frac{\nu_r\pi}{2}\right]\hat{w}[T_r(q,E_0)]$$

時間軸上の窓的関数

$$\hat{w}(T) = \int w(E)\exp\left[\frac{i}{\hbar}T(E-E_0)\right]dE$$

結局　$q \to q + \Delta q$

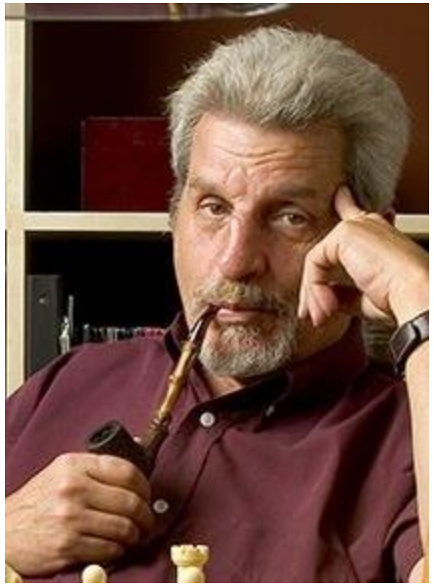
の変化で最も影響が
大きいのは作用部分

$$S_r(q+\Delta q, q+\Delta q, E_0) = S_r(q,q,E_0) + \Delta q\left(\frac{\partial S_r}{\partial q_B} + \frac{\partial S_r}{\partial q_A}\right)\Bigg|_{q_A=q_B=q}$$

$$= S_r(q,q,E_0) + \Delta q(p_B|_q - p_A|_q)$$

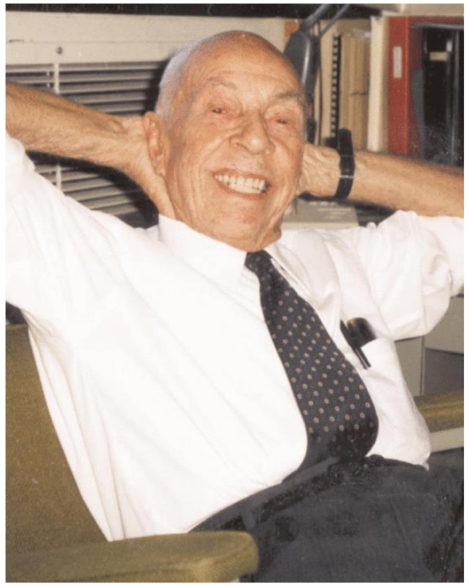
A, Bは，閉軌道のスタート，ゴールを示す：周期軌道以外では第２項が大きくなり，干渉で振幅は消滅

量子干渉回路中の量子ドット

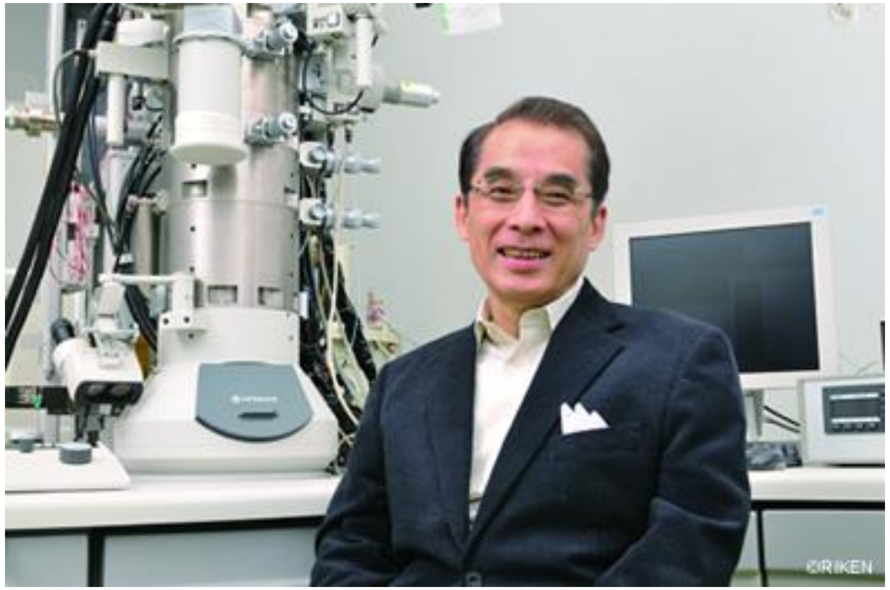


Yakir Aharonov
(1932-)

David Bohm
(1917-1992)



Ugo Fano
(1912-2001)



外村 彰
(1942-2012)

Markus Büttiker
(1950-2013)

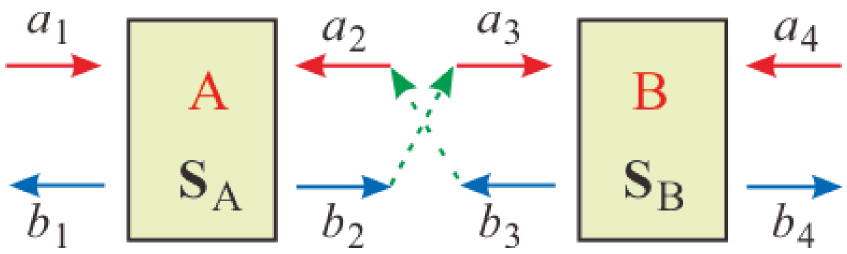
Rolf Landauer
(1927-1999)

Lars Onsager
(1903-1976)

Cascade connection of S-matrices

カスケード接続

条件: $b_2 = a_3, \quad a_2 = b_3$



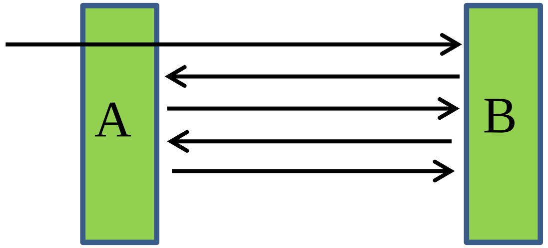
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = S_A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_{\rm L}^{(\rm A)} & t_{\rm R}^{(\rm A)} \\ t_{\rm L}^{(\rm A)} & r_{\rm R}^{(\rm A)} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

$$\begin{pmatrix} b_3 \\ b_4 \end{pmatrix} = S_B \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} r_{\rm L}^{(\rm B)} & t_{\rm R}^{(\rm B)} \\ t_{\rm L}^{(\rm B)} & r_{\rm R}^{(\rm B)} \end{pmatrix} \begin{pmatrix} a_3 \\ a_4 \end{pmatrix}$$

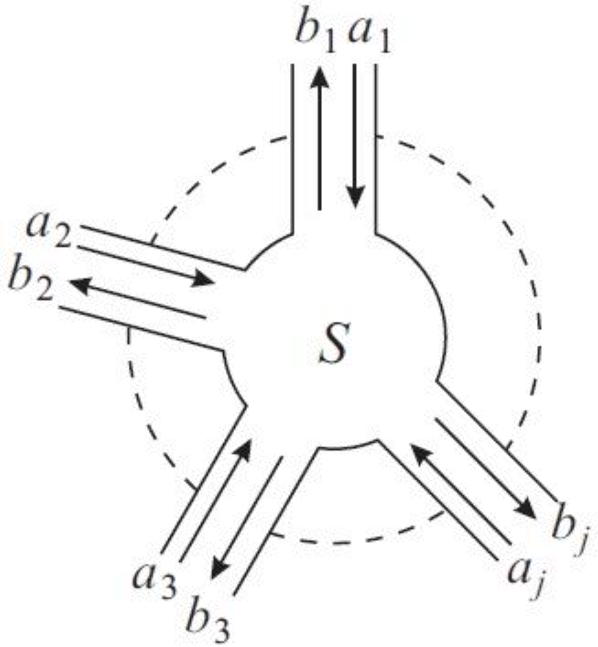
$$S_{\rm AB} = \begin{pmatrix} r_{\rm L}^{(\rm A)} + t_{\rm R}^{(\rm A)} r_{\rm L}^{(\rm B)} \left(I - r_{\rm R}^{(\rm A)} r_{\rm L}^{(\rm B)} \right)^{-1} t_{\rm L}^{(\rm A)} & t_{\rm R}^{(\rm A)} \left(I - r_{\rm L}^{(\rm B)} r_{\rm R}^{(\rm A)} \right)^{-1} t_{\rm R}^{(\rm B)} \\ t_{\rm L}^{(\rm B)} \left(I - r_{\rm R}^{(\rm A)} r_{\rm L}^{(\rm B)} \right)^{-1} t_{\rm L}^{(\rm A)} & r_{\rm R}^{(\rm B)} + t_{\rm L}^{(\rm B)} \left(I - r_{\rm R}^{(\rm A)} r_{\rm L}^{(\rm B)} \right)^{-1} r_{\rm R}^{(\rm A)} t_{\rm R}^{(\rm B)} \end{pmatrix}$$

Multi-terminal connection of S-matrices

$$\left(I - r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})}\right)^{-1} = I + r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})} + (r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})})^2 + (r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})})^3 + \cdots$$



Multi-channel



$$\boldsymbol{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & & & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix} = \mathrm{S}\boldsymbol{a}$$

Reciprocity $S_{ij} = S_{ji}$
(time-reversal symmetry)

Unitary $\sum_j S_{ji} S_{jk}^* = \delta_{ik}$

Onsager reciprocity

$$\left[\frac{(i\hbar\nabla + e\boldsymbol{A})^2}{2m} + V\right]\psi = E\psi$$ Complex conjugate and $\boldsymbol{A} \to -\boldsymbol{A}$

$$\left[\frac{(i\hbar\nabla + e\boldsymbol{A})^2}{2m} + V\right]\psi^* = E\psi^*$$ $\{\psi^*(-B)\} = \{\psi(B)\}$

Scattering solution: Sc{$a \to b$} $\mathrm{Sc}\{\boldsymbol{a}(B) \to \boldsymbol{b}(B)\} \in \{\psi(B)\}$, *i.e.*, $\boldsymbol{b}(B) = S(B)\boldsymbol{a}(B)$

$\boldsymbol{b}^*(B) = S^*(B)\boldsymbol{a}^*(B)$

$\mathrm{Sc}\{\boldsymbol{b}^*(-B) \to \boldsymbol{a}^*(-B)\} \in \{\psi^*(-B)\} = \{\psi(B)\}$ *i.e.* $\boldsymbol{a}^*(-B) = S(B)\boldsymbol{b}^*(-B)$

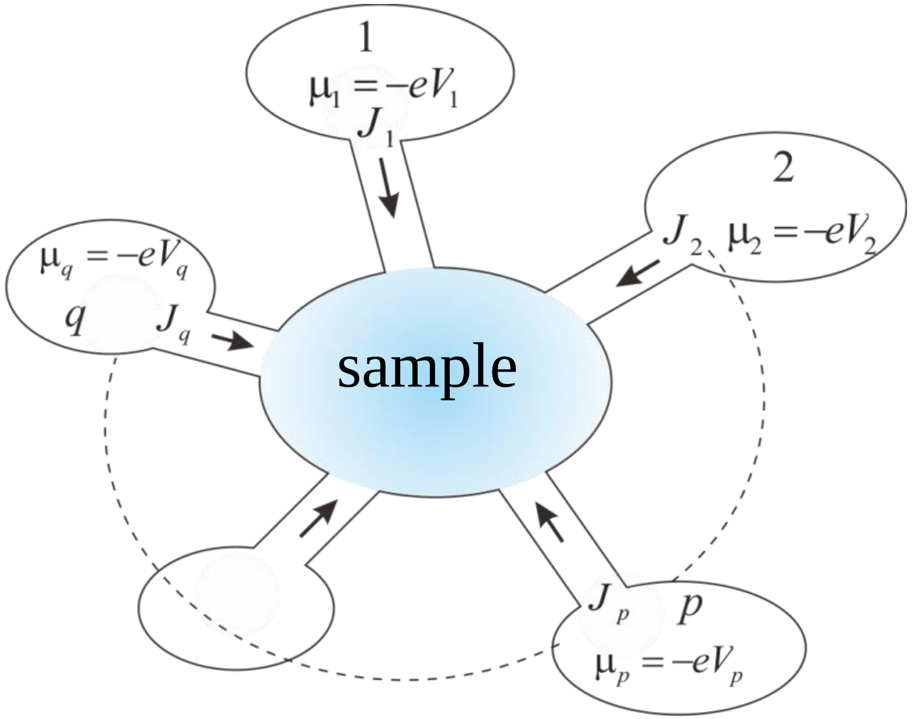
$$\boldsymbol{b}^*(B) = S^{-1}(-B)\boldsymbol{a}^*(B)$$

$S^*(B) = S^{-1}(-B) = S^\dagger(-B)$ (unitarity $SS^\dagger = S^\dagger S = I$)

$$\boxed{S(B) = {}^tS(-B) \qquad S_{ij}(B) = S_{ji}(-B)}$$

Landauer-Büttker formula

量子干渉回路中の量子ドット：S行列



$$J_p = -\frac{2e}{h}\sum_q [T_{q\leftarrow p}\mu_p - T_{p\leftarrow q}\mu_q]$$

$$\mathscr{T}_{pq} \equiv T_{p\leftarrow q} \quad (p \neq q), \qquad \mathscr{T}_{pp} \equiv -\sum_{q\neq p} T_{q\leftarrow p}$$

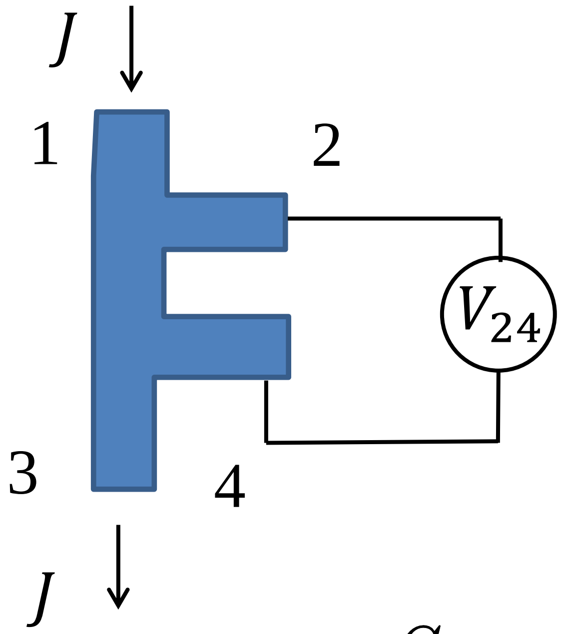
$$\boldsymbol{J} = {}^t(J_1, J_2, \cdots), \boldsymbol{\mu} = {}^t(\mu_1, \mu_2, \cdots)$$

$$\boldsymbol{J} = \frac{2e}{h}\mathscr{T}\boldsymbol{\mu}$$

$$V_q = \frac{\mu_q}{-e}, \qquad G_{pq} \equiv \frac{2e^2}{h}T_{p\leftarrow q} \ \text{ then } \ J_p = \sum_q [G_{qp}V_p - G_{pq}V_q]$$

$$\sum_q J_q = 0 \qquad \sum_q [G_{qp} - G_{pq}] = 0 \qquad G_{qp}(B) = G_{pq}(-B)$$

Landauer-Büttker formula: Application to 4-terminal measurement



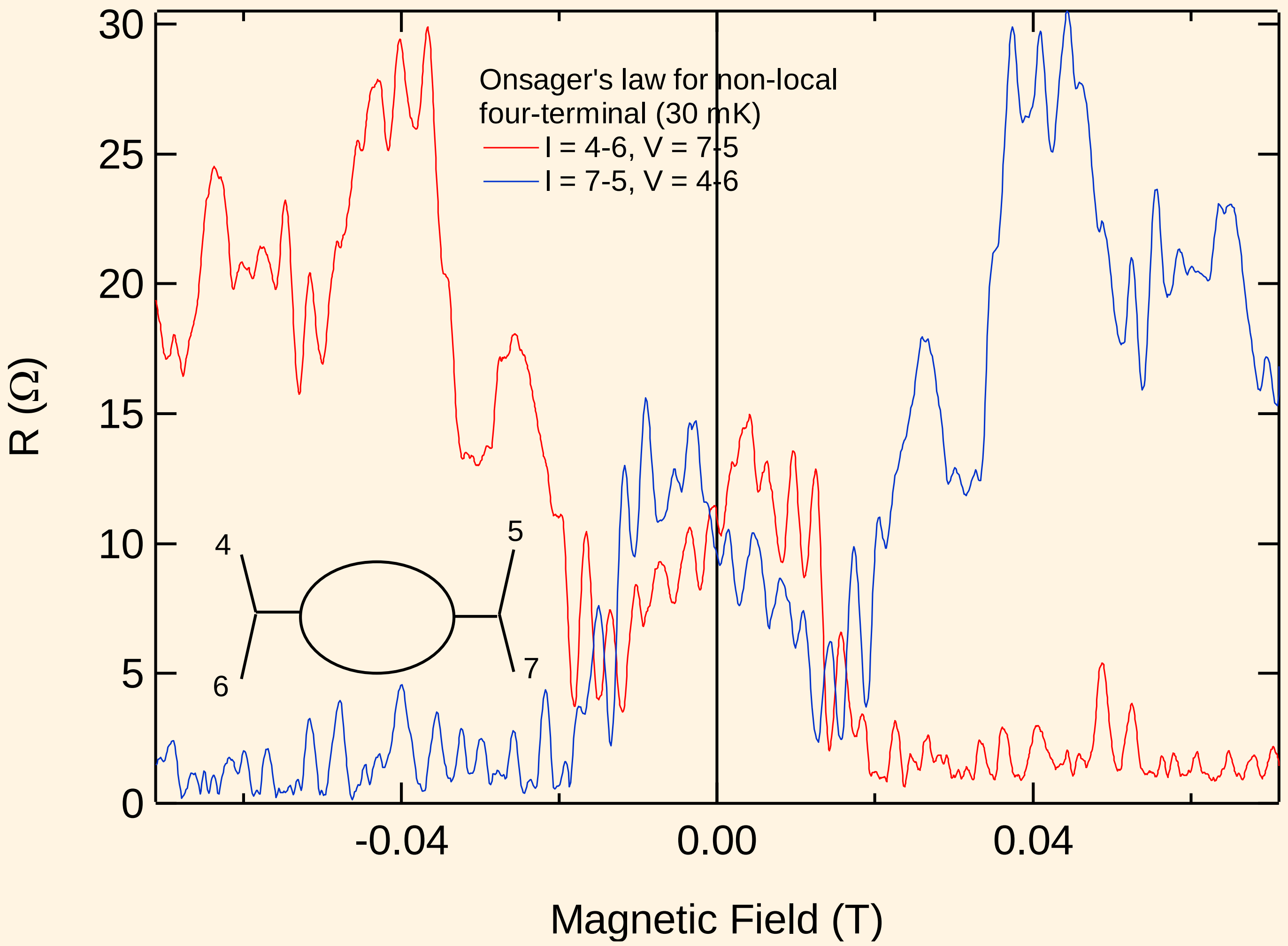
$$\alpha_{11} = 2G_q[-\mathscr{T}_{11} - S^{-1}(\mathscr{T}_{14} + \mathscr{T}_{12})(\mathscr{T}_{41} + \mathscr{T}_{21})]$$
$$\alpha_{12} = 2G_q S^{-1}(\mathscr{T}_{12}\mathscr{T}_{34} - \mathscr{T}_{14}\mathscr{T}_{32})$$
$$\alpha_{21} = 2G_q S^{-1}(\mathscr{T}_{21}\mathscr{T}_{43} - \mathscr{T}_{23}\mathscr{T}_{41})$$
$$\alpha_{22} = 2G_q[-\mathscr{T}_{22} - S^{-1}(\mathscr{T}_{21} - \mathscr{T}_{23})(\mathscr{T}_{32} + \mathscr{T}_{12})]$$

$$S = \mathscr{T}_{12} + \mathscr{T}_{14} + \mathscr{T}_{32} + \mathscr{T}_{34} = \mathscr{T}_{21} + \mathscr{T}_{41} + \mathscr{T}_{23} + \mathscr{T}_{43}$$

$$\mathcal{R}_{13,24} = \frac{V_2 - V_4}{J_1} = \frac{\alpha_{21}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}$$

$$\mathcal{R}_{mn,kl}(B) = -\mathcal{R}_{kl,mn}(-B)$$ Onsager reciprocity

Onsager reciprocity in AB ring

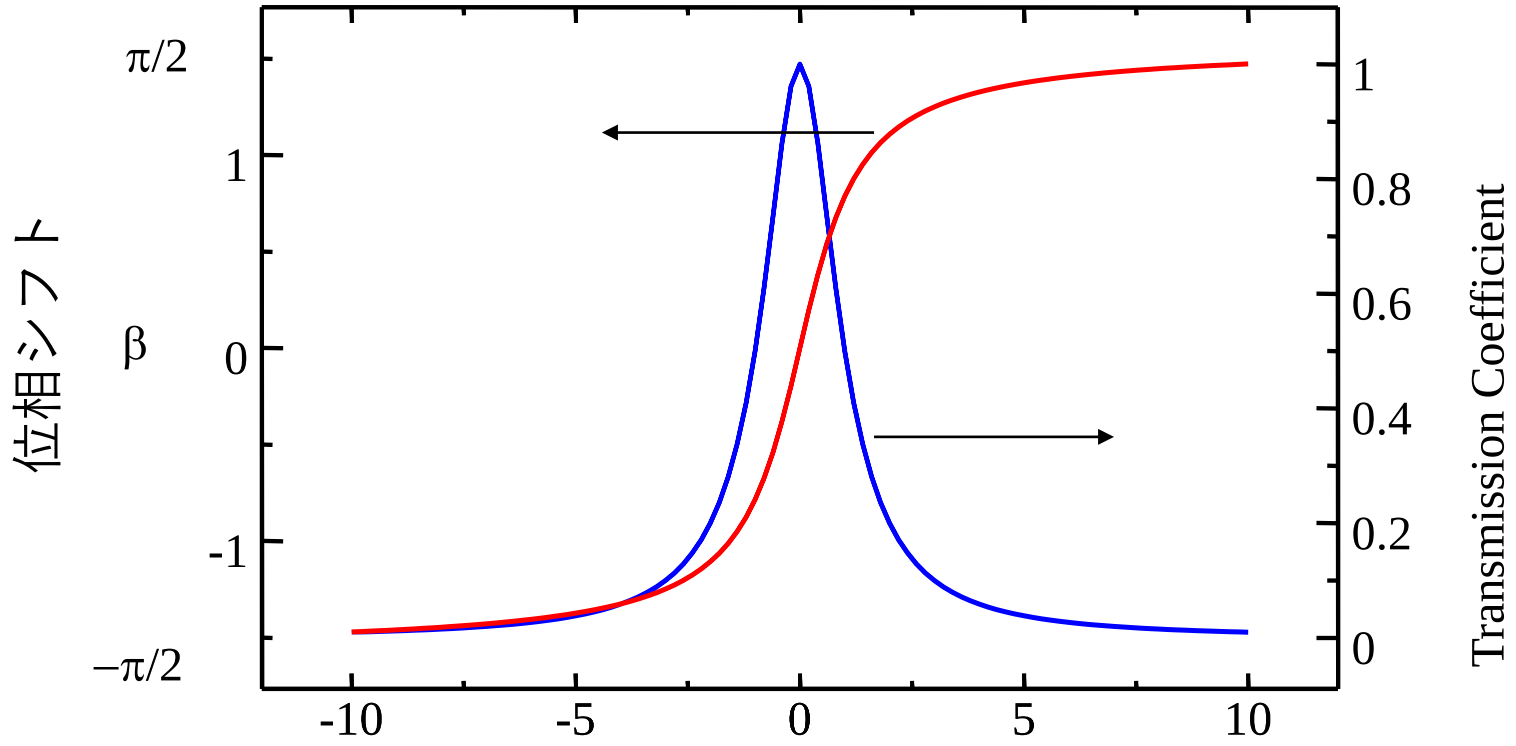


$$\mathcal{R}_{ij,kl}(B) = \mathcal{R}_{kl,ij}(-B)$$

In the case of two-terminal measurement

$$R(B) = R(-B)$$

Breit-Wigner form と位相シフト



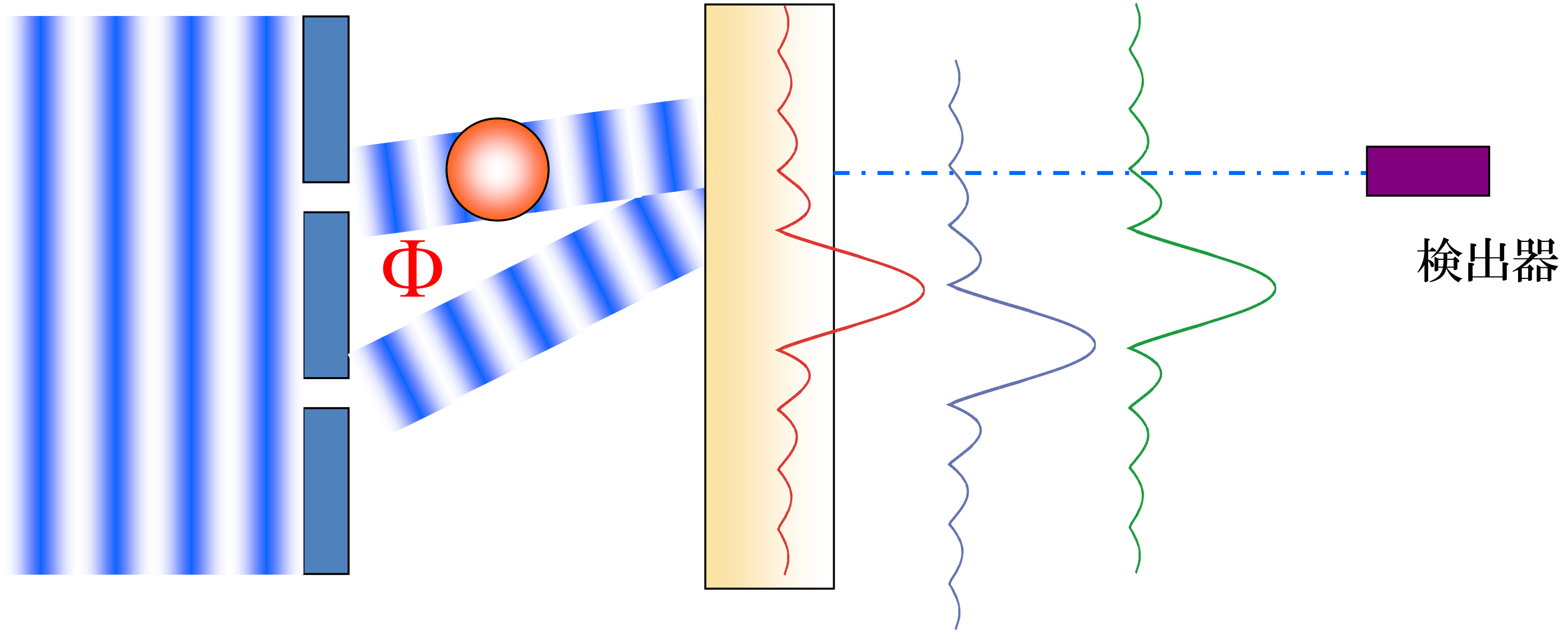
共鳴透過の一般的性質

共鳴の前後で，共鳴体を通した波の位相シフトが
πだけ連続的に変化

$$T(E)=\frac{4\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_2)^2}\cdot\frac{\Gamma^2/4}{(E-E_n)^2+(\Gamma/2)^2}$$

$$t=C\frac{\Gamma/2}{E-E_n+i\Gamma/2}$$

Phase shift measurement with interference (?)

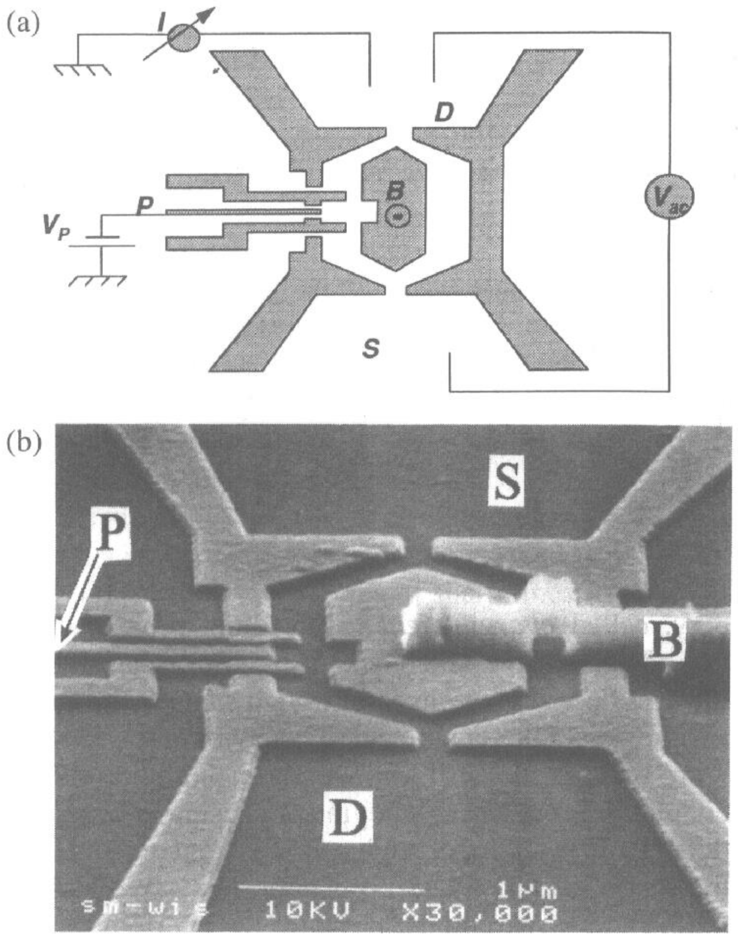


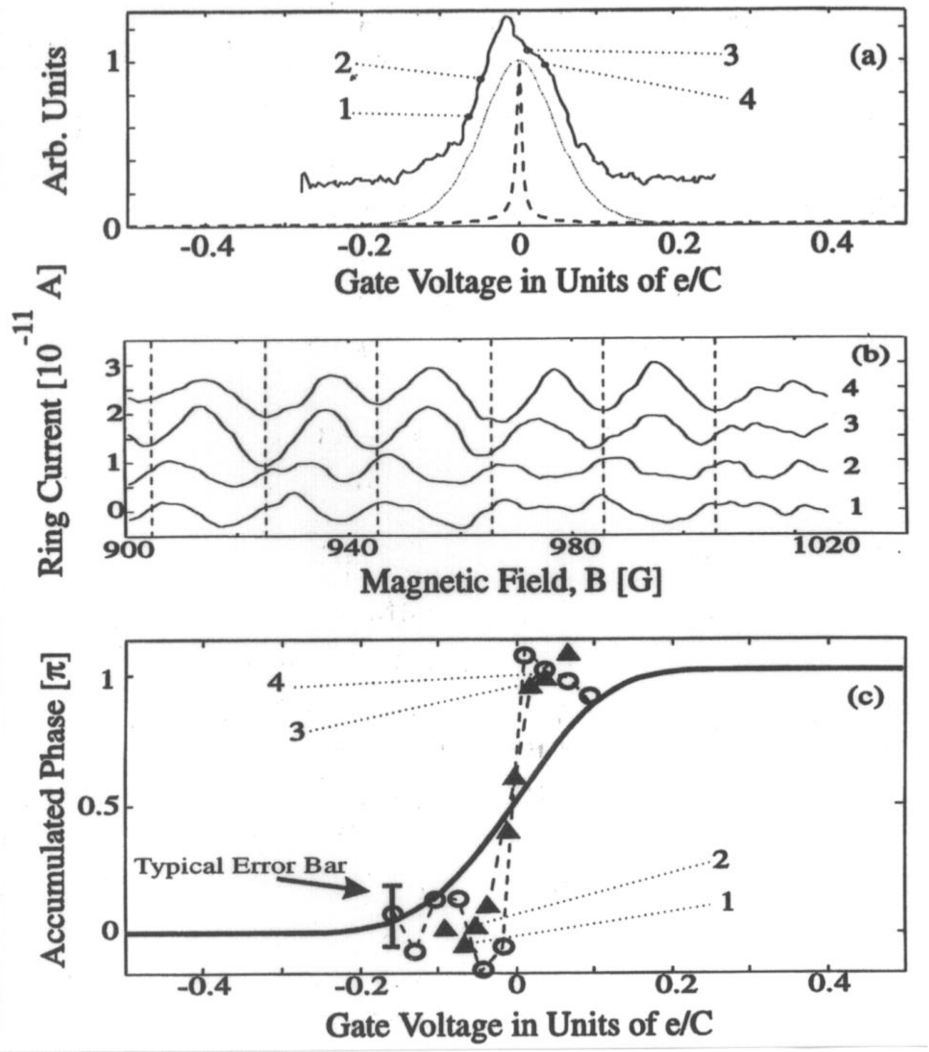
$$G = G_0 \cos\left(2\pi \frac{\phi}{\phi_0} + \theta\right)$$

$$\phi_0 \equiv \frac{h}{e}$$

Phase jump problem

A. Yacoby et al. Phys. Rev. Lett. **74**, 4047 (1995)





Breit-Wigner型に比べて明らかに位相シフトが急激に変化している

⇩

量子ドット内の多体効果によるものでは？

Phase rigidity

PHYSICAL REVIEW B VOLUME 53, NUMBER 15 15 APRIL 1996-I

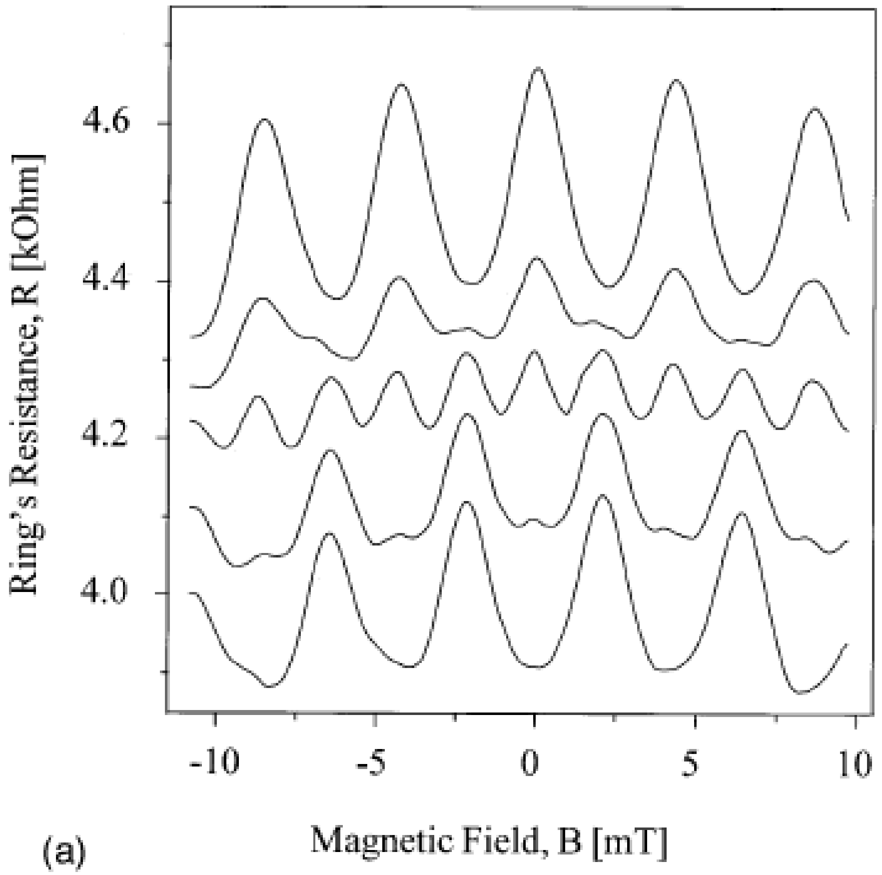
Phase rigidity and $h/2e$ oscillations in a single-ring Aharonov-Bohm experiment

A. Yacoby, R. Schuster, and M. Heiblum
Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

(Received 12 December 1995; revised manuscript received 24 January 1996)

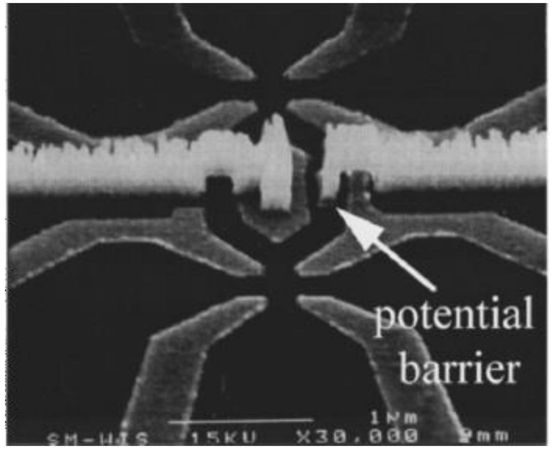
Phase rigidity問題

２端子素子では位相シフトは測れない



(a)

Gate voltage



$$R(B) = R_0 \cos\left(2\pi \frac{BS}{\phi_0} + \theta_0\right)$$

$$R(B) = R(-B)$$

$$\theta_0 = 0, \pi$$

Breaking of phase rigidity with multi-path effect

PHYSICAL REVIEW B **73**, 195329 (2006)

Breakdown of phase rigidity and variations of the Fano effect in closed Aharonov-Bohm interferometers

Amnon Aharony,[1,2,3] Ora Entin-Wohlman,[1,2,3] Tomohiro Otsuka,[1] Shingo Katsumoto,[1,*] Hisashi Aikawa,[1,†] and Kensuke Kobayashi[1,‡]
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[2]*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel*
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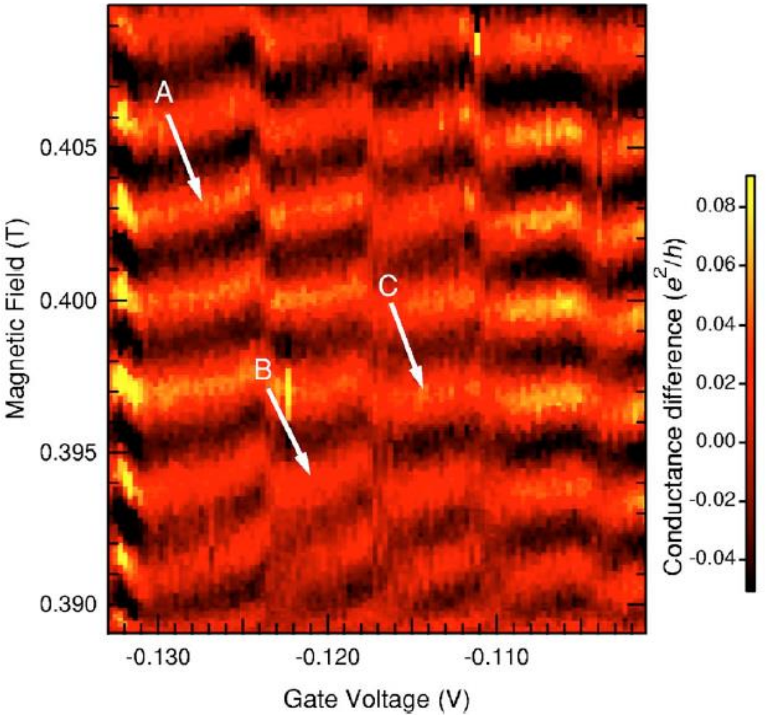
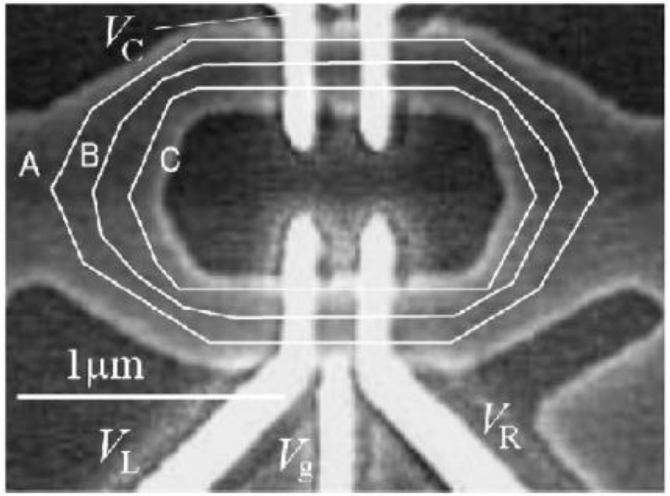
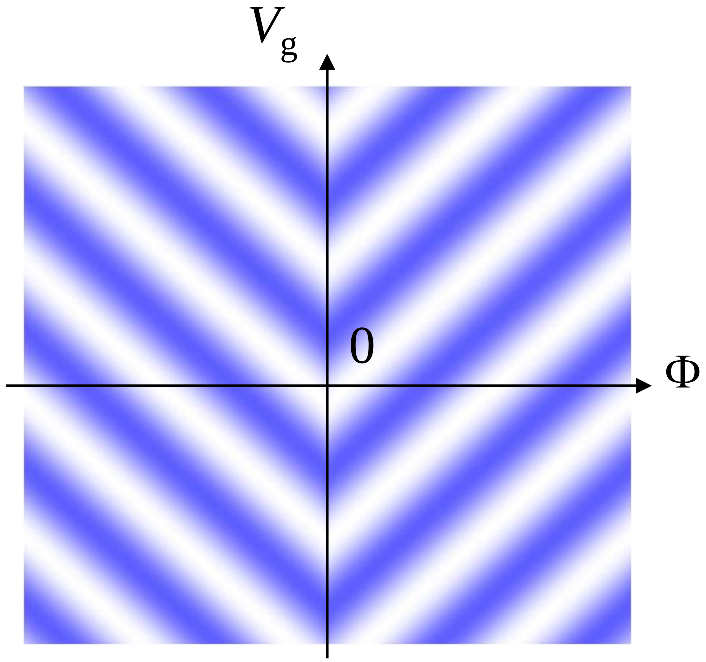
(Received 16 December 2005; revised manuscript received 16 February 2006; published 30 May 2006)

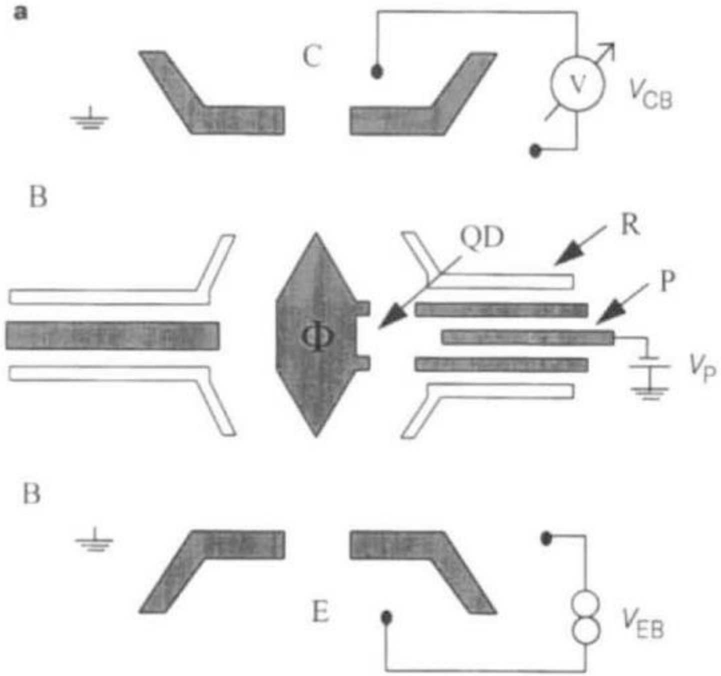


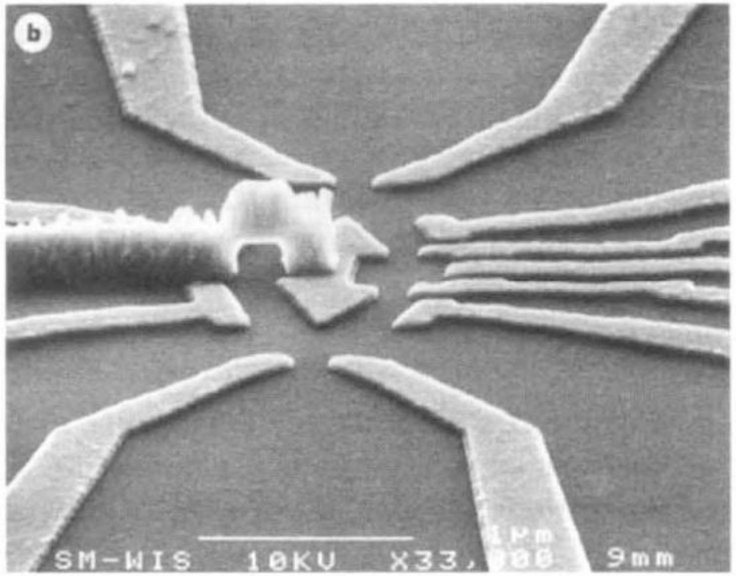


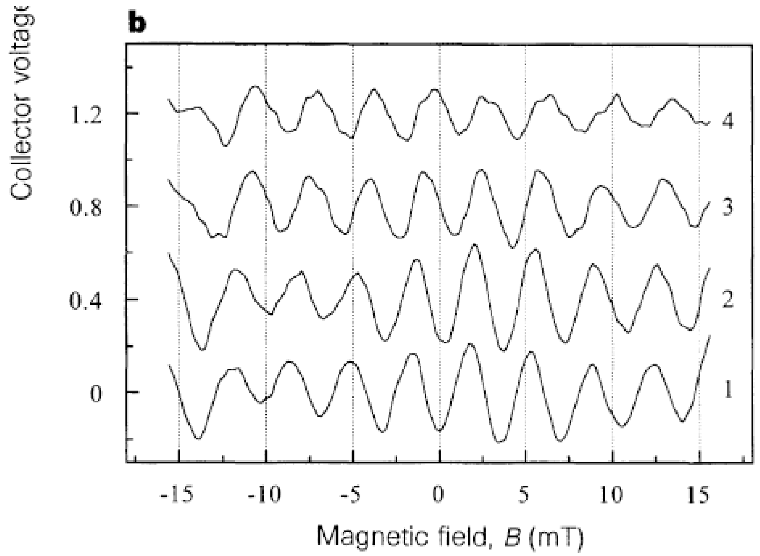


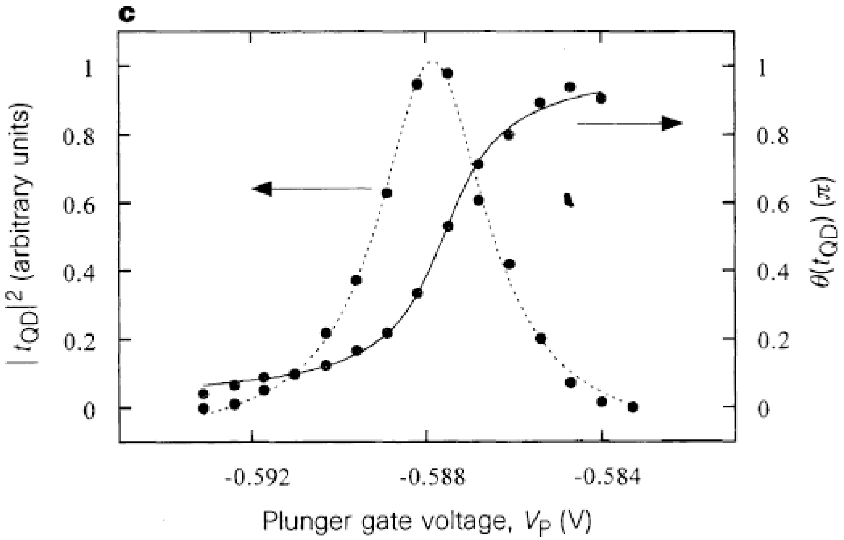
Phase measurement in a double-slit like interferometer

量子干渉回路中の量子ドット

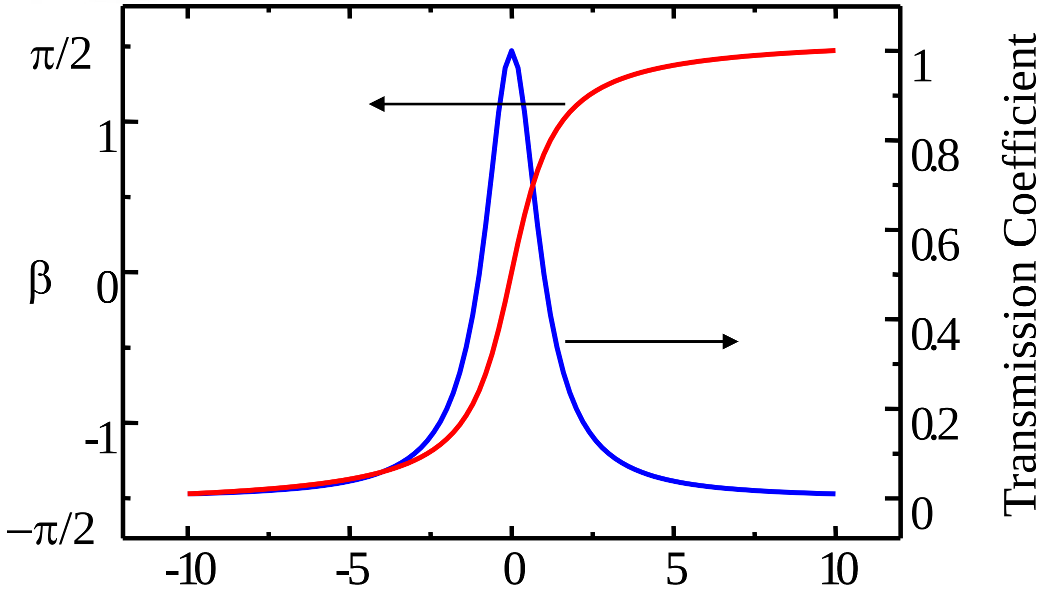






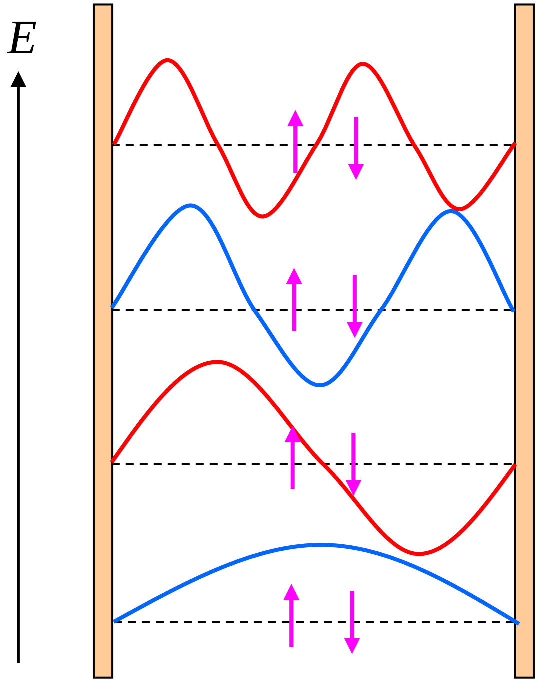


Schuster et al., Nature **385**, 417 (1997).



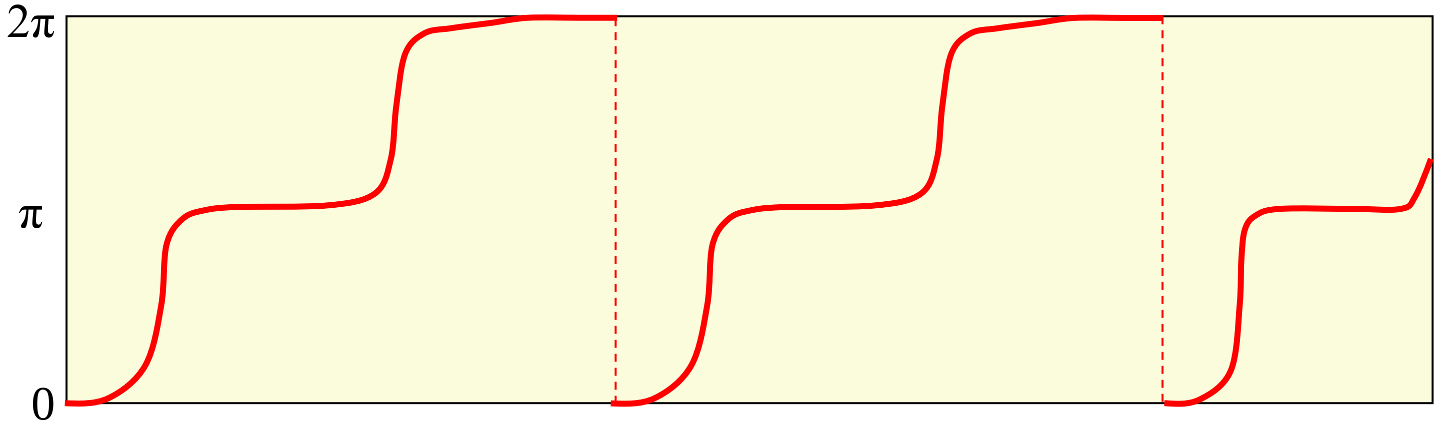
Another puzzle

量子干渉回路中の量子ドット



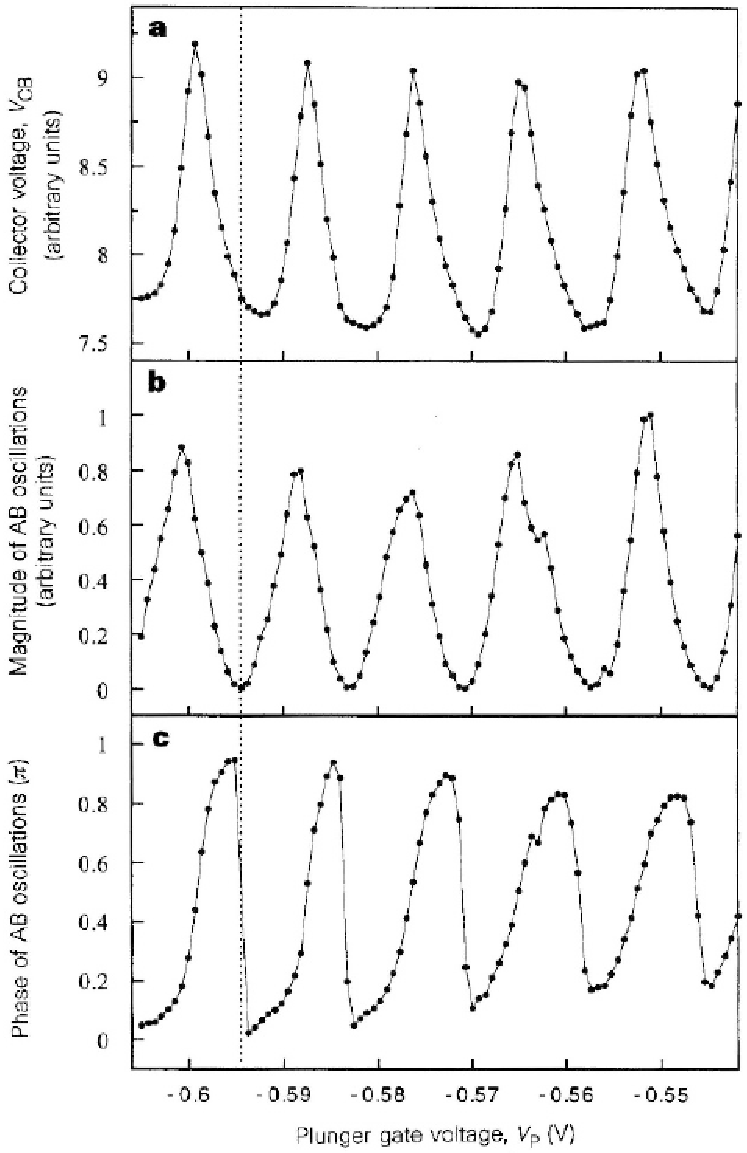
$$t(k) = \frac{1-r}{1-re^{2ikd}}e^{ikd}$$

このようになると期待される

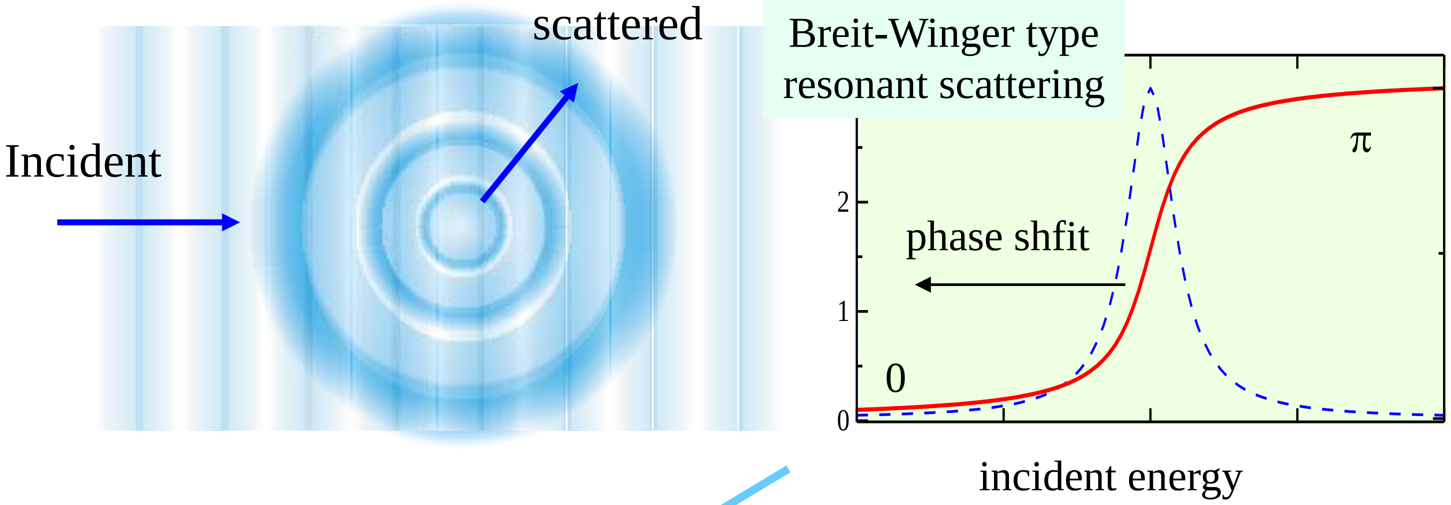


実験結果

Parity固定問題



Fano 効果

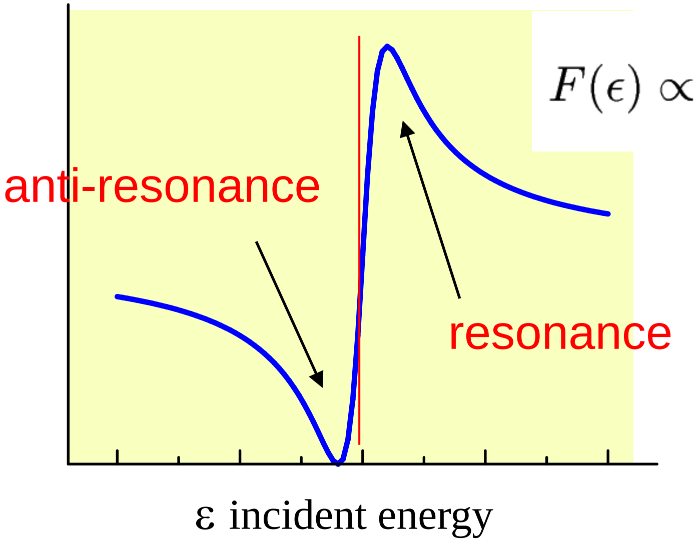


共鳴散乱の際に，散乱波の位相シフトが共鳴点付近でπ変化するため，散乱波と入射波の干渉によって共鳴線形がゆがむ効果

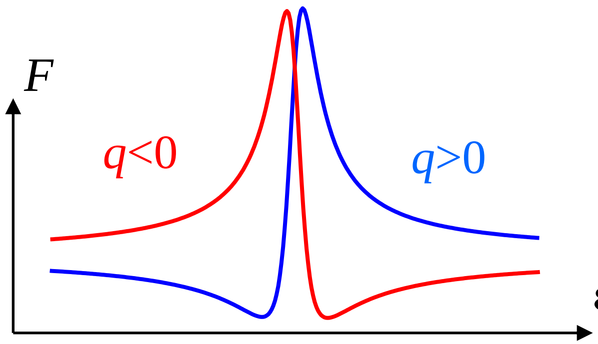
interference

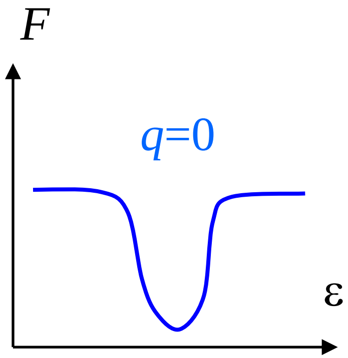
F (ε) : transition probability

Fano parameter

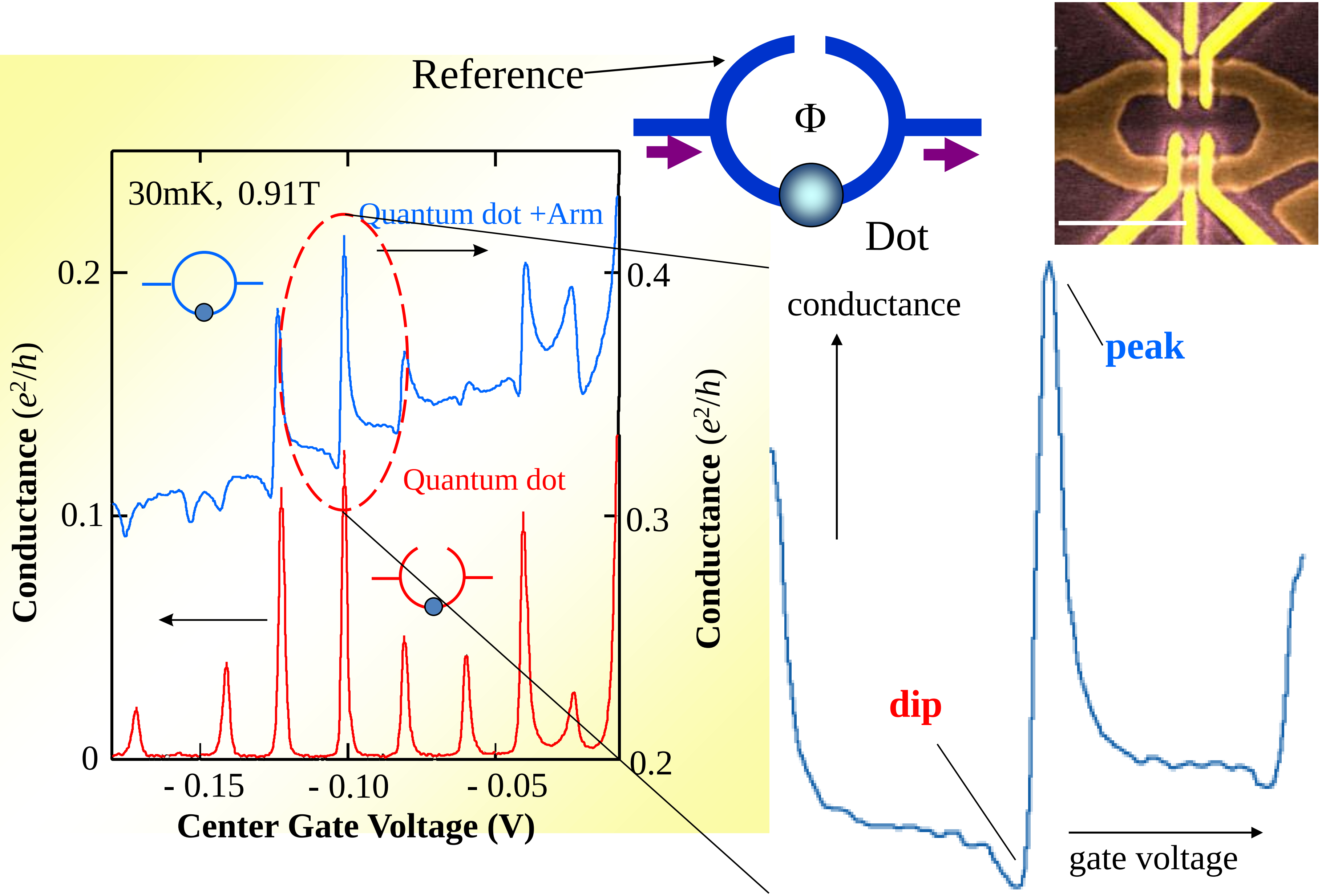


$$F(\epsilon) \propto \frac{(\tilde{\epsilon} + \boxed{q})^2}{(\tilde{\epsilon}^2 + 1)}, \qquad \tilde{\epsilon} = \frac{\epsilon - \epsilon_c}{\Gamma/2}$$



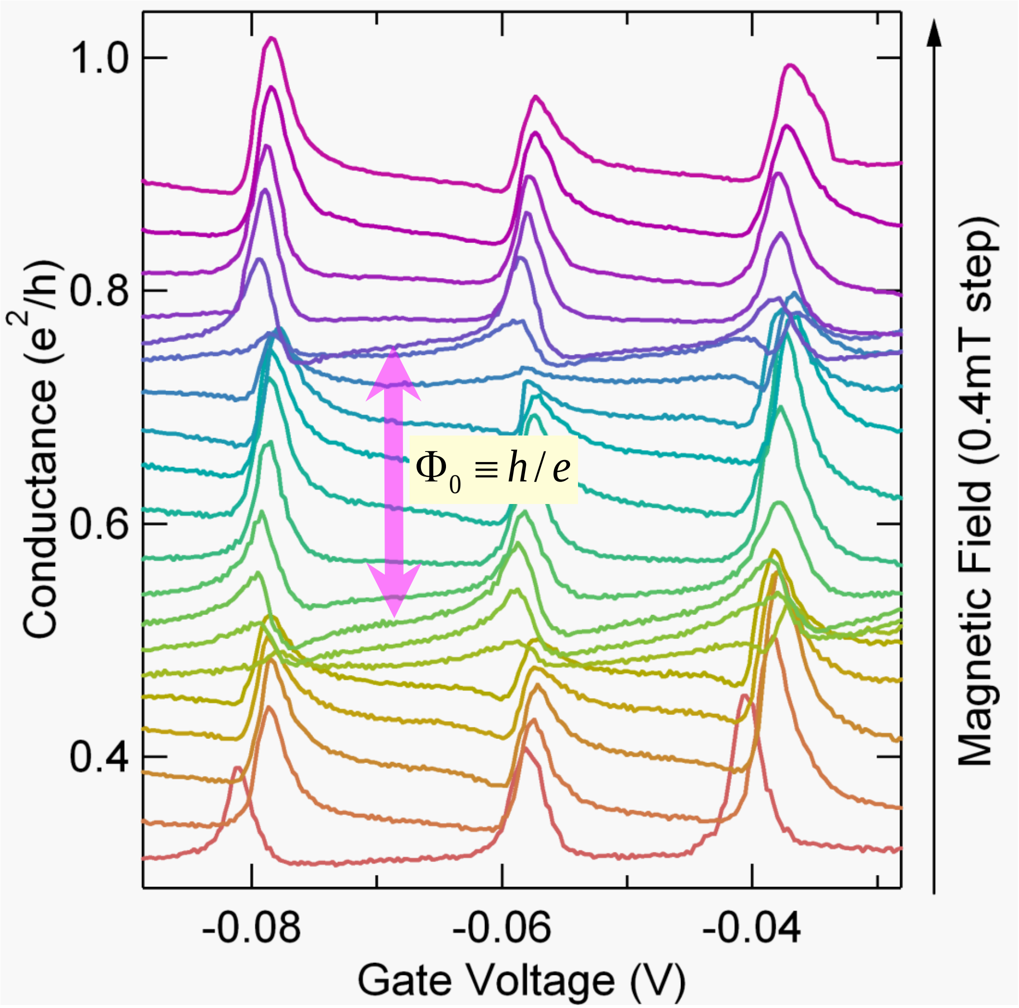


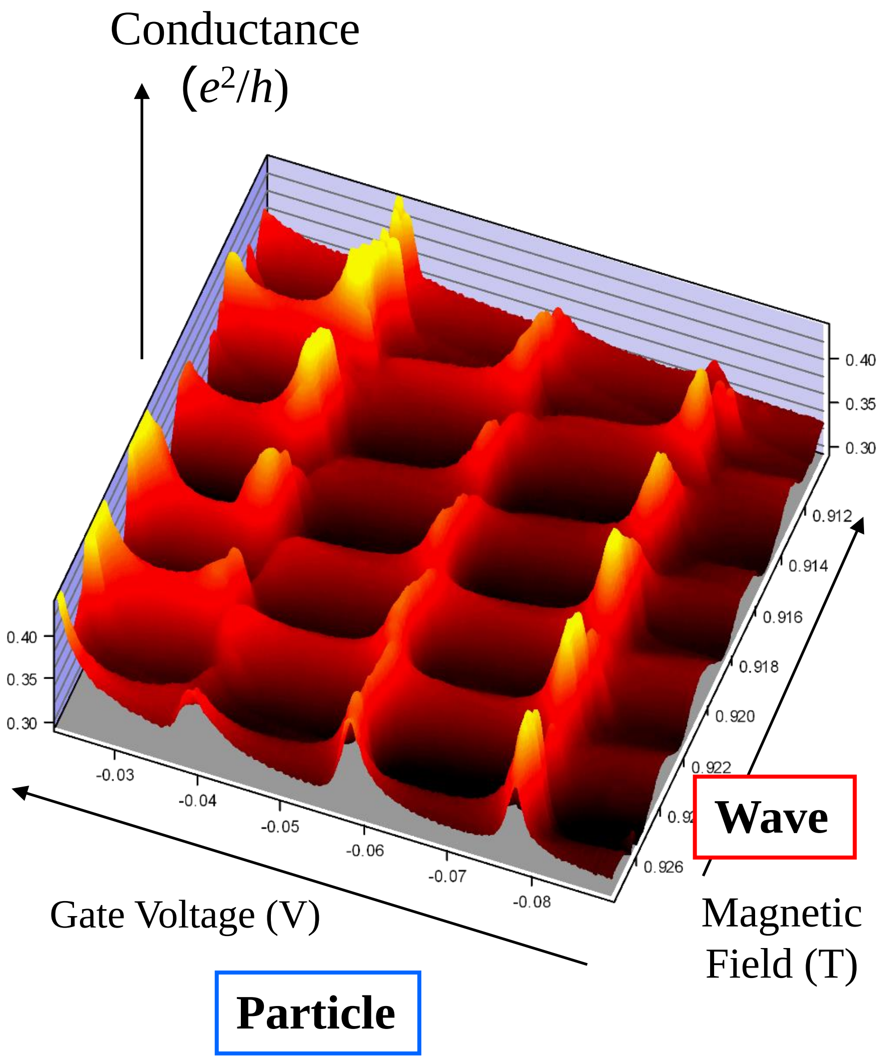
量子ドット+ABリング系のFano効果



量子ドット+ABリング系のFano効果 磁場の効果

量子干渉回路中の量子ドット

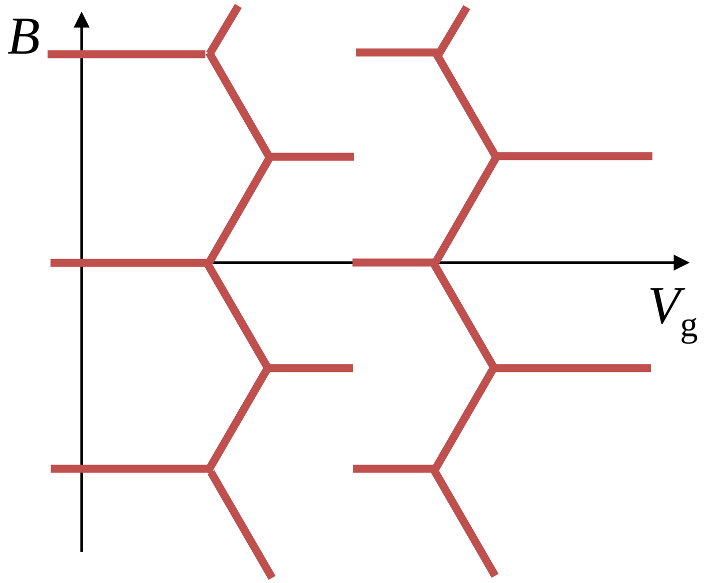


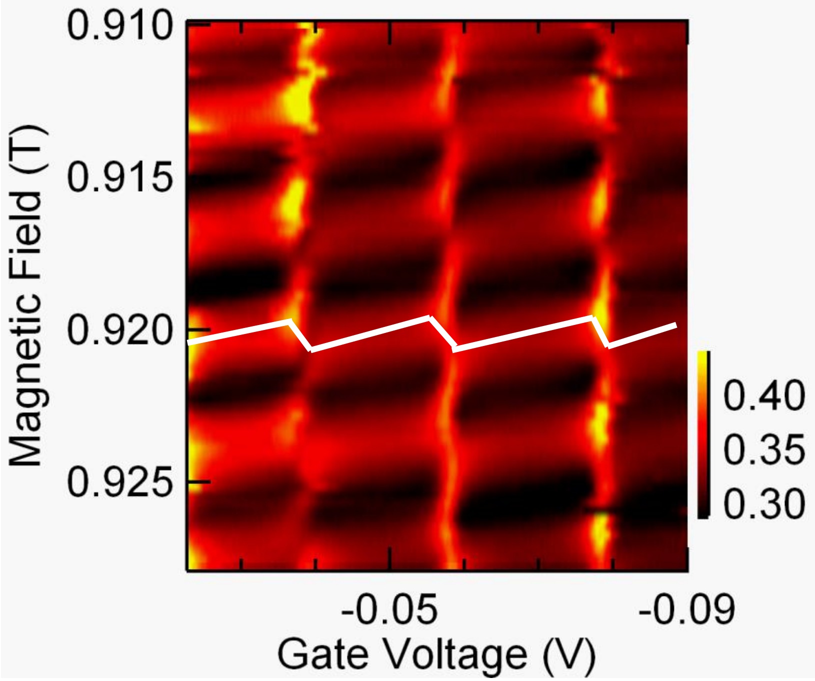


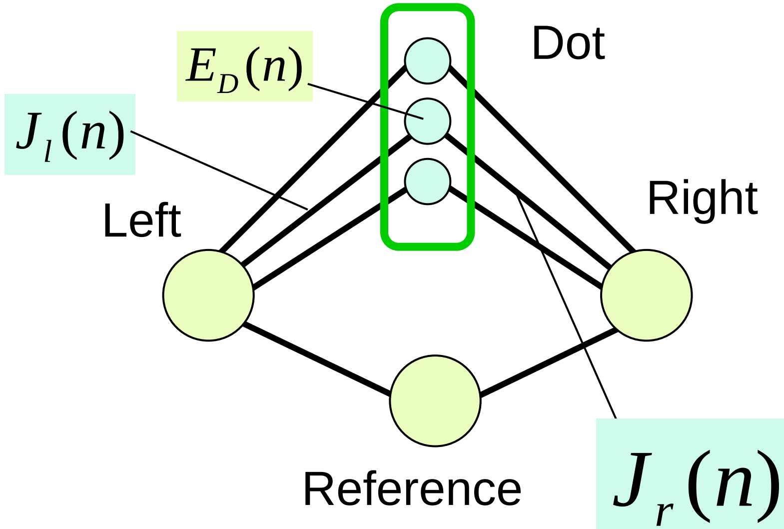
K. Kobayashi et al. PRL **88**, 256806 (`02)

Onsager相反性と多重パスの効果

量子干渉回路中の量子ドット

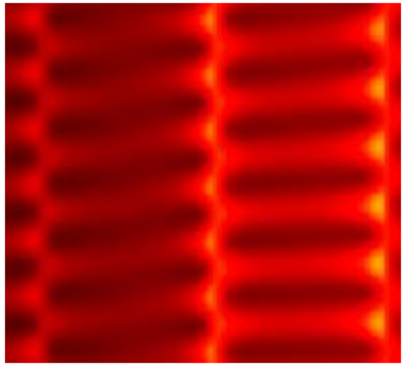






$$J_r(n) = J_r^0(n)e^{i\phi(n)}$$

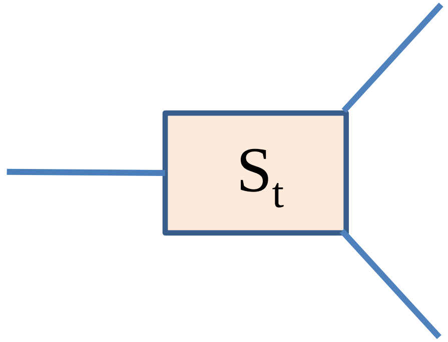
simulation



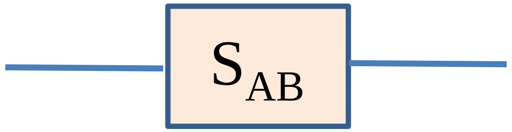
S行列法の量子ドット+ABリング系への応用

量子干渉回路中の量子ドット

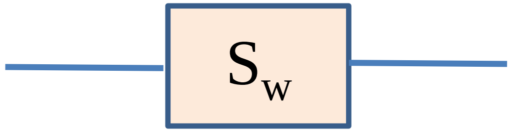
$$S_t = \begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix}$$

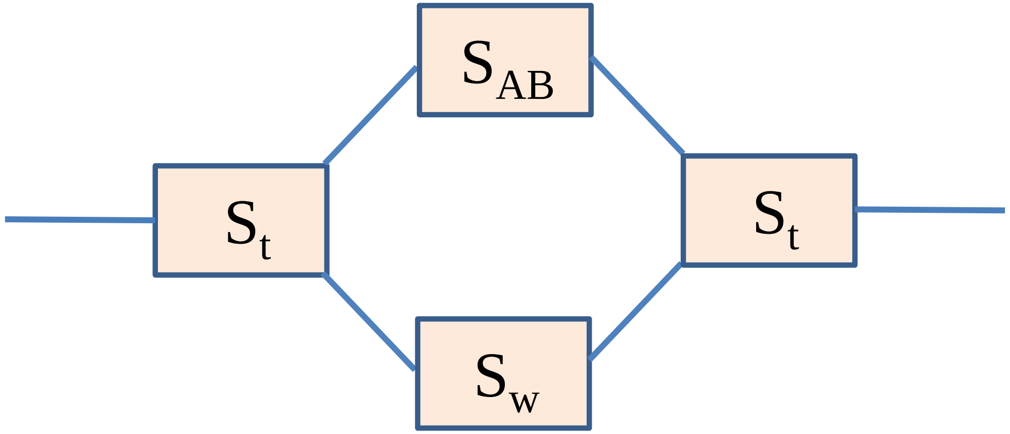


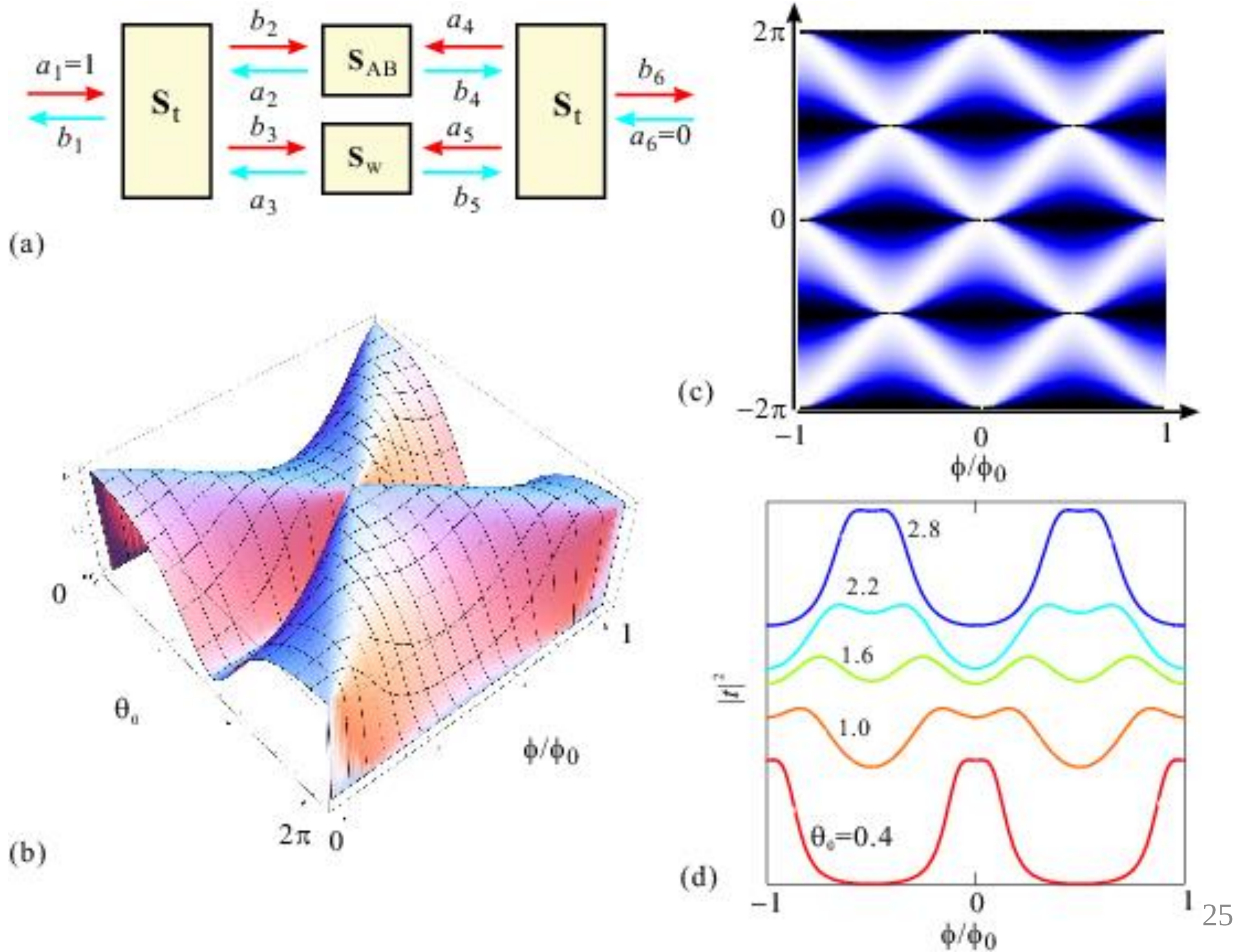
$$S_{\mathrm{AB}} = \begin{pmatrix} 0 & e^{i\theta_{\mathrm{AB}}} \\ e^{-i\theta_{\mathrm{AB}}} & 0 \end{pmatrix}, \qquad \theta \equiv 2\pi\frac{\phi}{\phi_0} = \frac{e}{\hbar}\phi$$



$$S_{\mathrm{w}} = \begin{pmatrix} 0 & e^{i\theta_0} \\ e^{i\theta 0} & 0 \end{pmatrix}$$

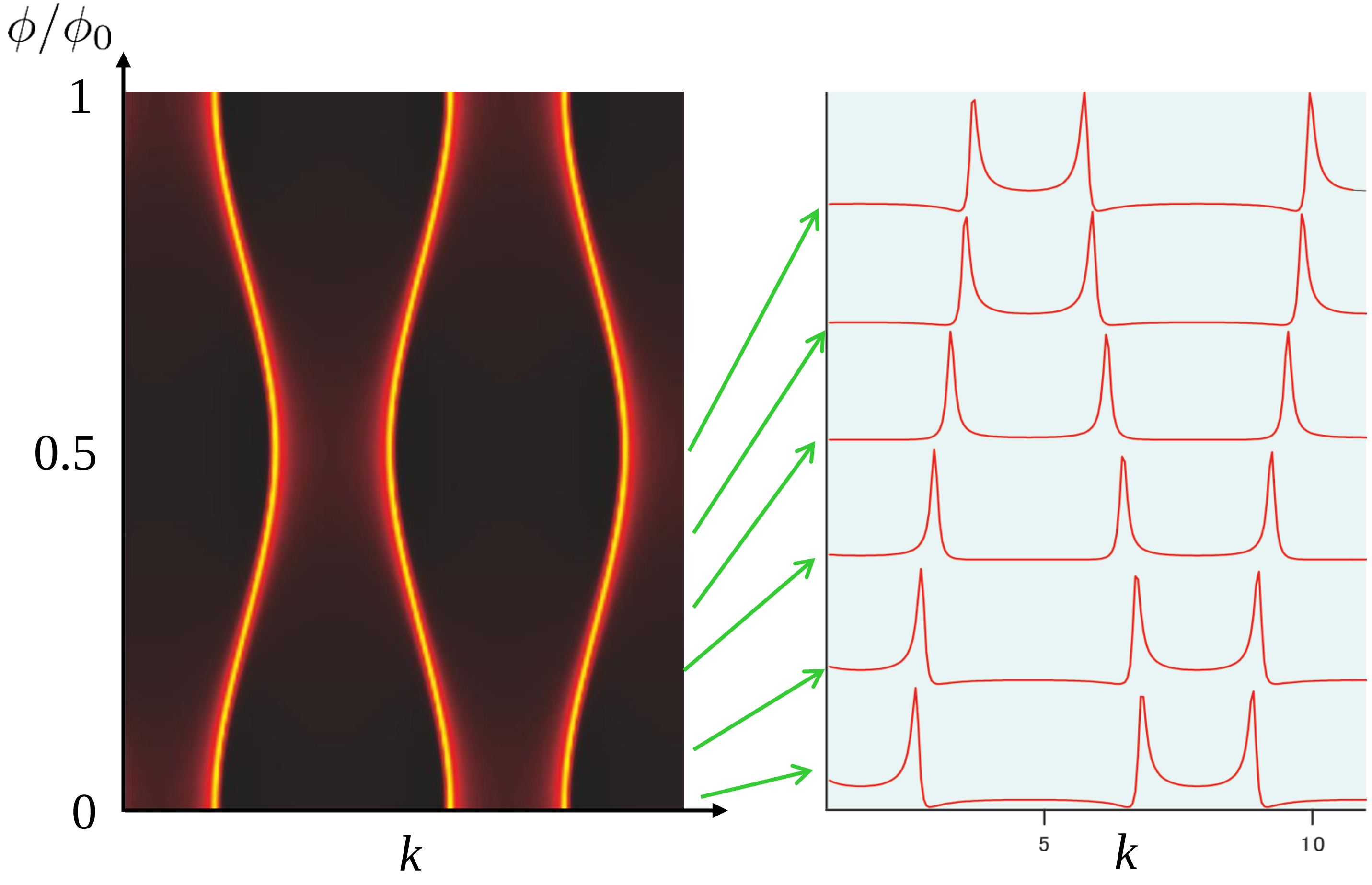






$$t = \frac{4\sin\theta_0}{1 + e^{i\theta_{\mathrm{AB}}}\left(e^{i\theta_{\mathrm{AB}}} + e^{i\theta_0} - 3e^{-i\theta_0}\right)}$$

S 行列法によるファノ線形の計算

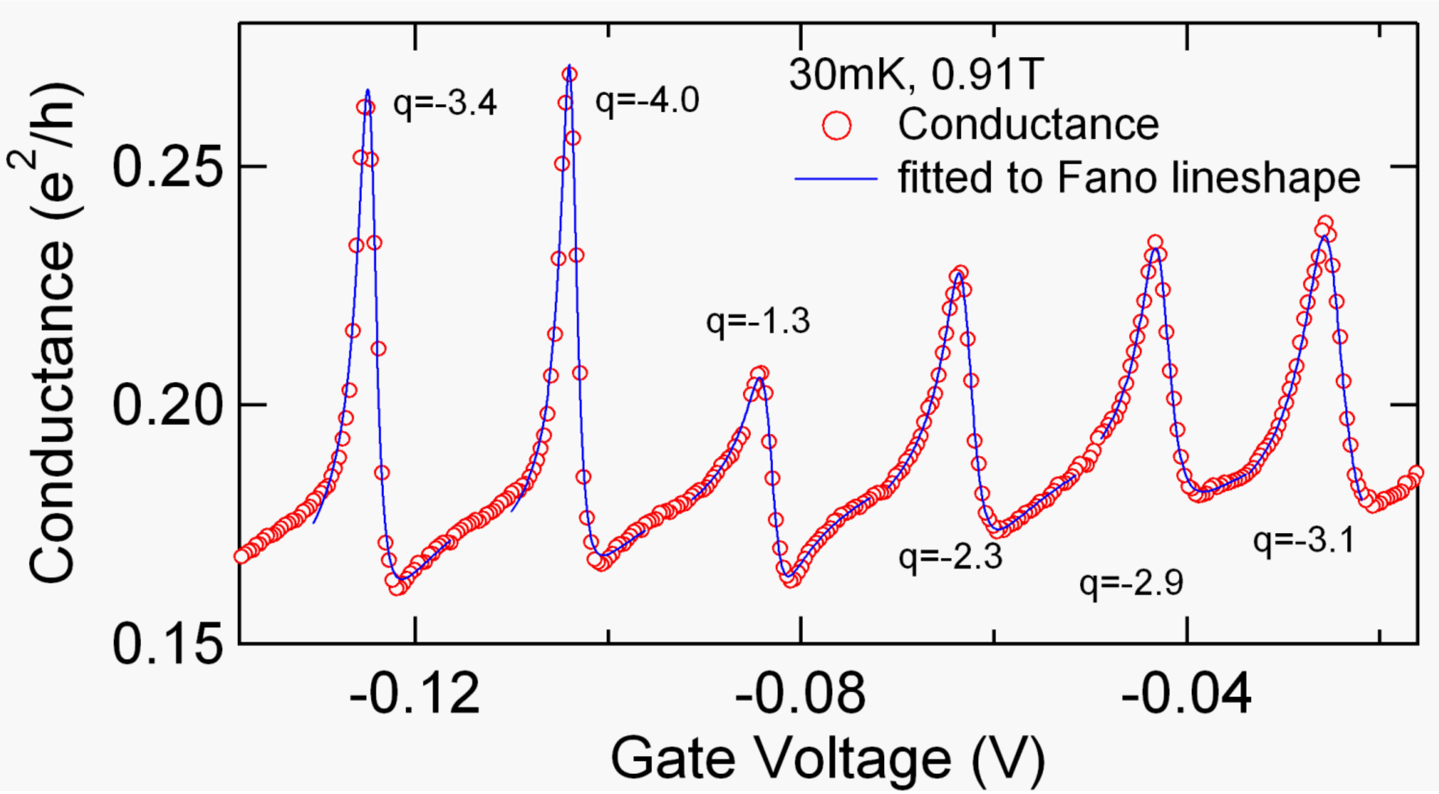


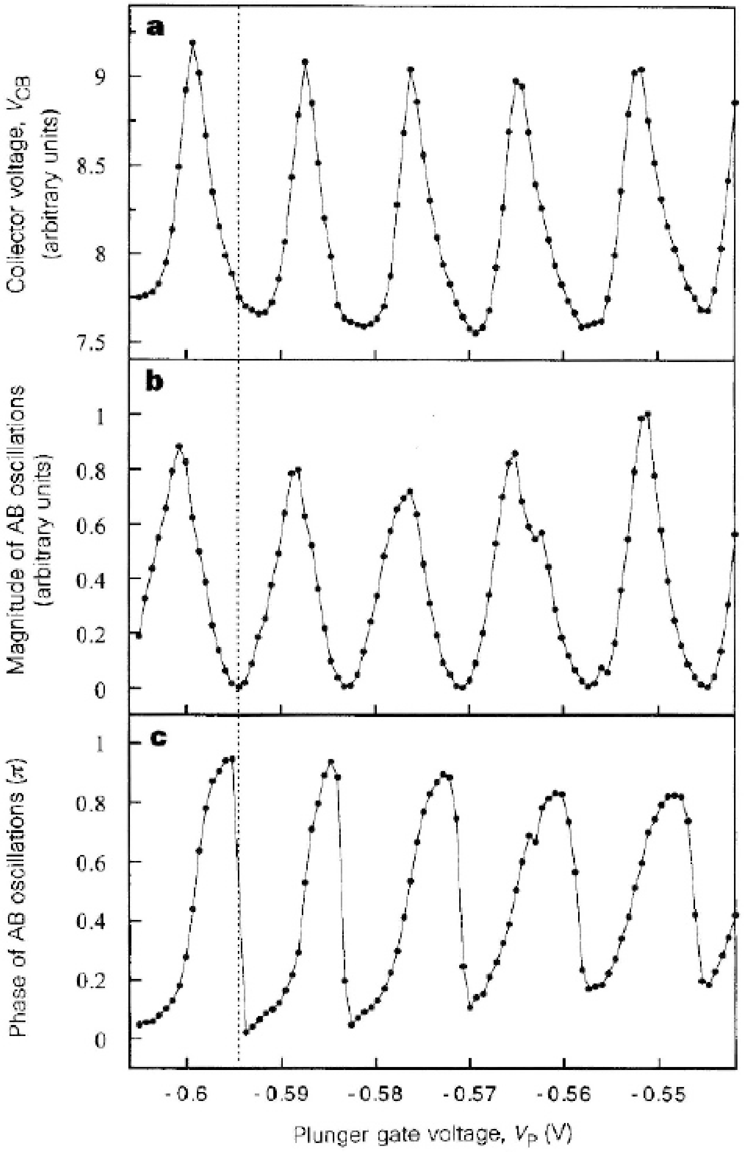
(ゲート電圧に相当)

波動関数のパリティ交代によってファノ歪みの向きが交互に変化する
(ファノ因子の符号が交互に変化)

Parity 固定問題

量子干渉回路中の量子ドット





Parity固定問題を再現

Further puzzle: interference without reference?

量子干渉回路中の量子ドット

PHYSICAL REVIEW B VOLUME 62, NUMBER 3 15 JULY 2000-I

Fano resonances in electronic transport through a single-electron transistor

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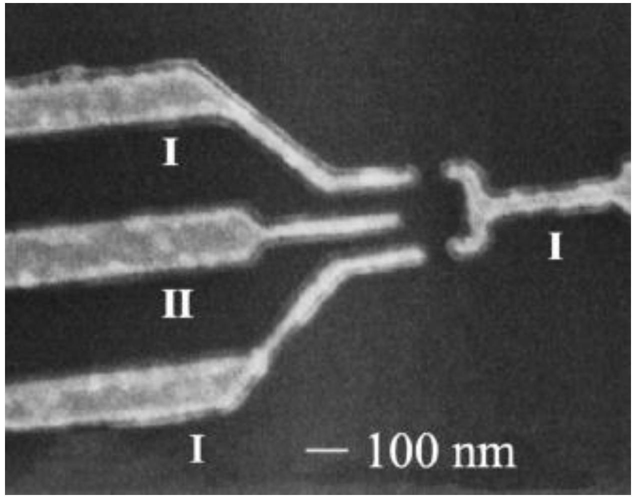
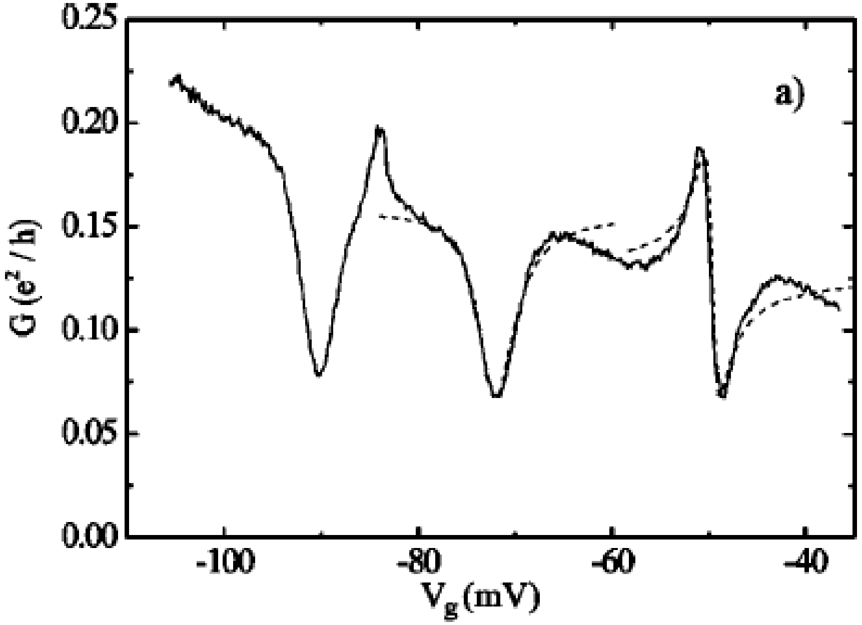
(Received 27 December 1999; revised manuscript received 15 March 2000)





「強結合」状態

量子干渉回路中の量子ドット

VOLUME 85, NUMBER 12 PHYSICAL REVIEW LETTERS 18 SEPTEMBER 2000

Towards an Explanation of the Mesoscopic Double-Slit Experiment: A New Model for Charging of a Quantum Dot

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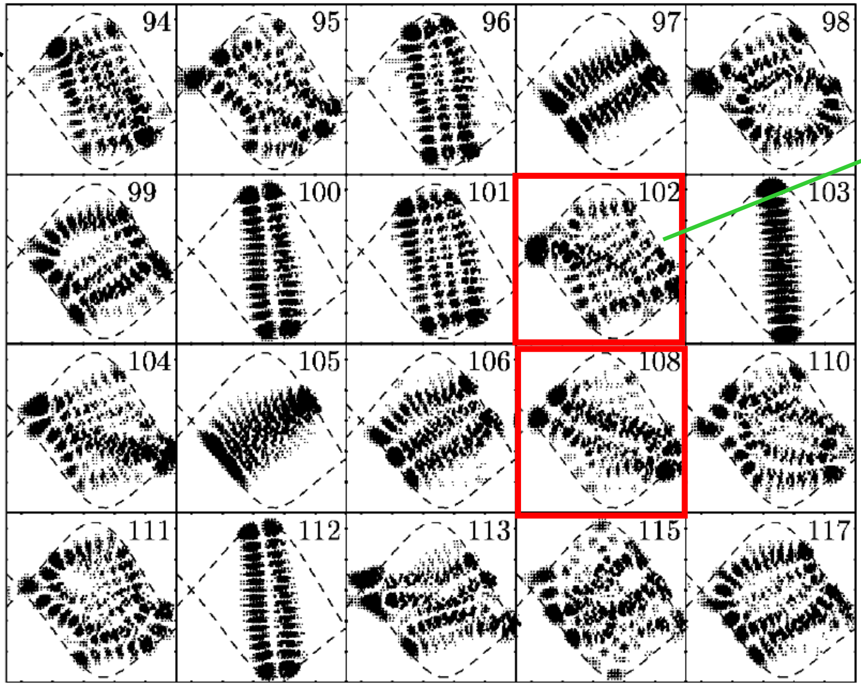
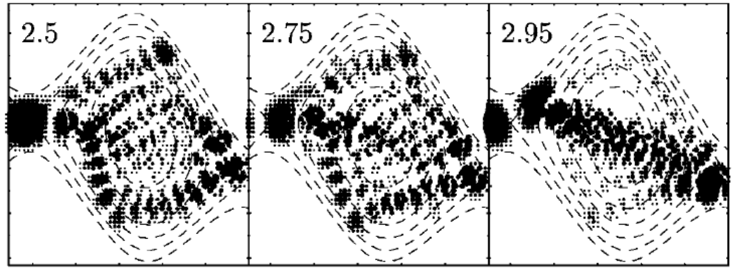
[2]*Weizmann Institute of Science, Rehovot 76100, Israel*
(Received 19 March 1999)

$|\psi|^2$

入り口



Appearance of Scar wavefunction !



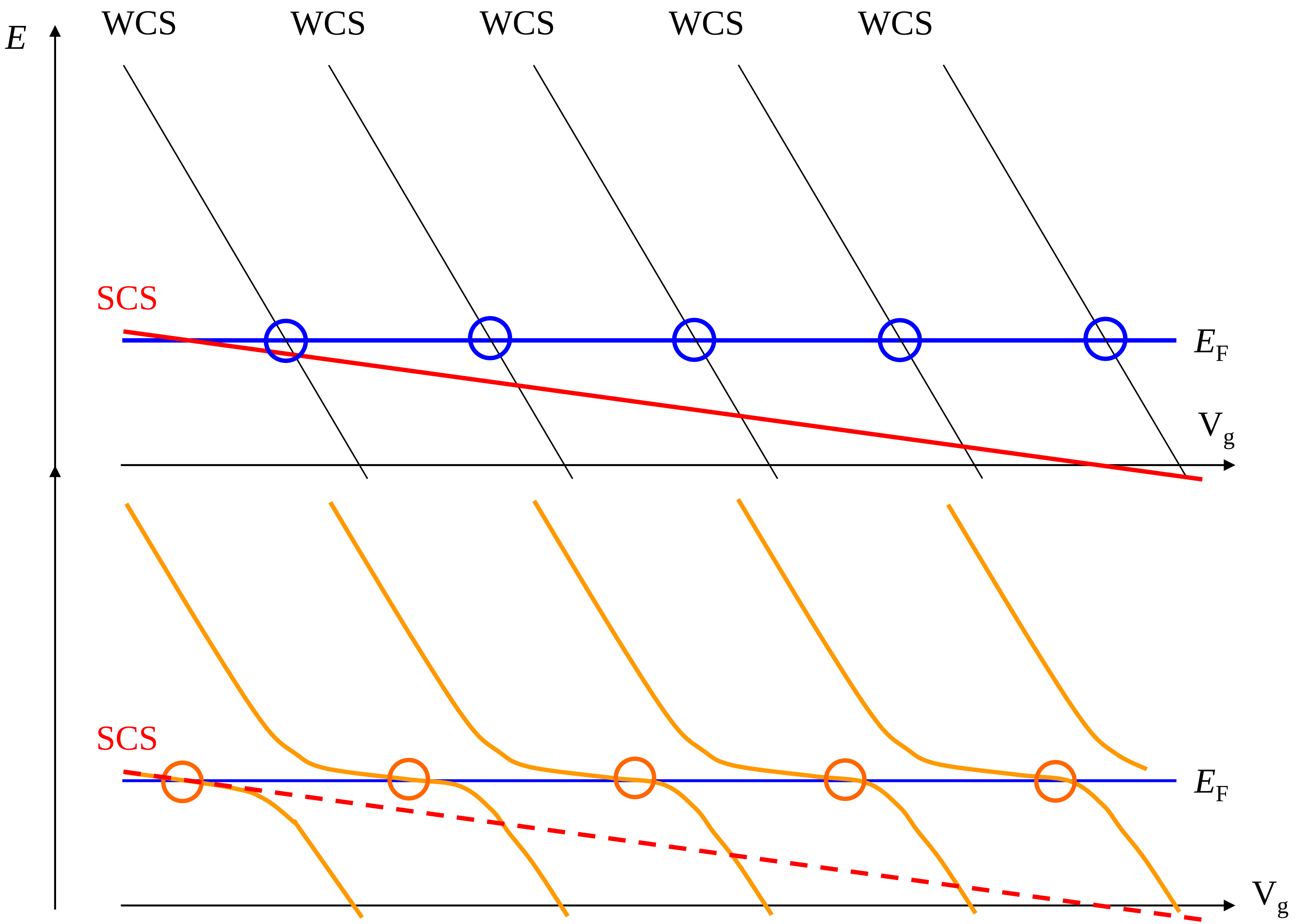
この計算では

強結合状態 $\Gamma/\Delta \sim 6$

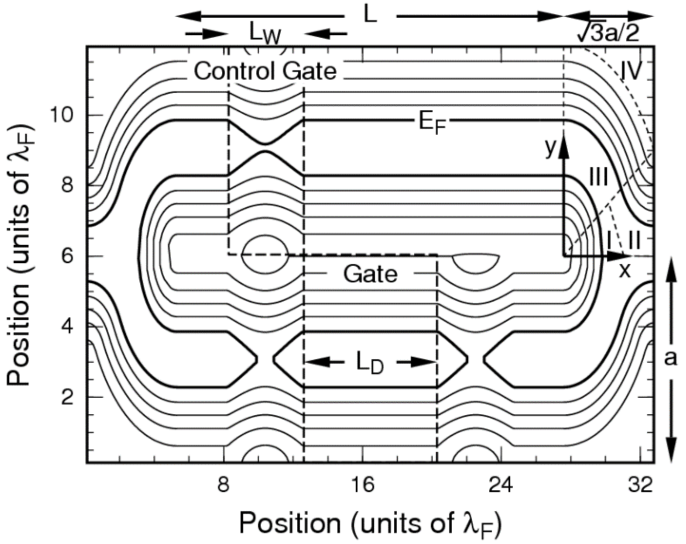
弱結合状態 $\Gamma/\Delta \sim 10^{-5}$

State hoverling

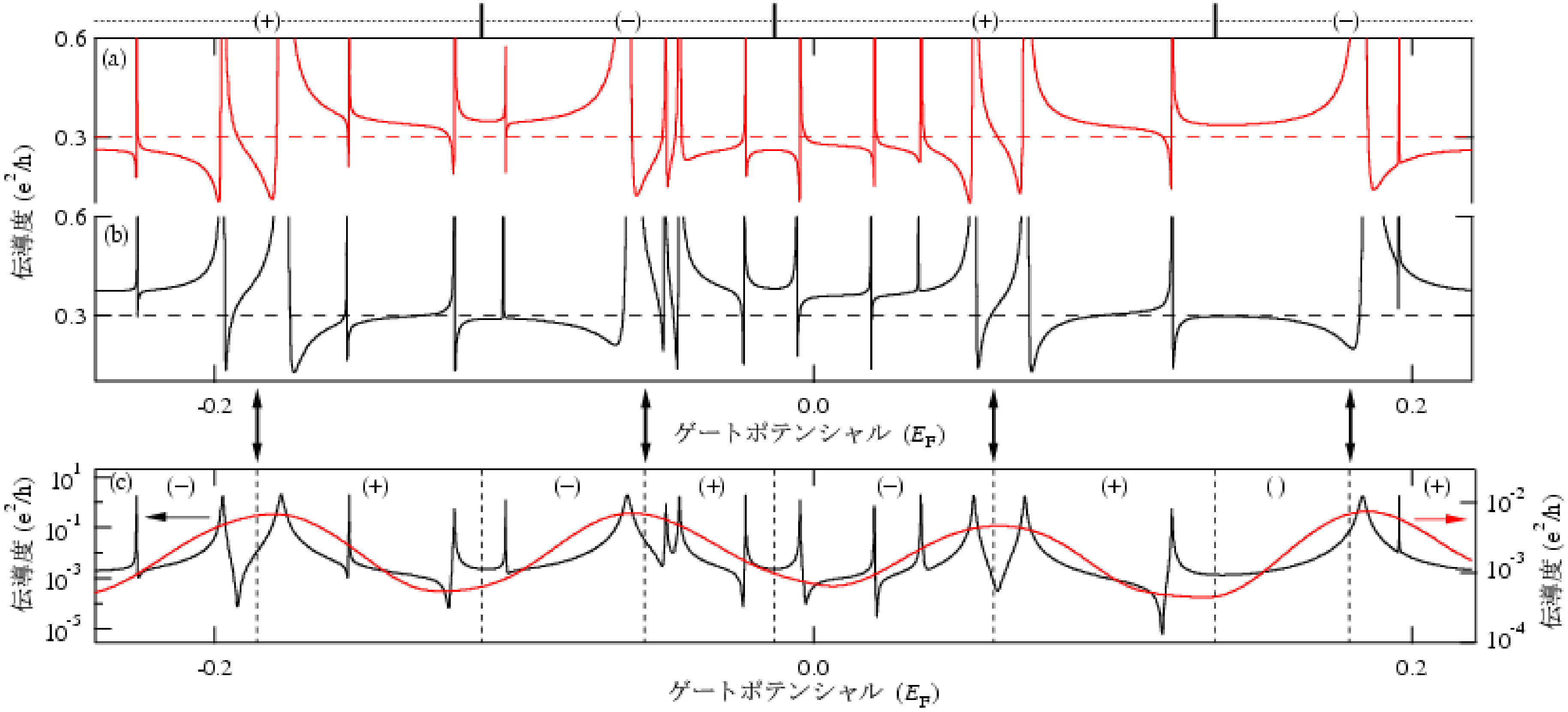
量子干渉回路中の量子ドット



Simulation in a practical model potential



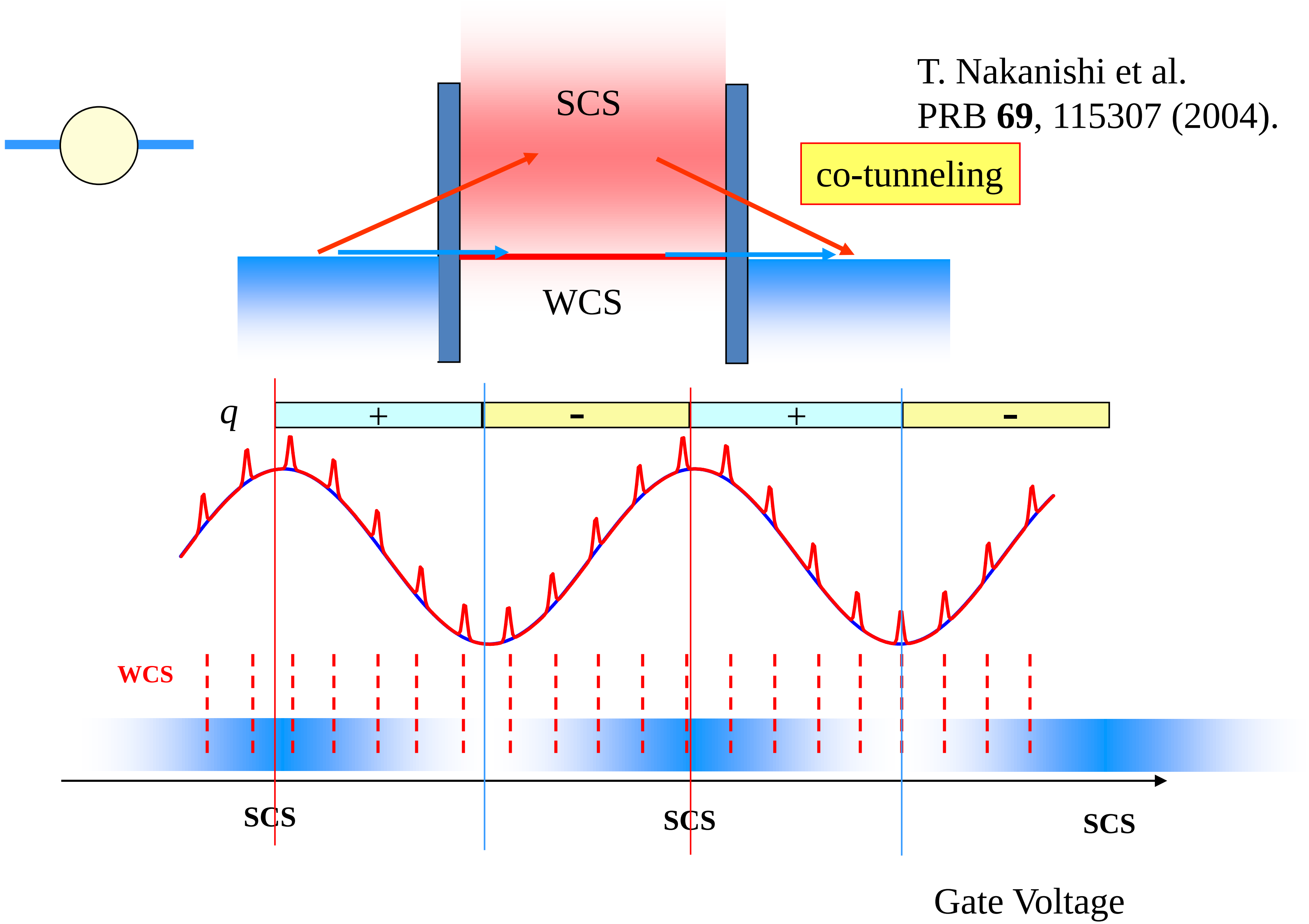
相川，小林，中西，日本物理学会誌 59, 682 (2004)



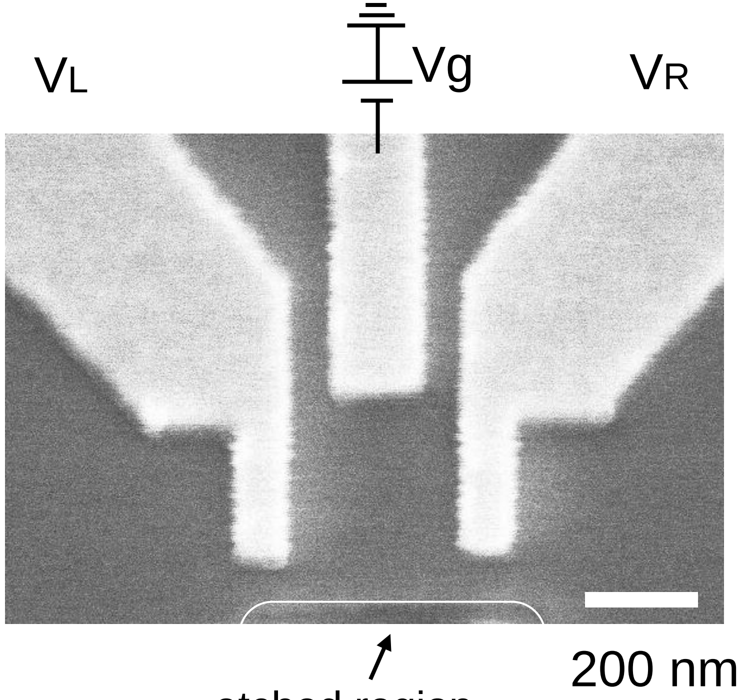
強結合状態を通した伝導によるファノ効果

量子干渉回路中の量子ドット

T. Nakanishi et al.
PRB **69**, 115307 (2004).



実験set up



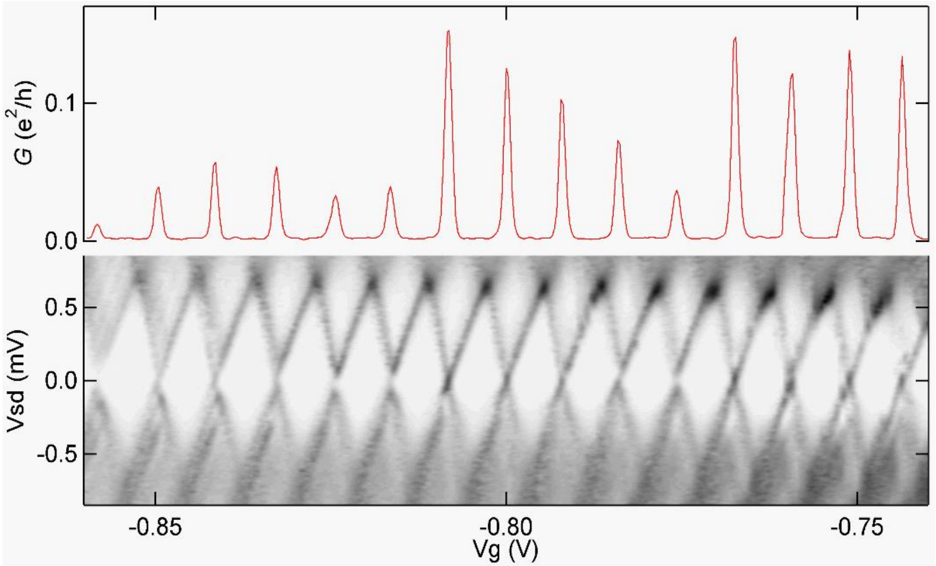
- wet etching
- Au/Ti metallic gate

- Dilution refrigerator (60 mK)
- Lock-in measurement (80 Hz)
- 2-terminal setup

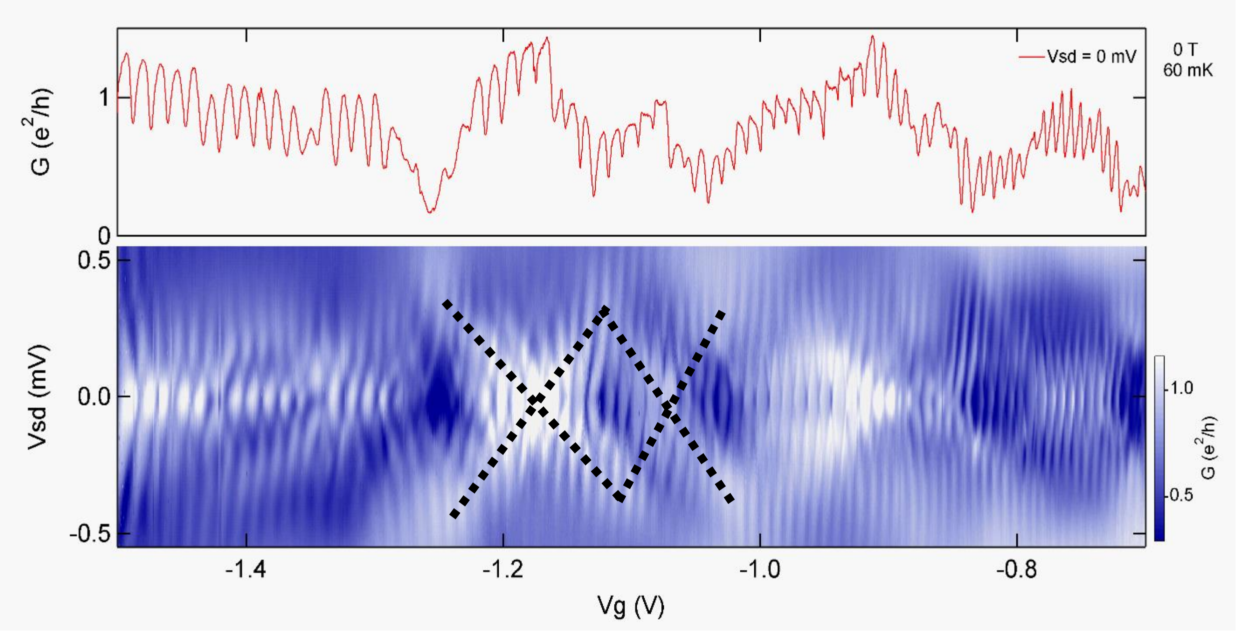
Property of 2DEG at 4.2 K

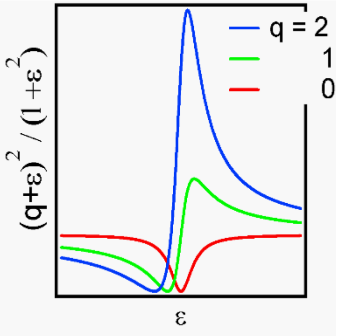
- electron mobility $\mu = 90\ m^2 / Vs$
- carrier density $n = 3.8 \times 10^{15}\ m^{-2}$
- mean free path $l_e \simeq 9\ \mu m$

Coulomb oscillation in CB regime



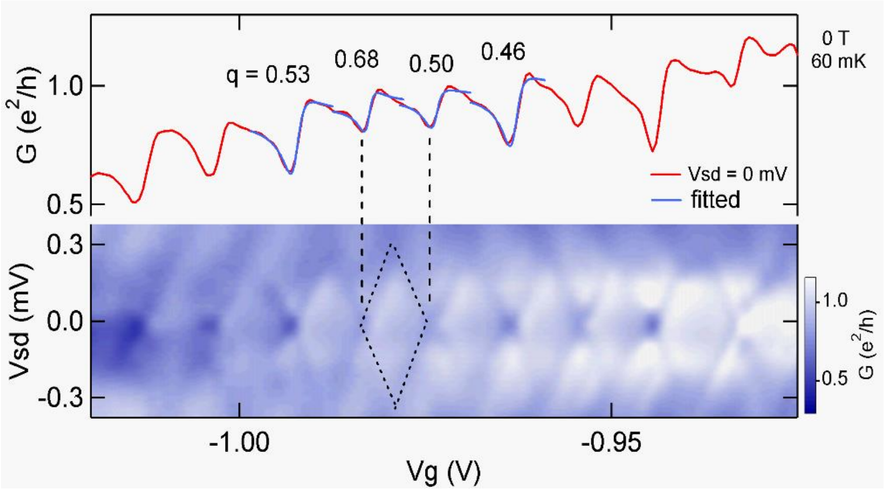
Quantum dot close to open condition





$$G \propto \frac{(\varepsilon+q)^2}{\varepsilon^2+1}$$

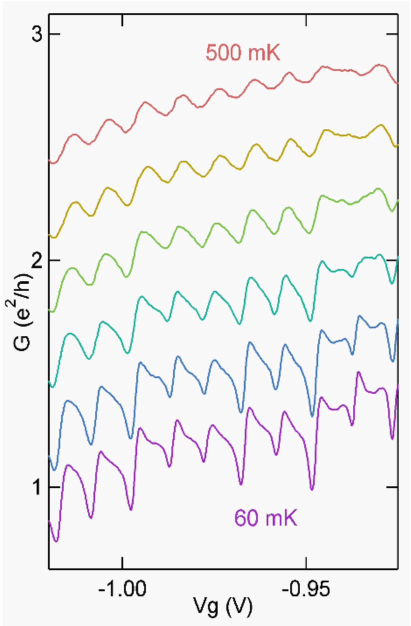
q : asymmetric parameter

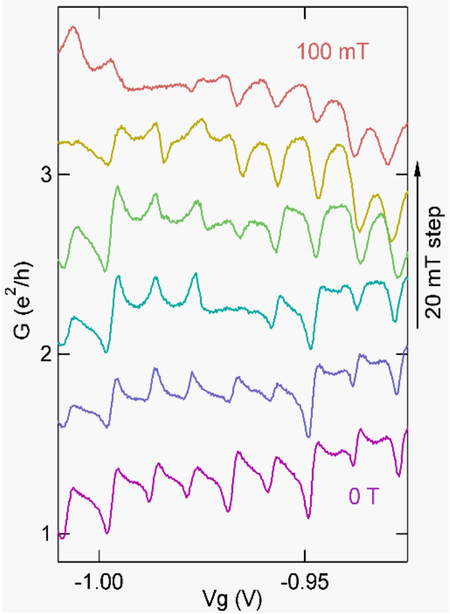


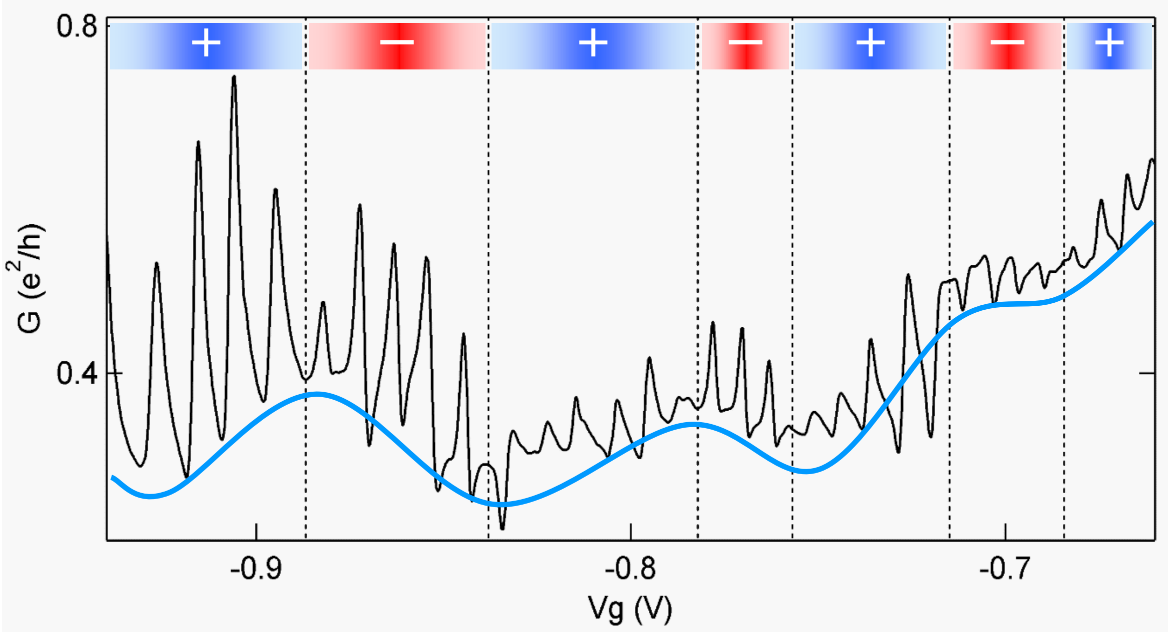
- Slow baseline oscillation
- Fano effect

Quantum dot close to open condition (2)

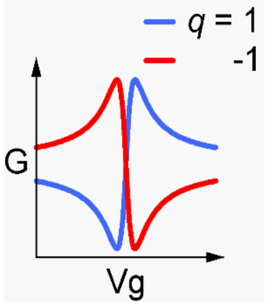
量子干渉回路中の量子ドット







●Reversal of the sign of *q* occurs at peaks / valleys of background oscillation.



Summary

1. Origin of “scar” wavefunction

2. S-matrix connection for makeup of interference circuits

3. Quantum dots in interference circuits

4. Phase rigidity problem

5. Strongly coupled state (originates from scar wavefunction)